[redacted]

[redacted]

THE STEAM ENGINE

CONSIDERED AS A HEAT ENGINE.

[redacted]

THE

STEAM ENGINE

CONSIDERED AS A HEAT ENGINE:

A TREATISE ON THE THEORY OF THE STEAM ENGINE,

ILLUSTRATED BY

DIAGRAMS, TABLES, AND EXAMPLES FROM PRACTICE.

BY

JAMES H. COTTERILL, M.A.,

PROFESSOR OF APPLIED MECHANICS IN THE ROYAL NAVAL COLLEGE.

LONDON:

E. & F. N. SPON, 46, CHARING CROSS.

NEW YORK:

446, BROOME STREET.

1878.

M.Sm.

PREFACE.

The present work is a second edition of some notes on the theory of the steam engine, published in 1871, with the object of facilitating the study of a difficult subject by students of engineering and others, interested in the working of steam engines in practice. The fragmentary character of the original notes, and the progress which the subject has made since the date of publication, have, however, rendered it necessary to re-write and enlarge the book, in doing which I have attempted to fulfil more completely its original object, and at the same time to employ the results of some of the valuable experiments which have been made in the last few years in comparing more precisely, than has hitherto been done, the working of the steam engine as it actually exists with the more or less ideal engine considered—and, in the first instance, necessarily considered—in theoretical investigations.

The object of the book being to study the process of the conversion of heat into work in steam engines, all questions relating to steam boilers, or to the strength and efficiency of the mechanism, or to the adaptation of the engine to the work it has to do, are excluded. Important as these questions are, they are quite independent of the efficiency of the engine, considered as a heat engine, that is to say, a machine for the conversion of heat into work.

In the first chapter the physical properties of steam are explained as determined by Regnault's experiments. I have not here attempted any detailed description of apparatus or of the numerous precautions necessary to secure accuracy: this would be out of place, especially as no real knowledge can be obtained except by a study of the original memoirs: and I have therefore only dwelt on certain points which are necessary to a right understanding of

Regnault's results. Preliminary information is also given on the density of steam.

The second chapter introduces the principle of the mutual equivalence of heat and work, and shows how the principle of work is applied to questions relating to heated bodies by extending the idea of work to changes carried on within the body itself: changes, the nature of which is unknown to us, but the magnitude of which can be inferred by observation of the energy required to produce them.

In the third chapter the theory of the steam engine is commenced by a preliminary investigation in which many of the circumstances, which complicate the question in practice, are left out of account, being reserved for discussion at a later period. I have, however, included the action of the sides of the cylinder by making some simple suppositions as to its effects and magnitude. I have assumed the expansion curve to be a common hyperbola, as experience shows is really approximately the case, and have further supposed that the amount of heat lost, by radiation to external bodies, and by transmission of heat to the exhaust steam, is known. The heat lost in these ways may be described as the heat "unaccounted for by the indicator," and for the sake of a name is called the "exhaust waste." On these suppositions the performance of an expansive engine supplied with dry steam is calculated, and the results as exhibited in the tables (pp. 58, 60, 62) are instructive. In the first place, if the consumption of steam be compared with that found by direct experiment in the best actual engines, it is found to be generally much inferior, showing that in actual engines there must either be a considerable exhaust waste or else a large amount of priming water in the steam supplied by the boiler. In the second place, if the useful work done be expressed in thermal units, it is found to be in all cases less than one-fifth the whole heat expended, and it is consequently necessary to consider the causes of the waste to see if a better result can be obtained by the use of a different kind of engine: it is this second question which is first considered.

A first important step is made by showing that no better result will be obtained by the use of any other machinery than the ordinary piston and cylinder, or by using many cylinders instead of one, provided that the steam be supplied with heat in the same way: the steam possessing during and afte[illegible] a certain definite amount of expansive energy which, [illegible]d, w[illegible]

do a certain definite amount of work: and it follows therefore that to obtain greater efficiency the treatment of the steam must be altered, or steam must be replaced by some other fluid. To see whether anything can be gained in this way it is necessary to consider, instead of steam, some body of more simple constitution, so that we may be able to construct a complete and exact theory of the action of heat upon it. Such a body is suggested by the properties of the permanent gases, all which possess certain characteristics so approximately, as to suggest to us the consideration of an ideal body called a "perfect gas," in which they shall be precisely realized. The fourth chapter is therefore occupied with the discussion of the action of heat on a perfect gas, and the result is utilized to study the operation of the simplest forms of air engines, when it is found that although in circumstances conceivable in practice, the efficiency is more than double that of the steam engine previously considered, yet still fully one-half the heat expended will be wasted. The cause of the waste, however, is remarkably different in the two cases; in the air engine, *all* the heat is in the first instance employed in doing work, and the waste is caused by the necessity which exists in air engines for a compressing apparatus, which requires so much power to work it that the larger part of the energy originally exerted on the working piston is thrown away: while in the steam engine, on the other hand, although some energy is employed in compression, represented by the back pressure on the working piston, yet the greater part of the waste arises from the great amount of internal work which it is necessary to do in order to generate steam, little of which can be utilized by the process of expansion.

Different, however, as the air engine and the steam engine are, it is possible to show that, if certain prescribed conditions are satisfied, the efficiency of the two must be the same, and further that no other engine can have a greater efficiency. This very important investigation occupies the fifth chapter, in which is introduced a second principle governing the operation of heat engines, not less important than the principle of the equivalence of heat and work. This principle is not probably in itself more difficult to understand and apply than the principle of work, but it involves new and abstract conceptions peculiar to the science of thermodynamics, and hence always offers great difficulties at first. Yet it is suggested by a fact not less familiar and certain than those which suggest that heat and work are mutually equivalent, and

that is, that the practical utility of a given quantity of heat depends upon its temperature. If the vast amount of heat discharged into the condenser of a steam engine could be utilized to produce fresh steam in the boiler, it is obvious that the efficiency of steam engines would at once be multiplied many times: now there is no reason why this should not be done, except that the temperature of the condenser is so much lower than that of the boiler. The second principle asserts that by no process of the nature of a perpetual motion is it possible to convert heat of low temperature into heat of high temperature, and it is certain that this principle cannot in any way be evaded so long as we are reduced to operate on matter in large masses; if we were able to operate on the ultimate molecules of which bodies are composed, the case might be different. Reasoning based on this principle leads to the conclusion that the practical utility for mechanical purposes of a given quantity of heat depends upon the limits of temperature within which it can be used, and shows that in designing a heat engine of any kind, the available difference of temperature must be carefully utilized as well as the available expansive energy of the fluid. Hence we derive the conception of a "perfect" engine being an engine in which no waste of heat occurs, except that which is theoretically unavoidable from the very nature of a heat engine. A table of the performance of perfect heat engines is given on p. 115, from which it appears that, with such temperatures as can be made use of in practice, two-thirds the whole heat expended is *necessarily* wasted, and thus the low efficiency found in the preliminary investigation is in great measure accounted for. The statement is still not unfrequently made, that the actual expenditure of heat in steam engines is ten times the theoretical expenditure; but in any legitimate sense of the word "theoretical," it would be much nearer the truth to say three instead of ten.

The remainder of the fifth chapter is occupied with the discussion of a perfect steam engine, and shows how the density of steam is found from Regnault's experiments on its latent heat of evaporation. The density of steam is not quite as certainly known as its other properties, but the extent of the uncertainty is often very much exaggerated by persons who are not fa[illegible] the mechanical theory of heat: the maximum probable [illegible] per cent., and perhaps much less. The sixth chapte[illegible] the generation of steam in a closed boil[illegible]

The expansion curve of steam depe[illegible] ıoı[illegible]

supplied during expansion, so that if an absolutely accurate indicator diagram were given, it would be possible to find the heat supplied at each step of the expansion: while, conversely, if the heat supplied be given, it is possible to find the expansive curve. The seventh chapter is occupied with this question, in dealing with which I have found it advisable to employ graphical methods to represent results, and, in some instances, to replace calculation. This is done by drawing a "curve of internal work," the area of which represents the work done in internal changes, just as the expansion curve represents the work done on external bodies, so that the heat supply is shown by the area intercepted between the two curves. The construction of this curve in special cases, from data furnished by calculation, has for some years formed a regular part of the course of instruction in the Royal Naval College. More recently the methods have been extended, so as to determine by graphic construction the heat-supply during expansion from data furnished by an indicator diagram. The two examples in Figs. 15, 16, were constructed to scale for me by Mr. T. Hearson, R.N., and results obtained closely agreeing with calculation. Besides illustrating methods, these examples call attention to the immense influence of the sides of the cylinder, for, although the application of the method to cases actually occurring requires great caution, it is probable that in the present instance the facts are in the main truly represented by the results of the calculation.

The graphical methods are likewise applicable in the converse case, where the supply of heat is known and the expansion curve required. In particular the adiabatic curve for a mixture of steam and water can be constructed, and the expansion curve for steam expanding in contact with a thin metallic plate which follows its temperature. Fig. 17—also constructed for me by Mr. Hearson —shows the process in detail.

The eighth chapter is a continuation of the fifth, and is, amongst other things, devoted to the discussion of the losses of heat which take place in steam engines in consequence of the non-utilization of the whole available difference of temperature and of the whole available expansive energy of the steam. The difference between the performance of the "perfect" steam engine of Chapter V. and the engine considered in the preliminary investigation of Chapter III. is thus completely accounted for.

The next question considered is the effect of clearance and wire-drawing, a part of the subject which cannot at present be

satisfactorily dealt with. In the first instance it is supposed—as is usual in such investigations—that the state of the steam is not materially changed, on which supposition the effect of clearance and compression is worked out without any difficulty, except that arising from the complexity of the details. In fact, however, the question is far more complex than might be supposed at first sight, for the energy "lost" by clearance and wire-drawing must in the first instance be represented by the kinetic energy of the violent eddying motions produced in the steam, which, if time enough elapse, will be converted into heat by fluid friction, and thus make the steam sensibly drier. Not only so, but the changes which take place within a steam cylinder proceed with such rapidity, that it is not certain that the kinetic energy in question can be completely absorbed in the way supposed till after cut off. A sensible part of the effects commonly attributed to the action of the sides of the cylinder may probably be in reality due to this cause: and till further exact experiments have been made, it is not possible to say how great that part may be.

The tenth chapter is occupied with the vexed question of the nature and magnitude of the action of the sides of the cylinder, a part of the subject upon which it is only within the last two or three years that it has been possible to form a definite opinion. If we suppose any considerable fraction of the steam to be condensed during admission, and re-evaporated during expansion and exhaust, it at once follows that the steam cylinder must be by far the most efficient condensing and evaporating apparatus at present known. A glance at the table on page 248 will show this, the smallest rate of transmission there given being equivalent to a condensation of nearly 100 lbs. of steam per square foot per hour. It is therefore not surprising that many should hesitate to accept a supposition involving such a conclusion, and should ascribe the results to errors in the experiments, or to large amounts of priming water in the steam supplied by the boiler, or to external radiation, or to the effects of clearance and wire-drawing just pointed out —except in the abnormal cases where large quantities of water accumulate in the cylinder. It may now, however, be considered certain that the sides of the cylinder have a powerful influence on the working of all condensing engines, and [illegible] the pr[illegible] carefully examined, it is seen that the ra[illegible] densation and re-evaporation has the effec[illegible] the action beyond what would be possi[illegible]

process of either kind. We may, however, safely adopt M. Hirn's conclusion that the action of the sides can only affect the particles of steam and water in immediate contact with them, the rest of the steam being practically uninfluenced, a supposition which enables the theory of the action of a thin metallic plate attached to the piston to be completely worked out with results closely analogous to known facts relating to the working of steam engines in practice. The nature of the influence of a steam jacket, and the effect of water remaining after exhaust, are also considered in this chapter.

The eleventh chapter, though placed at the end of the book, is intended to be referred to at an early period. I have here discussed some of the experiments made on steam engines, and especially those made in the last few years on American marine engines. The difficulties of experimenting accurately on steam engines are very considerable, and have been successfully overcome in very few instances, among which these experiments occupy a high place. In fact, they have only been rivalled by the experiments carried on nearly at the same time in France. If I have referred less frequently to the French experiments, it is not from underrating their importance, but from the circumstance that they either have been or will be fully discussed by the experimentalists: I have thought therefore, that it would be more useful to discuss an independent set of experiments, which lead to the same results. The chain of evidence, by which the magnitude of the action of the sides of the cylinder is demonstrated, is not, indeed, quite so complete as in the French experiments; but, on the other hand, they were made on five different engines, simple and compound, in a great variety of circumstances. A table of the distribution of heat in steam engines, as determined by these experiments, is given, from which it appears that the best condensing engines met with in ordinary practice—neglecting losses connected with the boiler, utilize about 50 per cent. of the heat supplied to them, the comparison being made with a perfect engine working within the same limits of temperature. In this chapter is also considered the mode of determining experimentally the magnitude of the losses due to various causes, and the table shows the results for the experiments in question.

I had originally proposed to add chapters devoted to certain special points in the working of compound engines and to the modern theories about superheated steam, but the book is already

[redacted]

too long. In the hope of rendering it more generally useful, I have given full numerical details, expressed in such units as may most probably be familiar to persons interested in steam engines, and I have added tables of the properties of saturated steam which will be found greatly to facilitate numerical computations.* With the same object I have avoided the use of the higher mathematics, and employed simple algebra and geometry. In this, as in other branches of applied mechanics, a constant reference to fundamental principles and much patient application to details are necessary, to make theoretical investigations of any use in practice; but advanced mathematical processes are not often required, and sometimes disguise the real difficulties of the subject under consideration. Though I have done my best to remove difficulties, yet I have not attempted to supply the preliminary knowledge necessary to render any treatment whatever of the subject intelligible: some knowledge of heat as a branch of physics, a thorough knowledge of the principle of work as applied to questions in practical mechanics, and some knowledge of the working of a steam engine in practice, and of the use of an indicator, are absolutely indispensable.

I have repeatedly acknowledged my obligations to other writers in the course of my work, but a special tribute is due to the memory of a great thinker whose loss mechanical science has had to lament since the appearance of the first edition of this book. In addition to other--perhaps even more important—claims, RANKINE will always be remembered in the history of science as one of the founders of the mechanical theory of heat, and as the author of the first treatise in which the theory of the steam engine was based on sound principles. I shall be glad if I have succeeded in facilitating the study of his work and—by the employment of fresh experimental evidence—in carrying it on a few steps beyond the point at which he left it.

* In revising the proofs of these tables I have been assisted by Mr. T. Hearson, R.N.

ROYAL NAVAL COLLEGE,
November, 1877

[redacted]

CONTENTS.

CHAPTER I.

PHYSICAL PROPERTIES OF STEAM.

CHAPTER II.

CONVERTIBILITY OF HEAT AND WORK. INTERNAL WORK.

CHAPTER III.

THEORY OF THE STEAM ENGINE. (PRELIMINARY.)

CHAPTER IV.

PHYSICAL PROPERTIES OF THE PERMANENT GASES. AIR ENGINES.

CHAPTER V.

PERFECT HEAT ENGINES.

CHAPTER VI.

GENERATION OF STEAM IN A CLOSED BOILER.

CHAPTER VII.

EXPANSION OF STEAM.

[redacted]

CHAPTER VIII.

ENGINES RECEIVING HEAT AT VARYING TEMPERATURE.

CHAPTER IX.

CLEARANCE AND WIRE-DRAWING.

CHAPTER X.

ACTION OF THE SIDES OF THE CYLINDER AND OF WATER REMAINING AFTER EXHAUST.

CHAPTER XI.

EXPERIMENTS ON STEAM ENGINES.

TABLES.

APPENDIX.

[redacted]

A TREATISE

ON THE

THEORY OF THE STEAM ENGINE.

CHAPTER I.

PHYSICAL PROPERTIES OF STEAM.

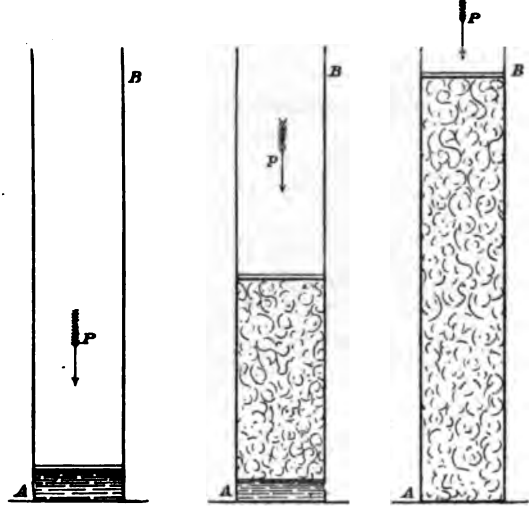
1. Our knowledge of the properties of steam is chiefly derived from experiments made by Regnault at the Paris Observatory, under the authority of the French Government, for the express purpose of ascertaining the numerical data necessary in calculations respecting steam and other heat engines. The experiments relating to steam are published in the twenty-first volume of the Memoirs of the Institute of France (Paris, 1847), to which the reader is referred for all details which it is not absolutely necessary for our purpose to mention: and such a reference is very desirable to obtain an idea of the immense labour and ingenuity employed in rendering the experiments as perfect as possible.

To understand precisely what Regnault ascertained, some preliminary explanations and definitions are requisite, as follows.

In the figure, A B is a cylinder open at the top, and containing a piston: the piston is loaded with weights, which with the atmospheric pressure are equivalent to P lbs. per square foot of the area of the piston, and rests on the surface of a mass of water placed below it; the quantity of water is

immaterial, but, for convenience, will be supposed 1 lb.: the temperature of the water is supposed that of melting ice or 32° on Fahrenheit's scale. If now heat be applied to the water, the temperature rises, becoming greater and greater the more heat is added, the piston remaining stationary

Fig. 1.



(save a very small rise due to the expansion of the water) until a limiting temperature has been attained, the value of which depends on the pressure: the temperature then remains stationary at that limit value, and the formation of steam commences, the piston rising as more and more of the water is evaporated; finally, when sufficient heat has been added to convert the whole of the water into steam, the temperature commences once more to rise, and may be raised to any amount if sufficient heat is added. These successive stages of the process are represented in the figure, which shows the piston in three positions, and for a complete theory of the steam engine a thorough knowledge of all three is indispensable. Such a thorough knowledge has not yet been attained as regards the third stage in which the

temperature of the steam is raised above that at which it was originally formed, but by aid of Regnault's experiments almost all the needful information can be obtained respecting the first two stages, to which we shall confine ourselves in the present chapter.

Connection between Pressure and Temperature.

2. It was stated above that evaporation takes place when the temperature reaches a certain value depending on the pressure; now if the constitution of fluid bodies was completely understood, it might be possible to determine the relation between pressure and temperature by theoretical considerations; at present, however, this cannot be done, and direct experiment is our only resource. Such experiments had been made by various experimentalists; but the results showed considerable discrepancy, and hence Regnault's first object was to set the question at rest by a thorough investigation. His apparatus consisted of a boiler containing, when half full, about thirty-three gallons of water, a condenser of suitable dimensions to condense the steam as fast as it was formed, and an air chamber three times the size of the boiler, provided with force pumps by means of which any desired pressure could be produced at pleasure. Pressures were measured by means of a column of mercury open to the atmosphere, an arrangement admitting of greater accuracy than the manometers of compressed air employed by others, but involving the manipulation of a column of mercury nearly 50 feet high at the greatest pressures experimented on. The air chamber and condenser enabled any desired pressure to be maintained for any length of time.

The principal difficulty to be overcome is, however, in the measurement of temperatures, which requires to be effected, especially at high pressures, with extreme accuracy. Now, a mercurial thermometer is not an exact measure of temperature unless it has been graduated by comparison with some

standard instrument: differences in the quality of the glass and the mode of construction producing sensible differences in the indications, especially at high temperatures, differences which were probably the most important cause of the discrepancy in the results of the earlier experiments on the elastic force of steam. Hence, in the measurement of temperature, Regnault employed as a standard, not a mercurial, but an air thermometer, an instrument which will be referred to further in a subsequent chapter.

Regnault's experiments at pressures above the atmospheric extended to pressures of twenty-eight atmospheres, or more than 400 lbs. per square inch, while those at pressures below the atmosphere made with a different apparatus extended not only to what is commonly called steam, but likewise to the vapour given off by water at all temperatures, even the lowest. His results are given in degrees centigrade, and millimetres of mercury, and are not merely stated in tables, but expressed graphically by means of a curve drawn on copper with extreme care and accuracy. In reducing them to English measures, it has to be remembered that 100° centigrade corresponds to the pressure 760 millimetres of mercury, or 29·922 inches, at the temperature 32°, and at a height of 60 metres in the latitude of Paris above the level of the sea: at any other level and in any other latitude, 100° centigrade will, on account of the variation of the force of gravity, correspond to a column of mercury of somewhat different height. Now, 212° on a British standard thermometer corresponds to 30 inches of mercury at the equator, 29·922 inches in the South of France, or 29·905 inches at London, and hence lies a little below 100° centigrade; so that 1° Fahrenheit is not exactly $\frac{5}{9}$ths of 1° centigrade, but is a little less; and thus the reduction from French to English measures requires considerable calculation. The reduction has been made with great care and accuracy by Professor Dixon in his valuable treatise

on Heat (Dublin 1849), and the table at the end of this book (Table I*a*) has been deduced from that given in his work, omitting the lower part of the table as not required for purposes connected with the theory of the steam engine, and converting inches of mercury into lbs. per square inch. In the reduction, it has been supposed that the British standard thermometer shows 212° at the pressure 14·7 lbs. per square inch, which is exact in the South of France, and near enough at any point of the earth's surface. The ratio which the pressure at any temperature bears to the pressure at 212° is the same everywhere, and it is this which for theoretical purposes it is important to know with accuracy. The table shows the pressure corresponding to each degree Fahrenheit, from 93° to 432° in lbs. per square inch. A supplementary table (Table I*b*), extracted directly from Dixon's work, shows the same pressure from 70° to 150° in inches of mercury. The third column in the principal table shows the rise of pressure consequent on an increment of temperature of 1°.

The general result of the experiments is to show that the pressure increases with the temperature, and that the more rapidly, the greater the pressure. For example, at 212° the pressure is 14·7 lbs. per square inch, and the increase of pressure for a rise of temperature of 1° is about ·29 lb.; at 247° the pressure has increased to 28·34, and the difference for 1° to about ½ lb.; at 300° the pressure reaches 67·22, and the difference 1 lb.; while at 432° the pressure is no less than 350·73, and the difference is 3·64. Thus, at 350 lbs. per square inch, the pressure increases about fourteen times as rapidly as it does at the atmospheric pressure.

Many formulæ have been devised for the purpose of representing algebraically the results of experiments on the elastic force of steam at a given temperature, a brief account of which will be found in the Appendix (Note A); for theoretical purposes a knowledge of these formulæ is some-

times necessary, and we shall occasionally use them hereafter. For practical purposes and numerical calculation the table is more useful; examples will be found attached to the table. It was mentioned above that Regnault constructed a curve graphically representing his results; this may be done by setting off the temperatures as abscissæ, and the corresponding pressures as ordinates; and the reader will find it a useful exercise to construct such a curve for himself, using the numerical values given in the table, so as to familiarize himself with the general character of the relation between pressure and temperature.

Before leaving this part of the subject, some circumstances must be noticed which modify the results now given in certain cases.

If perfectly quiescent water, perfectly free from air or other foreign substance, be heated in a clean glass vessel, the temperature may be raised far above 212° without occasioning ebullition; and when ebullition does take place it is effected, not regularly and quietly, but by fits and starts, producing what is called "bumping." This effect, which is much more manifest when sulphuric acid is used instead of water, is due to molecular cohesion; for particulars, the reader is referred to Professor Clerk Maxwell's treatise on the 'Theory of Heat,' page 269. If such an effect could be produced in the circumstances of an ordinary steam boiler it would be a great source of danger, for suppose a boiler constructed to work at a pressure of 67 lbs. per square inch absolute, say 52 lbs. above the atmosphere, this corresponds to a temperature of about 300° Fahr., if now the temperature could be raised to 320° Fahr. without the corresponding increase of pressure to 75 lbs. above the atmosphere taking place in the usual way, the slightest change of circumstances might produce explosive ebu[illegible]ompanied by a great and sudden increase of pres[illegible]ect requires further investigation, but althoug[illegible] possible that some

of the numerous cases of explosion which have occurred immediately after starting an engine may be accounted for in this way, yet the circumstances under which the effect is produced are rather those which occur in a laboratory than in actual practice.

Secondly, if a salt be dissolved in water the temperature of ebullition is varied; thus ordinary sea water contains one thirty-second part by weight of common salt, the temperature of the steam produced under the atmospheric pressure is not 212°, but 213°·2, and it is said that if more salt be added the boiling point of the brine is raised by 1°·2 for each thirty-second part of salt which is added. The steam in such cases is quite free from any admixture of salt, but probably has the temperature of the boiling brine, and is therefore to some degree "superheated," a term the meaning of which will be explained presently. (See Appendix, Note A.)

Subject to these observations, the elastic force of steam is always connected with its temperature, as shown by the table, so long as it remains in contact with water, no matter how the steam has been produced; thus if, instead of supposing the water confined in a cylinder provided with a piston which rises as the steam is formed, we suppose the steam to be produced in a closed steam boiler, then the temperature and pressure will keep rising as more and more heat is added, instead of remaining stationary; but the relation between pressure and temperature remains precisely the same so long as any water is left.

Specific Heat of Water.

3. Returning to our cylinder and piston, and considering the first stage of the process before the production of steam commences, we have now determined the limit temperature (t°) in terms of the pressure on the loaded piston, and we next consider, in order to complete our knowledge of the first stage, the quantity of heat which must be added to the water

in order to produce the change in question, or, in other words, to raise its temperature from 32° to $t°$. Now quantities of heat are measured in thermal units—that is, by the quantity of heat which is necessary to raise a lb. of water through 1° at its temperature of maximum density, or from 39° to 40° Fahr.; if then the same quantity of heat were required to raise a lb. of water through 1° at any other part of the scale, say from 212° to 213°, then the amount of heat required would be $t - 32$ simply, and this is what is usually assumed by practical writers on the subject of the steam engine.

Regnault, however, has shown, by a series of direct experiments, that the quantity of heat in question is always greater than $t - 32$, the difference becoming greater and greater as the temperature $t°$ is higher. His results changed into English measures are given in Table II*a* at the end of the book, for every 27° from 77° to 401°, in which the first column gives the temperature $t°$, and the second $t - 32$, while the third gives the quantity of heat in question as determined by Regnault's experiments. A knowledge of this quantity of heat is continually required in the course of our work, and hence a special symbol (h) is used for it, and it must be understood that h in this work always means the quantity of heat necessary to raise a lb. of water from 32° to $t°$, and is never used for any other purpose.

As just stated, the value of h is given by the table at the temperatures indicated in the first column, at any intermediate temperature interpolation is necessary, for which purpose the mean difference for 1° is given in the fourth column of the table. These numbers represent the mean quantities of heat required to produce a rise of temperature of 1°, or, in other words, the mean specific heat of water between the temperatures indicated, whence it will be seen that the specific heat of w[illegible] very considerably at high temperatures, b[illegible] at the tempera-

ture 375°, corresponding to a pressure of 185 lbs. per square inch. Examples of the process of interpolation will be found at the end of the tables.

Regnault's results require certain corrections in order to make them precisely applicable to our purpose, that is, to make them represent with absolute accuracy the heat expended in the first stage of the process we are considering. These corrections are, however, undoubtedly much less than the deviation of the specific heat of water from unity, and will not be considered here; for further information the reader is referred to the Appendix (Note B). Unless we have to do with steam of very high pressure the difference between h and $t - 32$ may frequently be safely disregarded, but much depends upon the particular question considered.

The quantity of heat requisite to raise a lb. of water through 1° has of late not unfrequently been called a "pound degree" by writers on the steam engine. If this expression be adopted, it must be remembered that it expresses a different quantity of heat for each particular temperature, so that for steam of 185 lbs. pressure it is about 4 per cent. greater than at low temperatures. Hence to make the "pound degree" a definite unit of measurement, the temperature employed as a standard must be indicated. There seems no advantage, however, in abandoning the well-understood term "thermal unit," used in measuring quantities of heat.

Total and Latent Heat of Evaporation.

4. We next go on to consider the second stage of the process, that is, the evaporation of the water, which takes place gradually as heat is added, the piston steadily rising, and the cylinder remaining at the constant temperature already investigated.

Let us first suppose that so much heat has been added, that every drop of the water is evaporated, and the cylinder

[redacted]

contains nothing but steam of the same constant pressure under which it was originally formed, then the first question to be considered is the quantity of heat required to evaporate the water as described. This quantity of heat is called the *latent heat of evaporation* of water, a term the origin of which will be explained hereafter. If further we consider the quantity of heat expended in the first and second stages together, that quantity of heat is called the *total heat of evaporation* of water. Thus the *total heat of evaporation of water is the quantity of heat requisite to raise a pound of water from* 32° *to a particular temperature, and evaporate it at that temperature,* while the *latent heat of evaporation of water is the quantity of heat requisite to evaporate a pound of water at a given temperature.* The first of these quantities will in this work invariably be denoted by H, and the second by L, symbols which will be used for no other purpose.

The values of H and L in the present state of our knowledge can only be determined by experiment, and as the results obtained by the earlier investigations of Watt and Southern were discrepant, a second principal object of Regnault's experiments was to set this question also at rest by a thorough investigation.

Regnault's apparatus consisted, as before, of a boiler condenser and air chamber, arranged so that a perfectly steady evaporation could be maintained for any length of time required under any desired pressure, the steam being conducted to the condenser, and condensed as fast as it was formed in the boiler, quite independently of the calorimeters, mentioned farther on, used to measure the heat given out in condensation. The steam pipe conducting the steam from the boiler to the condenser and calorimeters passed into the boiler below the water line, and after several convolutions terminated in the centre of the steam space, which was large; while outside the boiler the pipe was thoroughly

steam jacketed and clothed, and hence thoroughly dry steam was secured without any possibility of superheating.

Suitable steam of a given temperature being thus obtained, is conducted into a calorimeter consisting of a pair of copper globes surrounded by cold water. Condensation of the steam in the globes immediately takes place, the heat given out being abstracted by the cold water, the rise of temperature of which furnishes a measure of the quantity of heat, while the condensed water issuing from the globes gives the weight of steam condensed; hence the heat given out by each pound of condensing steam is fully determined.

Great care is necessary in conducting experiments of this kind to secure accuracy, a special difficulty being to find the quantity of heat lost by radiation from the calorimeter while the experiment is proceeding. For details I must refer to the original Memoirs. It is sufficient to say that all difficulties were overcome by Regnault, whose results are universally accepted as being as perfect as the nature of the case permits.

If now we carefully consider the way in which Regnault's experiments were made, it will be seen that the result he obtained is no other than the total heat of evaporation as defined above; for if, after the water has been completely evaporated by the application of heat to the cylinder, we imagine some cold body to be applied to take away the heat again, the steam will begin to condense and the piston to descend under its constant load, a process which will go on till all the steam is condensed and there remains nothing but water: a further abstraction of heat causes the temperature to fall till finally we have the pound of water at 32°, with which the process began. And the whole heat taken away when the steam is condensed is the same as the whole heat added when the water is evaporated. Now while the steam was being ...d in the globes of Regnault's calorimeter, a ... ure was maintained throughout

the apparatus during the whole period of the experiment, and thus we are sure that the circumstances of the experiment were just the same as in the case of our hypothetical cylinder and piston. The necessity for insisting on this point will be understood when we come to the next chapter; for the present it is sufficient to say that the values of H are certainly given to a great degree of accuracy by these experiments, which, together with the two other series already mentioned, form the experimental basis of the theory of the steam engine.

Regnault's experiments on the total heat of evaporation extended from a pressure of one-fifth of an atmosphere to a pressure of fourteen atmospheres, say 3 lbs. to 200 lbs. per square inch, and the general result is that H increases slowly with the temperature by ·305 thermal unit for each degree Fahr., so that it may be expressed by either of the formulæ

$$\begin{aligned} H &= 1091{\cdot}7 + {\cdot}305\,(t - 32), \\ &= 1082 \quad + {\cdot}305\,t, \\ &= 1146{\cdot}6 + {\cdot}305\,(t - 212). \end{aligned}$$

Below one-fifth of an atmosphere the difficulty of securing a regular steady ebullition prevented Regnault from obtaining thoroughly reliable results. It is, however, usual, to suppose that the same formula applies to all cases.

Table II*a* shows the results of the formula for every 27°, from 77° to 401°, and also at 32°, the fifth column giving the differences, which in this instance are constant. Intermediate values may be obtained either directly from the formula or by interpolation.

In our definition of the total heat of evaporation, it has been supposed that the temperature of the water was 32° when the heating commenced; in practice, however, it generally happens that the water originally has some other temperature t_0, we then speak of the total heat of evaporation *from* t_0 *at* t. The tabular values of H and h enable the

result in this case to be easily obtained, for if Q be the required quantity of heat, we shall have

$$Q = H - h_0,$$

where h_0 signifies the heat necessary to raise a lb. of water from 32° to the temperature t_0. For example, to find the total heat of evaporation of water *from* 104° *at* 293°: on referring to the table we find for the value of h at 104°, 72·09, and for the value of H at 293°, 1171·3, hence

$$Q = 1171 \cdot 3 - 72 \cdot 09 = 1099 \cdot 21.$$

In this case, if the temperatures are not those given in the tables, H and h_0 must be found separately by interpolation; but we may almost always simplify by using $t - 32$ for h. Thus in the present example the error of so doing is less than one-tenth of a thermal unit, a quantity which is inappreciable compared with the value of h.

From the total heat of evaporation H we can at once deduce the latent heat of evaporation L, for it is clear that

$$L = H - h;$$

so that we have only to subtract the tabular value of h from the tabular value of H in order to find the value of L. The seventh column of Table II*a* has been formed in this way, and shows the latent heat of evaporation of water for every 27° from 77° to 401°, while the eighth column shows the differences for 1°, from which the latent heat at any other temperature can be found by interpolation. The table shows that the latent heat L diminishes as the temperature increases, the rate of diminution not being exactly constant, but increasing with the temperature. Unless, however, special accuracy is necessary, the formula

$$L = 966 - \cdot 71\,(t - 212^\circ)$$

uich are amply sufficiently approximate,
the latent heat diminishes by rather

more than seven-tenths of a thermal unit for each degree Fahr., and is 966 thermal units at the temperature 212°.

Density of Steam.

5. To complete our knowledge of the first two stages of the process we are considering, it is now only necessary to know what is the volume of the resulting steam, or how high the piston will have risen at the instant when the last drop of water is evaporated. Unfortunately, in the present state of our knowledge, this question cannot be answered with the same degree of accuracy with which we know the elastic force of steam or the total heat of evaporation. Two distinct methods have been adopted: first, by a series of direct experiments; secondly, by a calculation based on the principles of thermodynamics from the data already given.

No experiments on the density of saturated steam have been published by Regnault, and the only investigation possessing any claim to be considered reliable was made by Messrs. Tate and Unwin, under the auspices of the late Sir W. Fairbairn, and published in a paper read by the latter before the Royal Society in 1860. An abridged account of these experiments is given in Fairbairn's 'Mills and Millwork,' Part I., p. 207, to which the reader is referred for details; we shall here only mention the principle of the investigation and its results.

A glass globe was provided with a long stem, say 32 inches long, which was filled with clean mercury, and inverted in a dish of mercury, the mercury being previously boiled to secure the absence of air. A bubble of glass containing the water to be experimented on was then introduced into the globe floating on the top of the mercury. If now heat be applied to the globe the water vaporizes, the mercurial column descends, and the capacity of the globe having been previously measured, furnishes the means of measuring the volume of steam produced from the known weight of water

in the bubble at a known pressure and temperature. Two special difficulties occur when attempting to measure the density of steam by such a method.

First, it is impossible to tell directly the exact instant at which all the water is evaporated, and it is clear that if the volume be observed before all the water is evaporated there will be no means of determining the weight of water turned into steam; while if the volume is observed after the water is all evaporated, the volume measured will not be that of steam in contact with water, but of steam which is more or less superheated. Now this first difficulty was overcome by replacing the dish of mercury by a long glass tube projecting below a copper boiler containing water, in which the glass globe was immersed. On heating the boiler, steam was produced outside the globe of the same temperature as the steam inside the globe, and hence so long as any water remained in the bubble the pressure outside and inside the globe was the same, as shown by the mercury standing at the same height in the stem of the glass globe and the outer tube connected with the boiler. The moment the steam inside the globe becomes superheated the pressure inside the globe becomes less than the pressure outside, and this is at once indicated by a rise of the mercurial column in the stem of the glass globe, and the precise instant at which the volume should be measured was thus determined; hence this first difficulty was successfully overcome.

The second special difficulty which besets experiments of this kind is the cohesive attraction between water and the glass vessels in which it is contained, in consequence of which a glass vessel may be heated considerably without being at once dried, and in all probability steam is condensed on a glass surface as hot or hotter than itself. This source of error remains in the results of these experiments, and it is probable that the densities determined by them are somewhat too large.

The experiments extended from 2½ lbs. on the square inch to 70 lbs. on the square inch, and their results are given in Table III. for a whole series of pressures. The fourth column of this table gives the weight in lbs. of a cubic foot of steam, at the pressure indicated in the first column, as determined by these experiments.

The second method of obtaining the density is by calculation on the principles of thermodynamics from the values of the latent heat of evaporation given above. This method will be fully considered hereafter; it is sufficient to say at present that certain needful numerical data are not as yet known with absolute exactness, and that consequently the results of the calculation are not free from possible error. The fifth column in Table III. gives the weight of a cubic foot of steam as determined by this method, and the approximate agreement with the results obtained by direct experiment shows that neither method can be very far wrong. The calculation values which are the smaller are to be preferred, and it is highly improbable that they can be so much as one per cent. in error.

The general results of calculation and experiment on the density of steam are that the weight (w) of a cubic foot increases nearly in proportion to the pressure (p), but at a somewhat slower rate, as shown by column 6, which gives the differences per lb. from which the values of w for pressures not given in the table can easily be found by interpolation.

The reciprocal of w the weight of a cubic foot is the volume in cubic feet occupied by 1 lb. of steam, a quantity which in this work will always be denoted by v when steam is under consideration. The value of v is given in the second column of the table for the pressures indicated in the first column; and we shall call v the specific volume of the steam.

It is common to compare the volume of the steam with the volume of the water from which it is produced, which

may be done by multiplying v by $62 \cdot 4$, which is nearly the weight of a cubic foot of water at ordinary temperatures; this may be called the relative volume of the steam, though by some writers the term "specific volume" is used in this sense.

Various formulæ have been devised for the purpose of connecting the pressure and density of steam, of which we shall here give two.

First, we place the formula given in Fairbairn's paper to represent his experiments, viz.

$$v = \cdot 41 + \frac{389}{p + \cdot 35},$$

altering the constants to suit the case when the pressure is given in lbs. per square inch, and the volume in cubic feet. This formula* gives the results obtained by direct experiment, and the values as stated above are probably somewhat too small for perfectly dry steam below 110 lbs. per square inch. At pressures much exceeding this limit the formula gives too large a result.

Secondly, the results of calculation are represented approximately by the formula

$$p\, v^{\frac{17}{16}} = \text{constant},$$

where the value of the constant for pressures in lbs. per square inch is about 475. This formula was employed by Rankine, and the elaborate calculations of Zeuner have fully confirmed it. Zeuner, however, shows that the index $\frac{17}{16} = 1 \cdot 0625$, used by Rankine for the sake of simplicity, may be replaced with advantage by the somewhat larger value $1 \cdot 0646$.†

* In Fairbairn's 'Mill Work,' Part I., p. 214, this formula is quoted with a wrong sign in the denominator of the fraction. The error has been copied by Rankine in his work on 'Shipbuilding,' where the formula has been applied to a numerical example, and by the present writer in the first edition of this work. In Rankine's 'Useful Rules and Tables' the formula is quoted correctly.

† 'Grundzüge der Mechanischen Wärmetheorie,' p. 294.

Neither of these formulæ rest on any theoretical basis, but both are empirical formulæ employed to represent in a simple form the results of calculation and experiment. The second formula is that which will be chiefly employed in this work, when a formula is necessary; for numerical calculations the table is preferable.

Partial Evaporation—Superheating.

6. In all that has been said in the two preceding divisions of this chapter it has been supposed that the process of evaporation has been carried on until every drop of the water has been evaporated, and there remains nothing but steam, while care has been taken to stop the application of heat at the instant the water has all disappeared, so that the temperature is still stationary.

In such a condition the steam has the greatest density possible in perfectly dry steam at that pressure, and is hence said to be steam of "maximum density," or otherwise it is said to be "saturated." Likewise the temperature of such steam is the lowest possible at that particular pressure. Steam, however, seldom exists in a perfectly dry and saturated condition; either it is more or less mixed with water, or else its temperature is greater than that corresponding to the saturated condition.

In the first place, the steam supplied by an ordinary steam boiler is probably rarely perfectly dry; much obscurity rests on this point from the absence of any easy means of testing steam so as to ascertain the proportion of suspended water, but there can be little doubt that if from any cause the ebullition is irregular, such as, for instance, is the case when the irregularity in the consumption of steam always existing is aggravated by small steam space and rapid evaporation—that water is carried over from the boiler along with the rushing steam. This effect is called "priming," and is often produced on a large scale by impurity of water and other causes which

we need not here consider. And even if the steam from the boiler be originally dry, it almost always condenses to a greater or less extent on entering the cylinder.

Thus in a theory of the steam engine it is not sufficient to confine ourselves to the consideration of dry steam, we must likewise consider stéam containing a certain amount f moisture. The amount of moisture in steam is estimated by the amount of pure steam (x) contained in a pound of the actual steam, then x is a fraction which is smaller the wetter the steam, and which may be called the dryness-fraction of the steam. Let s be the volume in cubic feet of 1 lb. of water, then since 1 lb. of the steam contains x lbs. of dry steam and $(1 - x)$ lbs. of water, it is clear that the specific volume V must be

$$V = v x + (1 - x) s,$$

where v as before is the specific volume of dry steam considered in the preceding section.

We may write this

$$V = x (v - s) + s.$$

Now s is a small fraction being $\cdot 016$ at ordinary temperatures and less than 20 per cent. greater (see Appendix, Note B) at the highest temperatures possible in practice, we may therefore safely neglect it; further, unless x be small, the remaining term s may be neglected, and we obtain simply

$$V \doteq v x.$$

These simplifications cannot be made at very high pressures, nor when (as is sometimes the case) we have to do with mixtures of steam and water, consisting chiefly of water. To show the amount of error involved, the value of $v - s$ is given in the third column of Table III.

So much for the density of moist steam. Next, for its total heat of evaporation, we have only to consider that for each lb. of such steam a lb. of water has been raised from 32° to

t, but only x lbs. have been evaporated; hence if Q be the heat expended,

$$Q = h + xL;$$

or if the water originally be at t_0 instead of 32°,

$$Q = h - h_0 + xL;$$

a formula which, by aid of the tabulated results given previously, enables us to find the total heat of evaporation very readily when x is known.

The heat necessary to produce dry steam from and at 212° is 966 thermal units, and the total heat of evaporation under any circumstances may conveniently be expressed by stating the equivalent evaporation from and at 212°. Suppose we call this E, then

$$E = \frac{Q}{966} = \frac{h - h_0 + xL}{966} \text{ per lb. of steam.}$$

For example, suppose a boiler to supply steam with 10 per cent. of suspended moisture, the evaporation taking place from 100 at 320°, then

$$E = \frac{220 + \cdot 9 \times 888}{966} = \frac{1019}{966} = 1 \cdot 055,$$

which is the factor by which the actual evaporation must be multiplied to obtain the equivalent evaporation from and at 212°.

The possibility of the steam generated by a boiler containing suspended moisture, and thus requiring less heat to produce it, should not be lost sight of when the efficiency of the boiler or the evaporative power of the fuel is being considered, for important errors may easily be produced in this way.

Not only may steam be wet, but it frequently happens, either by direct application of heat or by other causes to be considered hereafter, that its temperature is raised above the limit value, which is the lowest possible at the pressure considered, and in that case it is said to be superheated

Such is the case in the third stage of the process we have been considering, in which after the water has been all evaporated the application of heat is continued. The temperature then rises continually, instead of remaining stationary as before. Our experimental knowledge of this third stage is very imperfect, and the little that is known with certainty cannot advantageously be introduced here; I shall therefore now pass on in the succeeding chapter to explain the fundamental principle upon which, together with the results of experiment now given, all successful reasoning on the subject of the steam engine must necessarily be based.

CHAPTER II.

CONVERTIBILITY OF HEAT AND WORK. INTERNAL WORK.

7. When a resistance is overcome by the action of force, the effect of the force, considered as acting through space, is measured by the magnitude of the resistance multiplied by the distance traversed, estimated in the direction of the force. The force is then said to do work, and the work done is equal to the product of the resistance and the space.

The power of doing work is called energy, and a body or system of bodies possessing this power is said to "possess energy," an expression which implies that energy is conceived as separate from and independent of the bodies through which it is manifested, capable, like matter, of measurement in quantity, and, as we shall see presently, like matter, indestructible. In simple mechanics energy is of two kinds, energy of position and energy of motion, otherwise called potential energy and kinetic energy, exemplified by the simple cases of a raised weight and a rotating wheel, each of which possesses the power of doing work: the one, in virtue of its position at a certain height above the earth's surface; and the other, in virtue of its motion.

When the force applied is just sufficient and no more to overcome the resistance, the energy exerted is exactly equal to the work done, and this is true not merely of a single force applied directly so as to overcome the resistance, but to any number of forces applied by means of a machine of any degree of complexity, so that we may say in any case in which the forces just balance the resistance,

$$\text{Energy exerted} = \text{Work done}.$$

This is the principle of work as applied to balanced forces, and is identical with the older principle of virtual velocities explained and applied in all text books of elementary mechanics.

It will, however, rarely happen that the forces applied exactly balance the resistance overcome; let us suppose that the applied forces are the greater, then the unbalanced part of these forces takes effect by causing the parts of the machine to move quicker and quicker, and thus to increase their energy of motion; hence the energy exerted by the applied forces is not all employed in doing work, but partly takes effect in increasing the kinetic energy of the parts of the machine. In this case the principle of work is equivalent to the statement,

Energy exerted = Work done + Kinetic Energy accumulated in the moving parts of the machine;

a statement which, by supposing the accumulation of energy negative, will include also the case in which the applied forces are in themselves insufficient to overcome the resistance, so that a part of the work is done at the expense of the kinetic energy of the moving parts. Now, as the expression accumulated energy implies, the energy employed in altering the velocity of a particle or machine is not lost, but merely transferred to the particle or machine, existing there, in the form of kinetic energy, or energy of motion. And further, when the work done consists in raising weights or other similar operations, it is clear that the energy exerted is not lost, but exists in the weights raised, which possess on falling a capacity of doing work, or potential energy exactly equal to the work done in raising them; so that if we confine ourselves to such operations we may assert that energy when exerted is not destroyed, but simply transferred from one body to another.

In all cases, however, *some*, and in many cases *all*, the work done consists of mechanical operations in which energy

is to all appearance lost, a principal instance of this being friction. When one surface rubs against another work, is done in overcoming friction; and although no doubt there is a certain amount of wear so that some energy might be imagined to be regained by replacing the abraded particles in their original positions, yet it is certain that but a small amount of the whole energy exerted is thus accounted for. Thus, confining ourselves to simple mechanics, energy is not indestructible, and we cannot go farther than to say that energy cannot be created out of nothing, but must be obtained from some store of previously existing energy. To this extent the principle of work is equivalent to the statement that a "perpetual motion is impossible," an axiom which is actually forced on all those engaged in mechanical operations, and which was practically known to our great engineers of the last century long before the principle of work was formally stated by Poncelet in the *Mécanique Industrielle.*

We now, however, can go much farther than this, since we know that mechanical energy is only one of several forms in which energy may exist, and that the processes by which energy is to all appearance lost are really processes by means of which mechanical energy is transformed into one or other of those forms, and that when account is taken of all the results of the processes in question, we shall find that the energy which has disappeared is in nowise lost, but is merely transferred from one body to another, and altered in form, not substance.

The principal other form into which mechanical energy is capable of being transformed, and the only one which concerns us, is that powerful agent in producing physical changes which we call heat. It is unnecessary here to trace the steps by which the idea arose that heat and mechanical energy are quantities of the same kind, capable of conversion the one into the other; I shall content myself with enunciat-

ing the FIRST LAW of thermodynamics, that is to say, of the science of the relations between work and heat, as follows:—

Heat and mechanical energy are mutually convertible, a unit of heat corresponding to a certain fixed amount of work, called the mechanical equivalent of heat.

Thus, when mechanical energy is expended in overcoming friction, it is a matter of common experience that heat is produced, and the law just enunciated tells us that this heat is merely the energy expended in a different form. Assuming, which is very approximately the case, that the surfaces remain in the same state as before, the energy expended bears a fixed proportion to the heat generated, that fixed proportion being the mechanical equivalent of heat, and by comparing the heat generated with the energy expended the mechanical equivalent of heat may be determined.

The first attempt to establish a connection between work and heat was made by Count Rumford in his celebrated experiments on the heat generated during the boring of a cannon, but the first accurate determination of the mechanical equivalent of heat was made by Joule, and hence we commonly speak of Joule's equivalent. Joule experimented on the heat produced when a given amount of energy was employed in agitating water, and obtained the number 772, signifying that the heat necessary to raise a lb. of water from 39° to 40° would, if wholly converted into work, raise a lb. weight through 772 feet. It is certain that this number is very near the truth, although it is not probable that minute accuracy has yet been attained. The late Professor Rankine believed it to be within $\frac{1}{300}$th of the truth. Researches now proceeding may probably attain a still more approximate result, meanwhile we employ in this work the number 772 as the best value at present attainable. (See Appendix, Note F).

Since heat and mechanical energy are merely different forms of the same thing, it follows that quantities of heat

may be expressed in foot pounds, and conversely quantities of work may be expressed in thermal units; thus the total and latent heat of evaporation of water may be expressed in foot pounds, as is shown by Table II*b* derived from Table II*a*, by multiplication by 772. And again a horse-power of 33,000 foot pounds per minute is equivalent to $\frac{33,000}{772} = 42 \cdot 75$ thermal units per minute, or 2565 thermal units per hour, numbers convenient to remember in working examples.

Internal and External Work done during Evaporation under Constant Pressure.

8. Having now obtained a principle by means of which heat expended can be compared with work done, let us return to the case of the evaporation of water beneath a piston, which we considered at some length in the preceding chapter. It is clear that, when the piston rises during the evaporation, work is done in overcoming the pressure P with which the piston is loaded, and it is easy to find from the data of the last chapter the amount of that work. For let A be the area of the piston in square feet, y_0 the depth of water in the cylinder before evaporation begins, y the height of the piston above the bottom of the cylinder when the water is all turned into steam, then clearly $A\,y_0$ is the volume of the water in the cylinder, and $A\,y$ the volume of the resulting steam, or, using the symbols of the preceding chapter,

$$A\,y_0 = s; \qquad A\,y = v;$$

but the piston rises through the space $y - y_0$ overcoming a pressure of P lbs. per square foot, or a total pressure P A, hence the work done is given by

$$\text{Work done} = P\,A\,(y - y_0) = P\,(v - s).$$

Since both P and v are known, and s is the constant fraction ·016, the work done can be calculated for any pressure or temperature from the experimental data of the

preceding chapter. The results are exhibited in Table IV*a*, the fourth column of which gives in foot pounds the value of $P(v - s)$, or the work done, for every 27° of temperature from 104° to 401°, while the fifth column gives the differences per 1°, which enable the result for any other temperature to be readily found by interpolation. It appears from this table that the work done during evaporation of a lb. of water increases with the temperature, the rate being slow and slowly diminishing as the temperature rises. At the atmospheric pressure the result is 57,740 foot pounds.

Our next object is to compare this work with the amount of heat expended, in order to do which we have only to find the heat-equivalent of the work done, or else the work equivalent of the heat expended. Let us choose the first course, and obtain the heat-equivalent of the work done by dividing the values just found by 772, the results are given in the sixth column of the same table (Table IV*a*), while the seventh column shows the differences needed for interpolation. Now let us compare the result with the heat expended during evaporation, that is, with what we have previously called the latent heat of evaporation, and given in the seventh column of Table II*a*, and we are at once struck with the great difference which exists, the heat-equivalent of the work done forming but a small fraction of the heat expended. For example, take the temperature 293°: here the latent heat of evaporation is about 908 thermal units, and the heat-equivalent of the work done is only 78·6 thermal units, or hardly one-twelfth. The principle of work tells us that the work done cannot have been produced out of nothing, but must have been done at the expense of an equivalent amount of heat which has disappeared, while the numerical values, just obtained, show that the heat thus disappearing is comparatively small, and that the greater part of the heat must have been employed in producing changes within the water itself.

We are thus introduced to a conception of great importance, namely, the conception that work may be done, not only in raising weights or performing other visible operations in which the resistances overcome are manifest to our senses, but also in overcoming resistances caused by molecular forces invisible to us, if I may use the expression, and only manifest by their results. Thus, in the present example the difference between 908 and 78·6 is 829·4, which is the heat-equivalent of the work done in overcoming the molecular cohesion of the particles of water resisting its conversion into steam. Work done in this way is called "internal work," because the changes considered take place within the body itself, whereas in contradistinction the work done in raising the piston is called "external work," because the change considered takes place, not in the body itself, but in external bodies.

The internal work done during evaporation at constant temperature obtained by subtraction, as in the example just given, will be found in foot pounds and thermal units in the second and fourth columns of Table IV*b*; it is always denoted in this work by the symbol ρ. The differences needful for interpolation are given in the third and fifth columns, and examination of their values shows a nearly constant result, hence the value of ρ is given with very considerable accuracy in foot pounds and thermal units by the formulæ exact at 212°:

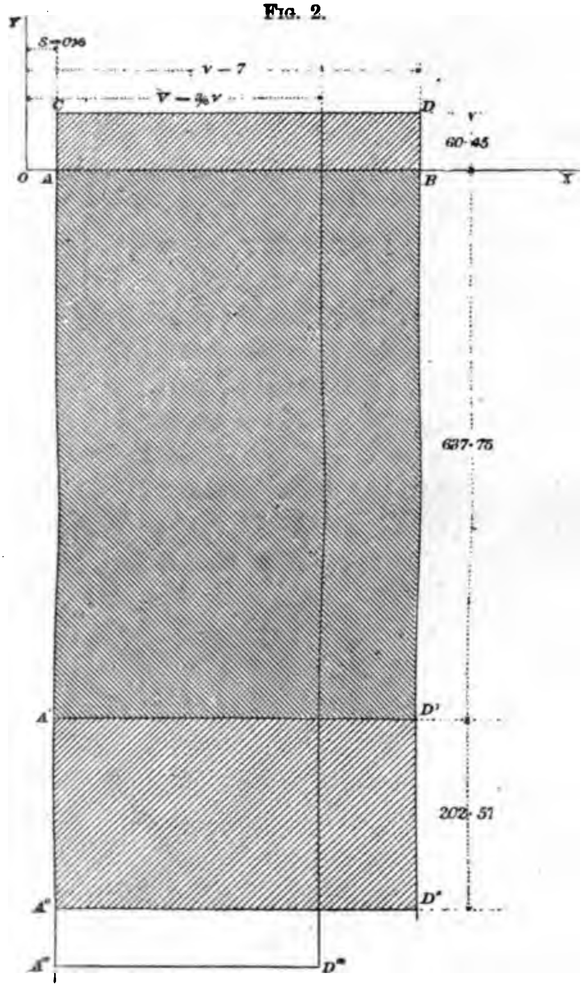
$$\rho = 819{,}330 - 611\,t \text{ foot pounds,}$$
$$\rho = 1061{\cdot}4 - {\cdot}792\,t \text{ thermal units.}$$

The sixth column shows the proportion (k) which the internal work bears to the external work, and exhibits in a striking manner the magnitude of the internal forces with which we have to deal, for we see that the internal work is from nine to sixteen times as great as the external.

9. We may with great advantage exhibit this graphically. In the figure (Fig. 2) O X is a line on which are measured

the volumes occupied by the steam and water during the process of evaporation, or, what is the same thing, the volumes swept out by the piston as it rises, on which line

Fig. 2.



we set off O A = s = ·016 cubic foot, and O B = v cubic feet. Let the ordinates parallel to O Y represent pressur and draw a horizontal line C D, the ordinate A C of

represents the pressure P, with which the piston is loaded; then if we complete the rectangle A D, its area will represent the external work done in raising the piston. Now to represent the internal work we have only to draw a corresponding rectangle below O X on the same base A B: the height of this rectangle will be k P, in order that its area may be k times the area of the upper rectangle. Thus the internal work is represented as the work which would be done in overcoming a pressure k P on the piston, an ideal pressure which may be called the pressure equivalent to the internal work, or, for brevity, "the internal-work-pressure." In practice pressures are always stated in lbs. per square inch, not lbs. per square foot, and the internal-work-pressure is consequently to be stated in like manner, as so many lbs. on the square inch. For example, take the temperature 293°, at which evaporation takes place under a pressure of 60·45 lbs. per square inch: the molecular resistance to evaporation is equivalent to a pressure 10·55 times as great, say to a pressure of 637·75 lbs. per square inch of the piston area. The figure (Fig. 2) is drawn to scale for this case, except that O A, being only ·016 cubic foot, is for clearness set off on a much larger scale than O B, which represents 7 cubic feet, the volume of dry steam at the pressure 60·45 lbs. on the square inch.

For any other pressure of evaporation, a corresponding internal-work-pressure exists, usually denoted by $\overline{P}$, which may be found from the tables already given by the method just indicated. But this pressure being frequently required, a special table has been calculated (by a different method) which gives it for any desired pressure. In Table V. the internal-work-pressure is given in lbs. per square foot, and in lbs. per square inch for pressures ranging from 4 lbs. per square inch to 250 lbs. per square inch, together with the differences necessary for interpolation.

Since the heat expended is ... f the internal

work and the external work taken together, it appears that the heat expended may be represented as overcoming a pressure on the piston equal to $(k+1)$ P: in the numerical example this pressure is 698·2 lbs. on the square inch; it may be called the pressure equivalent to the expenditure of heat, or, more briefly, the heat-pressure. The last column of the table (Table V.) shows the heat-pressure. In the figure the heat expended is represented by the whole area CD^1 of the two rectangles. When quantities of heat are represented by rectangles, the heights of which are pressures in lbs. per square inch and the bases volumes in cubic feet, the numerical value of the area is to be multiplied by 144, to express the quantity of heat in foot pounds, or divided by $\frac{772}{144}$, that is to say, 5·36 to express it in thermal units.

Internal Work done during Rise of Temperature.

10. Hitherto we have considered exclusively the second stage of the process, namely, that in which water at a given temperature is converted into steam of the same temperature under the corresponding constant pressure; but internal work is also done during the first stage in which the temperature of the water is raised from some lower temperature to the temperature at which evaporation commences. This is shown by the heat expended, which must be expended in producing molecular change of some kind; the nature of which it is needless for us to inquire into. Strictly speaking, indeed, external work is also done, for the water expands as its temperature rises and so raises the piston, but the amount of this is so small as to be quite insensible as compared with the heat expended, the value of which can be found in foot pounds or thermal units from Table II., as previously explained. Hence practically the whole heat expended is employed in producing molecular changes, or, as we express it, in doing internal work. (Comp. Art. 15.) Thus, in the numerical example given above, in which the steam is formed

at 60·45 lbs. per square inch, the heat required to raise the water from 32° to 293° is 263·4 thermal units, or 203,300 foot pounds, all of which is spent in internal changes.

The proportion which the internal work so done bears to the external work done during evaporation is obviously got by dividing one by the other: thus, in the example, the ratio is

$$k' = \frac{263 \cdot 4}{78 \cdot 6} = 3 \cdot 351;$$

and hence we may represent this internal work also as equivalent to raising the piston through its whole height against a pressure equal to k' times the actual piston load. In the example this pressure is $3 \cdot 351 \times 60 \cdot 45$, or 202·57 lbs. on the square inch.

To treat the question graphically, we have only to prolong B D′ (Fig. 2) to D″, making D′ D″ equal to 202·57, and complete the rectangle A′ D″, then the area of that rectangle represents the internal work done during the rise of temperature of the water from 32°.

The height of the rectangle may, however, conveniently be found thus, without the calculation of k', for any initial temperature of the water. The heat requisite to raise a lb. of water from t_0 to t_1 is $h_1 - h_0$, calculated as previously explained, but may usually be taken as $t_1 - t_0$ thermal units. Let P′ be the corresponding pressure in lbs. per square foot, which would do an equivalent amount of work upon the piston, then

$$P' . (v - s) = (t_1 - t_0)\, 772;$$

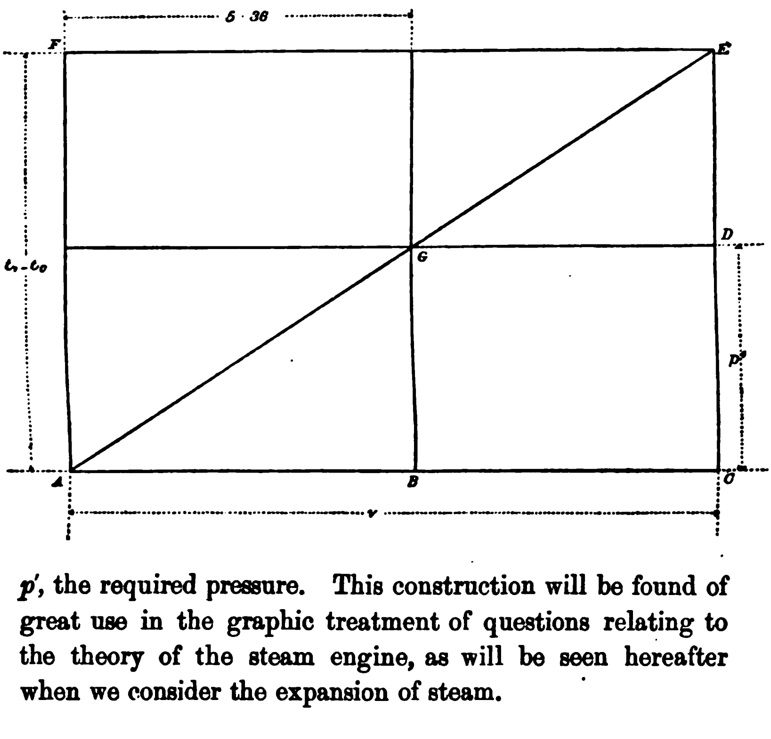
or if p' be the same pressure in lbs. per square inch,

$$p' . (v - s) = \frac{772}{144} . (t_1 - t_0) = 5 \cdot 36\, (t_1 - t_0).$$

In Fig. 3 set off A B = 5·36 cubic feet and A F equal to $t_1 - t_0$ reckoned as lbs. per square inch, and complete the rectangle B F. Now set off A C $= v - s$ cubic feet, complete

the rectangle A E, and join A E; further complete the rectangle A D by drawing a parallel through G, then the rectangles B F and A D are equal; therefore C D represents

FIG. 3.



p', the required pressure. This construction will be found of great use in the graphic treatment of questions relating to the theory of the steam engine, as will be seen hereafter when we consider the expansion of steam.

Total Internal Work.

11. The total amount of internal work done is of course obtained by adding the two results together: hence

$$\text{Internal Work} = h + \rho = h + L - P(v - s)$$
$$= H - P(v - s);$$

thus in the numerical example for steam at 293° the internal work done during the rise of temperature is 263·4 thermal units, and during evaporation 829·3 thermal units; hence the whole amount of internal work done

turning water at 32° into steam at 293° is 1092·7 thermal units, a result which may also be obtained by subtracting the heat-equivalent of the external work (78·6) from the total heat of evaporation (1171·3). Table IV*c* shows the results of this simple calculation for the same range of temperature as in the previous tables: these results are given in foot pounds and thermal units, together with the differences, which facilitate interpolation and serve other purposes.

These results show that the whole heat expended in internal changes increases with the temperature, though at a rate which is still slower than that of the total heat of evaporation.

Partial Evaporation.

12. In the three preceding sections it has been supposed that the evaporation is complete, so that the result of the operation is dry saturated steam. Let us now imagine the evaporation stopped when x lbs. of water have been evaporated, as in Art. 6, Chapter I.

In this case the specific volume is

$$V = x(v - s) + s;$$

and by reasoning similar to that in Art. 8, it is clear that

$$\text{External Work} = P(V - s)$$
$$= xP(v - s);$$

thus the external work done is simply x times what it would have been had the evaporation been complete. The heat expended during evaporation is clearly x L, and therefore bears the same proportion to the external work as if the evaporation had been complete: in the same way the internal work will be ρx during evaporation, but will be h, as before, during the rise of temperature. Thus the whole amount of internal work is given by

$$I = h + \rho x \text{ (reckoned from water at 32°).}$$

Or we may express our result [illegible]ms of the internal-

work-pressure $\overline{P} = k\,P$, for just as the external work is $P\,(V - S)$, so we shall have during evaporation

$$\text{Internal Work} = \overline{P}\,(V - s);$$
$$\therefore\ \text{Total Internal Work} = h + \overline{P}\,(V - s).$$

In graphically representing the process of evaporation, the rectangle representing the internal work during evaporation will be of the same height as before, but described on the base V instead of the base v; while the rectangle which refers to the rise of temperature is not of the same height, but of the same area as before: it is constructed as in the last article, except that the base is V instead of v.

In the figure (Fig. 2, Art. 9) the construction is shown supposing the volume V three-fourths that of dry steam at the same pressure.

Formation of Steam in a Closed Vessel.

13. We have now thoroughly considered the whole process of evaporation, when conducted in a cylinder beneath a loaded piston, which rises as the evaporation proceeds, and we proceed to a different case, by supposing that the evaporation takes place not beneath a rising piston, but in a closed vessel of given capacity.

Let a lb. of water at the temperature 32° be placed in a closed vessel of known capacity, and let heat be gradually applied, then the temperature of the water will gradually rise as before, but instead of the formation of steam commencing at some definite temperature, as in the previous case, steam will be formed at once, and the pressure will keep rising as more and more steam is formed. The pressure is still connected with the temperature by the same invariable law as before, but the evaporation now takes place at a gradually rising temperature, instead of a certain fixed temperature. If sufficient heat be applied, every dr

of the water will at length be evaporated, and we shall h

nothing but steam: the pressure of that steam will depend on the volume of the vessel, which must be supposed large to avoid excessive pressure. The magnitude of the pressure is found by reference to the table of density (Table III.): thus, for instance, suppose that the volume of the vessel is 4 cubic feet, then the lb. of dry saturated steam occupies 4 cubic feet, therefore its pressure must be that corresponding to a volume of 4 cubic feet, which a reference to the table shows to be almost exactly 110 lbs. on the square inch. On reference to the temperature table (Table I*a*) the corresponding temperature is found to be about $334\frac{1}{2}^\circ$, which is the temperature to which the vessel has risen when every drop of the water is evaporated. If the application of heat be still continued, the steam will become superheated; we, however, suppose it stopped before this takes place, and the question proposed for consideration is to find how much heat is spent in evaporating the water under these circumstances.

Now the essential difference between the two cases is, that in the first case, work is done by raising the loaded piston, while, in the second case, no work is done, and hence, in the first case, we have done two things instead of one; not only has the water been evaporated, but a certain amount of work has been done on external bodies: this amount of work cannot have been produced out of nothing, but must have been obtained at the expense of the energy applied to the water in the shape of heat. Hence, unless some difference be imagined in the amount of heat requisite to produce internal changes, we shall be obliged to conclude that the heat expended in the second case is less than the heat expended in the first case by the exact amount of the heat-equivalent of the external work, which *is* done in the first case, and *is not* done in the second. There is, however, no reason to believe that any such difference can exist; the steam produced is the same in both cases, and the water from which the steam is formed is likewise the same, and

consequently the amount of internal change must be precisely the same, and we are entitled to conclude that when water is evaporated in a closed vessel, the heat expended is the same as that expended in internal work, when the water is evaporated beneath a loaded piston. It is therefore given by the table of internal work just calculated (Table IV*c*).

The reader will now understand why, in defining the total heat of evaporation, it is necessary to specify that the evaporation is supposed conducted at a fixed temperature, and why it was necessary to examine the method in which Regnault carried out his experiments to see if the prescribed condition was satisfied. If that condition be not satisfied, it will be necessary, in order to find the heat expended in the production of steam, to consider how much external work has been done during its formation, and to add its heat-equivalent to the heat just found to be necessary to produce the steam itself. In fact, whenever external work is done in the formation of steam, heat flows out of the steam under the form of mechanical energy, just as really as when it escapes in the form of heat by the process called radiation.

Similarly when steam is condensed, the heat which it gives out is not always the same, but depends upon the circumstances under which the steam is condensed: if it be condensed under constant pressure, the heat given out will be what we have previously defined as the total heat of evaporation, and will include not only the energy given out by the steam during its contraction into water, but likewise the mechanical energy exerted by the pressure of the piston, which will appear in the form of heat in the condenser. But if it be condensed under any other circumstances, the heat given out will be different, because a different amount of energy will be supplied from external sources. In the next chapter we shall have ample illustrations of this in the working of a steam engine.

Internal Work in General.

14. As in the case of steam, so in the most general case of the action of heat on any body whatever: we must always separate the external and visible work, done on external bodies, from the internal and invisible work, done in changing the state of the body. The second part, which we call the internal work, depends upon the change of state alone, and not upon the way in which the change of state is produced, while the first part represents energy, which has passed out of the heated body into external bodies, and may have any value according to the way in which the change is accomplished. The heat expended is the sum of these two amounts of work, as expressed by the equation:

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work};$$

which is the general statement of the principle of work as applied to such cases.

In the case of a heat engine it frequently happens, that the change considered is of such a kind that heat is added to the steam or other fluid during one part of the process, and taken away during another part: the heat added is then usually called the "heat expended," and the heat taken away, the "heat rejected," and the statement of the principle takes the form:

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work} + \text{Heat Rejected}.$$

If, moreover, the change considered be such that the steam or other fluid, after going through any number of intermediate changes, finally returns to its original state, then the internal work done is evidently zero, and we write simply:

$$\text{Heat Expended} = \text{External Work} + \text{Heat Rejected}.$$

Thus, for example, in the steam engine, if the feed water be taken from the condenser, forced by the feed pump into the boiler, there evaporated, and finally, after passing through

the cylinder, be returned to the condenser in the shape of water of the same temperature as before, the internal work done in the whole operation is zero, and we shall have:

Heat Expended = Useful Work done + Heat Rejected into the condensation water.

A change of this latter kind is called a "cycle of operations," or a "circular process," because the fluid goes through a cycle of changes, returning to its original state. The conception of a cycle of operations is due to Carnot, and was an important step towards the true theory of heat engines. Whenever, from deficiency of experimental information, we have no means of telling directly or indirectly the work spent in internal changes, we are obliged to resort to a cycle of operations as being the only case in which we can find the relation between heat expended and work done. In the case of saturated steam and permanent gases, the internal work is known with tolerable certainty, and we are not obliged to confine ourselves to the consideration of cycles of operation; but when superheated steam, for instance, is in question, we cannot reason with certainty except in this way, because our experimental knowledge of superheated steam is still very scanty, so that we have no means of knowing with certainty the work done in internal changes.

Thus, in a steam engine, it will sometimes happen that the condensed steam is not at the same temperature as the feed water from which it was originally produced; then the original state of the steam was water of temperature t_1 say, and its final state is water of temperature t_2, so that the internal work done in changing from the original state to the final state is not zero but $t_2 - t_1$ thermal units, whence we see that the principle of work takes the form:

Heat Expended = $t_2 - t_1$ + External Work + Heat Rejected;

the external work being expressed by its heat-equivalent in thermal units. For an example the reader is referred to the discussion of Mr. Donkin's experiments in Chapter XI.

15. When the temperature of a body remains constant during the application of heat, that heat is said to be "latent." So long as heat was supposed to be a material substance, such an expression as the "latent heat of steam" was strictly appropriate; but, if used now, it must be distinctly understood that a part of the heat is latent, not in the steam, but in external bodies in the form of mechanical energy. In this work the term will only be used in the phrase "latent heat of evaporation," which has the well-understood conventional meaning defined in Art. 4.

In contradistinction to "latent," the word "sensible" was formerly applied to heat which is effective in raising the temperature of the heated body: while the sum of the "sensible" and the "latent" heat was the *total* heat. It is now advisable to employ the expression "total heat" to signify the sum of the quantities of heat expended in internal and external work respectively; and when steam is formed in any way I shall call the whole heat expended its TOTAL HEAT OF FORMATION, while the phrase "total heat of evaporation" will always be used, in accordance with the definition already given, for the particular case in which the steam is formed under constant pressure.

The expression "internal work" has been employed throughout to signify energy expended in internal changes without any distinction between different kinds of internal change. Writers on thermodynamics, however, often distinguish between internal changes consequent on change of temperature and internal changes consequent on change of molecular position, and confine the term "internal work" to the latter kind of changes only. It seems better, however, to use the term in a sense capable of exact explanation, without any hypothesis as to the nature of the changes considered. The whole of the reasoning in this chapter depends solely on the principle that heat and mechanical energy are merely different forms of the same thing, and are, therefore,

completely interchangeable. No hypothesis is involved as to the ultimate nature of heat or the constitution of matter. Another very expressive term was introduced by Rankine, which is directly applicable to the case in which a heated body is a source of energy. Every such body possesses, in virtue of the heat which has been applied to it, a store of energy which is precisely equal to the internal work done during the heating, and which consequently depends upon the state of the heated body alone, and not upon the circumstances under which the body was heated, or upon its relation to external bodies. This store of energy is therefore called the *intrinsic* energy of the body, and when the body returns to its original state it is always given out either wholly as heat, or partly as heat and partly as external work done upon a piston during expansion. Thus, intrinsic energy and internal work are the same, differing only in sign. For example, the total internal work done in producing dry saturated steam from water at 32° (given in Table IV*c*) may likewise be considered as the intrinsic energy of that steam. I shall occasionally use this term hereafter, but "internal" energy may also conveniently be used in the same sense.

CHAPTER III.

THEORY OF THE STEAM ENGINE (PRELIMINARY).

16. THE principle of work is not, by itself, sufficient to answer many of the most important questions which arise respecting the operation of heat engines; but I shall nevertheless go on at once to consider such parts of the theory of the steam engine as can be conveniently treated here, reserving other parts, which are complex, or which require the application of a second equally important principle, till a later period.

In studying a difficult problem of any kind it is necessary to commence with the most simple cases, and pass gradually on to the more complex, in order that we may be enabled to deal with the difficulties of the subject one at a time. These simple cases are ideal, being formed by abstracting, from cases actually occurring, a number of disturbing causes which complicate the problem in practice. When once such cases are thoroughly understood, it is comparatively easy to estimate the effect of each disturbing cause separately in modifying the result of the preliminary investigation.

In dealing with the steam engine then, I, in the first instance, make certain suppositions, never exactly, and sometimes not nearly, realized in practice, as follows:

(1) In the first place, the supply of steam is supposed uniform, which cannot be the case in practice on account of the varying speed with which the piston moves. At the beginning and end of the stroke no steam passes from the boiler to the cylinder, and in expansive engines this stoppage lasts during a considerable part of the stroke: hence the

evaporation is necessarily irregular, and that the more so, the greater the expansion and the smaller the steam space in the boiler as compared with the capacity of the cylinder. No attempt has yet been made to estimate quantitatively the effect of irregular ebullition, but there is no reason to think it important, except as a cause of the production of moist steam.

(2) The effect of clearance is neglected and also that of wire drawing during the passage from the boiler to the cylinder. Both these disturbing causes always exist and exert considerable influence on the working of the engine: they will consequently be considered hereafter.

(3) The exhaust is supposed to open suddenly, exactly at the end of the stroke, and the mean value of the "back pressure" always existing behind the piston is supposed given. In practice some lead is usually given to the exhaust, and the back pressure depends on various complicated circumstances not yet reduced to a complete theory.

(4) The action of the sides of the cylinder is either neglected altogether, or some simple supposition is made respecting it. In practice this action has a most important prejudicial influence, and hence will form hereafter the subject of a special chapter.

Subject to these observations, I proceed to discuss various cases, commencing with the simplest.

Non-expansive Engines.

17. When the steam port is open throughout the stroke the engine works without expansion, the pressure remaining constantly that of the boiler (Art. 15, [2]). In this case the process of evaporation is the same as in the simple case of cylinder and piston considered in the two preceding chapters; the only difference being that the evaporation of the water takes place in a boiler connected with the cylinder by a pipe instead of in the cylinder itself, and that the piston,

instead of moving continuously in one direction, moves backwards and forwards. Neither of these circumstances has any influence on the heat expended on, or the energy exerted by, each pound of steam, which are accordingly given by the preceding rules.

The energy exerted in driving the piston is, however, somewhat greater, as is seen thus. Let x be the length in feet described by the piston in a given time, say 1′, A the area of the piston in square feet, then A x is the volume swept through by the piston per 1′ in cubic feet. If P be the pressure in lbs. per square feet, P A x will be the work done per 1′, and supposing the engine uses N lb. of steam per 1′, the volume of each of which is v,

$$N v = A x,$$

since for each cubic foot swept through by the piston a cubic foot of steam must pass from the boiler to the cylinder. Thus the work done per lb. of steam in driving the piston is P v instead of P $(v - s)$ which is the true value of the energy exerted by 1 lb. of steam during evaporation. The reason of this is that part of the energy exerted in driving the piston is obtained by the action of the feed pump, which for each lb. of steam used forces 1 lb. of water into the boiler, and, in doing so, does an amount of work represented by P s. Thus the true energy exerted by 1 lb. of steam is the difference between the energy exerted in driving the steam piston and the piston of the feed pump. This distinction, though theoretically interesting, is unimportant in practice, so far as the steam engine is concerned, on account of the smallness of s, as compared with v, in consequence of which, except at very high pressures, P v is sensibly equal to P $(v - s)$ as is shown in Table IVa by the values of these quantities there given.

The energy exerted in driving the steam piston is, however, by no means the same as the useful work done by the

engine; this is always less, and often much less, on account of "back pressure." Back pressure consists of three parts; (1), the pressure corresponding to the temperature of the condenser, or the pressure of the atmosphere if there be no condenser; (2), the pressure of the air always contained in the water of the condenser or present through leakage; (3), the difference of pressure between the cylinder and condenser. The first may be taken on the average as 1 lb. per square inch where there is a condenser, and 14·7 where there is none; the other two depend on the speed of the piston, the state of the steam, the size of the ports, and other circumstances, but are probably seldom less than 1 or (under normal circumstances) more than say 3 lbs. on the square inch. Thus for non-condensing engines the back pressure may range from 16 to 18 lbs. on the square inch, and for condensing engines from 2 to 4 lbs. on the square inch, but these values may be much increased by improper construction and management.

Let now P_b be the back pressure, then the effective pressure is $P - P_b$, and the useful work done per lb. of steam is $(P - P_b)\,v$, which is less than if there were no back pressure in the proportion $P - P_b : P$, a fraction which is at most ·9 in non-condensing engines and ·95 in condensing engines. The annexed table gives the effective work of 1 lb. of steam in an engine working without expansion at various boiler pressures. The results are given in thermal units and foot pounds in columns 2 and 3, while the fourth column shows the number of pounds of steam required per indicated horse-power per hour, which is readily obtained by dividing 1,980,000 by the work done by 1 lb. of steam.

The expenditure of heat per lb. of steam is simply the total heat of evaporation *from* the temperature of the feed water *at* the temperature of the boiler; it is given in column 5 in thermal units, while column 6 shows the heat expended, in thermal units per indicated horse-power per 1′, a

mode of stating the expenditure of heat, which is often convenient; the circumstances chosen are described in the table.

PERFORMANCE OF A NON-EXPANSIVE ENGINE.

Remarks.	Pressure (absolute) lbs. per square inch.	Effective work per lb.		Lbs. Steam per I.H.P. per hour.	Heat expended in thermal units.		Efficiency.
		Thermal units.	Foot pounds.		Per lb. of Steam.	Per I.H.P. per 1'.	
Non-condensing, back pressure 16 lbs. Feed heated to 212°.	160	74·8	57,700	34·3	1012	578	·074
	80	64·	49,400	40·1	996	668	·064
	55	55·6	42,900	46·2	989	763	·056
Condensing, back pressure 2 lbs. Feed taken from condenser at 104°.	60	76·2	58,800	33·7	1099	620	·069
	30	70·1	57,100	36·6	1086	668	·064
	20	66·2	51,100	38·8	1079	701	·061

The ratio which the useful work done bears to the heat expended is called the *efficiency* of the steam, and is given in the last column of the table, from which it appears that only from $5\frac{1}{2}$ to $7\frac{1}{2}$ per cent. of the heat expended is converted into useful work, the remainder being dissipated in the atmosphere or condenser. Yet the performance indicated is better than will actually be realized in practice, on account of the disturbing causes mentioned above, which always have a more or less prejudicial influence, though to a less extent, in the present case, than in cases where the engine works at a high rate of expansion.

18. Since the useful work amounts to from $5\frac{1}{2}$ to $7\frac{1}{2}$ per cent., the remainder of the heat expended is from $94\frac{1}{2}$ to $92\frac{1}{2}$ per cent. of the whole. This remainder is conveniently called the "heat rejected," and appears in the condenser, where there is one, in the form of heat. When an injection condenser is used, this heat is employed in raising the temperature of the injection water up to the temperature of the condenser. Let θ be the rise of temperature, n the

number of pounds of injection water per lb. of steam, then each n pounds of that water abstracts from 1 lb. of steam $n\,\theta$ thermal units nearly, whence

$$n = \frac{\text{Heat rejected}}{\theta},$$

which determines the amount of condensation water per lb. of steam, from which it is easy to deduce the amount per indicated horse-power per hour. The annexed table shows the result of such a calculation for the same pressures as before, assuming a rise of temperature of 40°.

CONDENSATION WATER.

Pressure (absolute) p.	Heat rejected.		Lbs. of condensation water.	
	Th. units per lb.	Th. units per I.H.P. per 1′.	Per lb. of steam.	Per I.H.P. per 1′.
60	1023	577	25·6	14·4
30	1016	625	25·4	15·6
20	1013	658	25·3	16·4

It is here supposed that the feed water is (as usual) taken from the condenser, which, in the numerical calculations, was supposed, as before, to be at the temperature 104°; if this be not the case, let $\Delta\,t$ be the difference of temperature, then Δt must be subtracted from the heat rejected, when the temperature of the feed is lower than that of the condenser, and added if it be higher, as will be seen on referring to Art. 14. Again, when a surface condenser is used, the temperature of the condensing steam is higher than that of the condensation water, and unless the feed be taken from the condensed steam a correction will, strictly speaking, be necessary. Minute accuracy here, however, is only useful for the sake of practice in reasoning correctly on these

[redacted]

questions, since, in practice, ample margin must be allowed for leakage and contingencies.

Measurement of the quantity of heat given out in the condensation water of an engine furnishes an excellent test, which has of late been practically introduced, of its efficiency. I shall return to this in a later chapter; for the present it is sufficient to say that the difference between the heat expended, and the work done as shown by the indicator, a quantity which I have here called the heat rejected, and tabulated in the table last given, is nearly the same as the heat discharged from the condenser, differing from it only on account of piston friction and radiation, and the difference of temperature (if any) of the feed water from that of the water *entering* the condenser. (See Chapter XI.)

The size of cylinder required for a given power is expressed by the number of cubic feet which the piston must sweep through per 1′ for each indicated horse-power: this is easily obtained from the consumption of steam per hour, or directly by the formula

$$C = \frac{33000}{P - P_b} = \frac{229}{p - p_b}$$

varying inversely as the effective pressure of the steam. The actual dimensions of the cylinder will then be fixed by the piston speed and the stroke. The capacity of the feed pump (when single-acting) will be to the capacity of the cylinder in the proportion which unity bears to half the relative volume of steam at the boiler pressure, but allowance of course has to be made for leakage and contingencies.

20. The calculations made in Arts. 17, 18, pre-suppose that the boiler supplies dry steam; if this be not the case, then the total heat of evaporation, and the useful work done, must be calculated according to the method explained in Arts. 6 and 12. The general effect is that both the work done, and the heat expended, per lb. of steam, will be diminished, but the former in a greater proportion than the

latter; hence the number of pounds of steam per indicated horse-power per hour is increased, and the efficiency diminished, but the diminution of efficiency is but small, being occasioned solely by the fact that *all* the feed water must be raised up to the temperature of the boiler, though only part of it is afterwards evaporated. If, however, liquefaction take place after admission to the cylinder, then the work is diminished, while the heat expended remains unaltered, and the efficiency is diminished in precise proportion to the liquefaction.

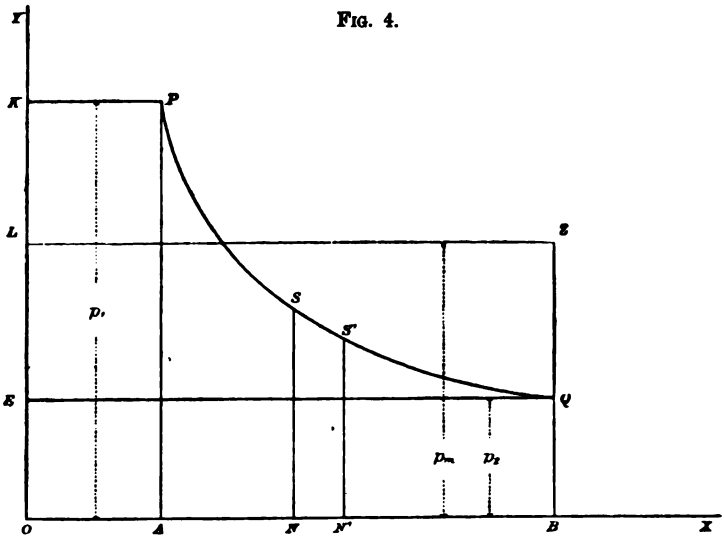
Work done during Expansion.

21. The calculations just made respecting the performance of an engine working without expansion show that some 40 lbs. of steam will be required per indicated horse-power per hour, and that in the most favourable circumstances hardly 7 per cent. of the heat expended will be utilized. An additional amount of work may, however, be obtained without increasing the heat expended in the same proportion, by cutting off the steam when a certain part of the stroke has been performed, and allowing it to expand, thus obtaining from each pound of the steam energy which would otherwise be uselessly dissipated in the condenser.

In the figure (Fig. 4) O X is a line on which are set off the volumes of the whole amount of steam shut up in the cylinder, the corresponding ordinates representing pressures which in the first instance are supposed expressed in pounds per square foot. O A represents the volume, and P A the pressure at the beginning of the expansion, that is to say, at the instant when the steam is cut off; while O B, B Q represent the same quantities at the end of the stroke. The cut-off is supposed instantaneous, and the pressure then falls regularly from P to Q; the ratio O B : O A is the ratio of expansion. At any intermediate point of the stroke the volume and pressure will have intermediate values O N, N S: the point S lying on the expansion curve P S Q, a

curve which is the same as that drawn by the pencil of a good ordinary indicator, when properly set, because the volumes of the steam are proportional to the spaces tra-

FIG. 4.



versed by the piston. It has already been shown that, if the pressure be constant, the work done in driving a piston is P V, where V is the volume swept out by the piston (Art. 17): take now two points N N′ very near together, then N N′ is the increase of volume of the steam, that is, the volume swept through by the piston as it advances through a small space, and therefore P.N N′ is the work done if P be supposed constant. But P may be made as nearly constant as we please by sufficiently diminishing N N′, and hence the area of the strip S N′ represents the work done during the small advance of the piston considered. Whence, dividing the whole area into similar strips, it appears that that area represents the whole work done, during the expansion, by the whole mass of steam shut up in the cylinder. By similar reasoning the rectangle A K represents the work done during admission, and the whole area K P Q B O con-

sequently represents the total work done by the steam during admission and expansion.

The work done in driving a piston is usually and very conveniently expressed by finding the *mean* pressure of the steam, that is, the pressure which, if it remained constant throughout the stroke, would do the same amount of work as actually is done. In the figure set up B Z so that

$$\text{Rectangle } OZ = \text{Area } OKPQB,$$

then B Z is the mean pressure required; it is usually denoted by P_m or p_m, according as it is given in pounds per square foot, or pounds per square inch, and called the "mean *forward* pressure," to distinguish it from the "mean *effective* pressure," $P_m - P_b$, found by subtracting the mean back pressure P_b. If the horse-power of the engine be calculated, by use of the mean forward pressure instead of the mean effective pressure, the result is what is sometimes called the *total* horse-power of the engine, as distinguished from the *indicated* horse-power.

22. The reasoning of the last article is supposed to be already familiar to every reader of this work, and is here repeated for the sake of pointing out that the question is one of pure mechanics, and not at all respecting the nature of the fluid employed, or the circumstances of the expansion. The result of the calculation depends solely on the form of the curve, and the total volume of the cylinder in which the expansion takes place.

When, however, we wish to discuss questions relating to efficiency by comparing work done with heat expended, it is indispensably necessary to know directly or indirectly not merely the work done by a given *volume* of the fluid, but the work done by a given *weight*, for simplicity 1 lb. For this purpose let N be the number of lbs. weight of steam contained in the cylinder: then the work done by 1 lb. will be $\frac{1}{N}$th the total amount, and the volume occupied by 1 "

will be $\frac{1}{N}$th the total volume; hence if N be known, a simple alteration of the scale on which volumes are measured will cause the diagram to represent the relation between volume and pressure, and the work done by 1 lb. of steam. It only remains to determine N, and that is to be done either by direct measurement of the consumption of steam by the engine, or by an independent knowledge of the state of the steam at some one point of the stroke. Suppose, for instance, that we know that, at the end of the stroke, the steam contains a given percentage of suspended moisture, then by Art. 6 the volume of 1 lb. of that steam can be calculated, and if the result be compared with the total volume of the cylinder, the value of N will manifestly be determined.

For the purposes of this preliminary investigation it will be supposed, in the first instance, that the steam is dry at the end of the stroke, a condition which ought always to be aimed at, for reasons which will afterwards appear, but which cannot generally be realized in practice, especially at high rates of expansion. The value of N is then found by dividing the whole volume of the cylinder by the specific volume (v) of dry steam at the known terminal pressure; and we may now take the diagram as representing the changes of volume and pressure of, and the work done by, 1 lb. of steam. In the second place, it will be supposed that the steam contains a given percentage of moisture at the end of the stroke, determined by considerations to be explained presently. The sole effect of this is to alter the scale on which volumes are measured, so that when the diagram represents 1 lb. of steam, it is smaller in the proportion $x : 1$ where x is the weight of pure steam in 1 lb. of the actual steam. The area of the diagram, and consequently the work done per lb. of steam, is then diminished in the same proportion. Experience tells us that the expansion curve does not differ wid[illegible] [illegible]mon rectangular hyperbola, the product of [illegible] volume re-

maining nearly constant. This implies (Art. 55) that the steam is initially moist and becomes drier and drier as it expands, an effect due mainly to the action of the sides of the cylinder, which condenses on its relatively cold surface a part of the steam admitted, which then forms a film of moisture on the sides of the cylinder, afterwards re-evaporated during the expansion. Without here entering on this question, I shall suppose that the curve is a common hyperbola, and leave other cases to be discussed in a later chapter.

23. In the common hyperbola P Q (Fig. 4) the rectangles O P, O Q are equal, while the area P Q B A is found by multiplying either rectangle by $\log._\epsilon r$, where r is the ratio of expansion O B ÷ O A, and the logarithm is of the kind called hyperbolic, from this very property of the hyperbola, found either by multiplying the common logarithm by 2·3026, or by a special table such as is given at the end of this book. Hence, taking first the case of dry steam, since O B is now v_2, the volume of dry steam at the known terminal pressure p_2, the area of the whole figure, or the energy exerted by each pound of steam, in driving the piston is

$$\text{Energy Exerted} = P_2 v_2 \{ 1 + \log._\epsilon r \}. \qquad (1)$$

The effective work is of course somewhat less, being found by subtracting from the foregoing result the quantity $P_b v_2$, where P_b is as before the back pressure, and consequently,

$$\text{Effective Work} = P_2 v_2 \left\{ 1 + \log._\epsilon r - \frac{p_b}{p_2} \right\}. \qquad (2)$$

Secondly, if the steam be moist at the end of the stroke, let $1 - x_2$ be the weight of moisture in 1 lb. of the steam, as in Art. 6, then by that Art., neglecting s (=·016) the specific volume is $x_2 v_2$, and hence the energy exerted on the piston is

$$\text{Energy Exerted} = P_2 x_2 v_2 (1 + \log._\epsilon r), \qquad (3)$$

while the effective work becomes

$$\text{Effective Work} = P_2 x_2 v_2 . \left\{ 1 + \log._\epsilon r - \frac{p_b}{p_2} \right\}. \qquad ($$

[redacted]

The greater the expansion, other things being equal, the greater the amount of work done by the steam, until it is carried so far, that the terminal pressure has fallen to the back pressure; in that case the expansion may be said to be *complete*. The effective work done per lb. in complete expansion is evidently $P_2\, x_2\, v_2,\ \log._e r$ where x_2 is unity when the steam is dry. In practice, the prejudicial action of the sides of the cylinder, and other causes, render a moderate amount of expansion preferable, as will be seen hereafter.

The mean forward pressure in hyperbolic expansion is given (whatever be the state of the steam) by the formula

$$p_m = p_1 \cdot \frac{1 + \log._e r}{r},$$

where $p_1 = r\, p_2$ is the initial pressure.

Expenditure of Heat in an Expansive Engine.

24. The heat expended in an engine is, of course, all primarily employed in the evaporation of water in the boiler, but it is nevertheless not all used in the same way. When a steam jacket is used a part of the steam is condensed in the jacket, and the steam so used represents heat expended, although none of it passes through the engine and does work. Again, without anticipating what will be said in a subsequent chapter respecting the action of the sides of the cylinder, it is clear that heat will be transmitted to the exhaust steam as it leaves the cylinder, and that heat will be radiated continually to external bodies by the hot cylinder. Of these two causes of loss of heat the last is comparatively unimportant, when the cylinder is well clothed; but the first being chiefly due to evaporation of a film of moisture deposited on the internal surface of the cylinder, is frequently great. I shall therefore call the loss of heat occasioned in these ways the "EXHAUST WASTE."

The cylinder, then, is continually being cooled by the exhaust waste, and heated by the steam iacket, if there is one;

and the difference between the two must necessarily be subtracted or added to the steam, during its passage from the boiler to the end of the stroke, according as the exhaust waste, or the supply of heat by the jacket, is the greater. Hence it is not sufficient to consider the heat expended on the working steam during evaporation in the boiler, but we must also consider the heat added or subtracted from that steam during its passage from the boiler to the end of the stroke. Now if the state of the steam at the end of the stroke be known, it will be possible to find the heat so added or subtracted, and thus the heat supplied by the jacket over and above the exhaust waste will be known; and, conversely, if the heat added or subtracted be known, it will be possible to find the state of the steam at the end of the stroke.

25. In the first place, suppose that the steam is dry at the end of the stroke; then if that steam were formed in a boiler by evaporation at constant pressure the heat expended would be simply the total heat of evaporation (as defined in Chapter I.) *from* the temperature of the feed water *at* the temperature corresponding to the given pressure. The external work then done would be $P_2 v_2$, and the heat-equivalent of that work is included in the total heat of evaporation.

But now the steam is formed in a much more complicated way. It is evaporated in the boiler at a much higher pressure than that at which we find it at the end of the stroke; it passes from the boiler to the cylinder and expands, while, as it does so, heat is added or subtracted. During this process external work is done by it, in driving the piston, represented by $P_2 v_2$, the heat-equivalent of which must form part of its total heat of formation. Hence (comp. Art. 13) its actual total heat of formation must be greater than the total heat of evaporation at P_2 by the heat-equivalent of the difference between $P_m v_2$ and $P_2 v_2$. We must therefore have

$$\text{Total Heat of Formation} = H_2 - h_0 + (P_m - P_2) v_2,$$

[redacted]

where $H_2 - h_0$ is, as in Chapter I., the total heat of evaporation of water *from* t_0 *at* t_2 and $(P_m - P_2)\, v_2$ is expressed in thermal units. Or if Q be the total heat of formation,

$$Q = H_2 - h_0 + \left(\frac{p_m}{p_2} - 1\right) P_2 v_2,$$

where $P_2 v_2$ is given in thermal units by Table IV*a*.

The same results may be reached by comparing this case with the case of a non-expansive engine, working with the same *terminal* pressure, when it will be seen that, although the circumstances under which the steam is *formed* are different, the circumstances under which it is *condensed* are identical. To fix our ideas, imagine an engine working at 60 lbs. pressure (absolute), and cutting off at one-tenth, assuming the common law of expansion, the terminal pressure is 6 lbs. per square inch, and the steam is dry by hypothesis, therefore, the cylinder at the end of the stroke is full of dry steam, of pressure 6 lbs. per square inch, which, when the exhaust is opened, rushes out into the condenser and is there condensed.

Now compare this with the case of an engine, working without expansion, at the pressure 6 lbs. per square inch with the same back pressure, a case which, though not occurring in practice, we are entitled to assume for our purpose; then at the end of the stroke, in this case, also we have a cylinder full of dry steam at pressure 6 lbs. per square inch, which rushes into the condenser when the exhaust is opened, and is there condensed. Clearly it is impossible to distinguish the two cases. They are identical, and the reader who has carefully considered Chapter II. will perceive that we are entitled to conclude that the heat *rejected* is the same in the two cases; there is, in fact, no possible reason for any difference.

The heat expended, however, is not the same, because the steam is formed by a different process in the two cases; in the first case it is generated in the boiler at 60 lbs., passes

into the cylinder, and there expands till its pressure has fallen to 6 lbs., while in the second case it is generated in the boiler at 6 lbs. But since we know the heat rejected by the rule previously given for a non-expansive engine, we can find the heat expended in the expansive engine by simply adding to that rejected heat the useful work done, for the two together make up the whole heat expended.

Using the same notation as before (comp. Art. 18), the heat rejected is given by

$$R = H_2 - h_0 - (P_2 - P_b)\, v_2,$$

while the useful work done is $(P \;\; - P_b)\, v_2$: adding which two together we find as before,

$$Q = H_2 - h_0 + (P_m - P_2)\, v_2.$$

This general formula is applicable to any case in which the steam is dry at the end of the stroke, and from it we can find how much heat is to be supplied to the jacket, independently of the exhaust waste, which for dry steam is probably small. For $H_1 - h_0$ is the heat supplied in the boiler to each pound of the working steam, assuming the boiler to supply dry steam; therefore if J be the jacket heat per lb. of working steam,

$$J = H_2 - H_1 + (P_m - P_2)\, v_2$$

$$= P_2 v_2 \left(\frac{p_m}{p_2} - 1\right) - \cdot 305\,(t_1 - t_2).$$

By division by $H_1 - h_0$ we find how much steam must be supplied to the steam jacket besides that required to provide for the exhaust waste, in which is included the waste heat of the liquefied steam discharged from the jacket.

Whatever the form of the expansion curve, this must be true, but if we suppose, as before, that curve to be an hyperbola, we get

$$J = P_2\, v_2 \,.\, \log._e r - \cdot 305\,(t_1 - t_2),$$

which will give the required results, in conjunction with formula (2), of Art. 22, for any ratio of expansion r: taking $P_s v_s$ in thermal units from Table IV*a*, and remembering that $t_1 - t_2$ is the difference of temperature at the beginning and end of the expansion found by Table I*a* from the corresponding pressures.

PERFORMANCE OF AN EXPANSIVE ENGINE. CASE I.

Expansion.	Effective work per lb. in th. units.	Lbs. of Steam per I.H.P. per hour.			Heat expended per I.H.P. per 1′.	Efficiency.
		Working.	Jacket.	Total.		
None.	79·0	32·5	0	32·5	594	·072
$r = 2$	126·3	20·3	·73	21·0	382	·112
$r = 5$	180·1	14·2	1·15	15·3	280	·153
$r = 9$	206·7	12·4	1·33	13·7	250	·171
$r = 13$	218	11·8	1·44	13·2	242	·177
Complete.	228	11·2	1·77	13·0	236	·181

Remarks.—Condensing engine, initial pressure 95 lbs., back pressure 3 lbs. absolute. Steam dry at the end of the stroke. Exhaust waste neglected.

The annexed table has been calculated to show the performance of a condensing engine of initial pressure 95 lbs. per square inch, working at various rates of expansion, with back pressure 3 lbs. The results show that the efficiency may be more than doubled by expanding the steam five times, and still further increased, though only to a small extent, by a further increase in the expansion. Not all the gain here shown can be realized in practice, chiefly on account of the action of the sides of the cylinder, which occasions greater and greater loss as the ratio of expansion is increased; till, at a certain limit, there is no longer any advantage but a positive loss in expansion. The reason of this cannot be discussed here. (See Chapter X.)

26. The amount of heat indicated by the table as necessary to be supplied to the steam, together with that required for the exhaust waste, is so considerable, that the steam jacket will, in most cases, be unable to supply it, and, in that case, the steam cannot be dry at the end of the stroke, but must contain a certain portion of moisture, either distributed over the internal surface of the cylinder or suspended throughout the whole mass of steam. In the latter case it will rush out into the condenser with the exhausting steam, but in the former it will remain on the internal surface of the cylinder and piston during the exhaust, and will be evaporated by heat abstracted from the cylinder. Thus the exhaust waste will in general be much increased when the steam is wet at the end of the stroke, and will increase the difficulty of supplying enough heat by the jacket. I shall now consider two cases in which the steam is wet. First, I shall imagine the cylinder jacketed, but that the jacket supplies only just enough heat to provide for the exhaust waste, so that the steam obtains none, and expands as in a non-conducting cylinder, except that the expansion curve is still supposed approximately an hyperbola. This case sometimes occurs in practice.

The steam is then moist at the end of the stroke; let the dryness-fraction be x_2, then the total heat of evaporation of the steam in its terminal state is by Chapter I.,

$$\text{Total Heat of Evaporation} = h_2 - h_0 + x_2 L_2,$$

of which $P_2 x_2 v_2$ is due to external work: while during formation, in the present case, by Art. 21, the work done in driving the piston is $P_m x_2 v_2$, hence the total heat of formation must be

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) x_2 v_2 .$$

Now in the present case the steam obtains no heat during its passage from the boiler to the end of the stroke, and

[redacted]

consequently its total heat of formation must be equal to the heat supplied in the boiler, therefore

$$H_1 - h_0 = h_2 + x_2 L_2 - h_0 + (P_m - P_2) x_2 v_2,$$

or remembering that the expansion curve is supposed an hyperbola,

$$H_1 = h_2 + x_2 L_2 + x_2 P_2 v_2 . \log._e r,$$

an equation which enables us to find out how much steam is liquefied at the end of the stroke: for

$$x_2 = \frac{H_1 - h_2}{L_2 + P_2 v_2 . \log._e r}.$$

Having found x_2, the useful work done can now be found by formula (4) Art. 23, and the efficiency is deduced by dividing by $H_1 - h_0$.

The annexed table shows the results of this calculation under the same circumstances as in the preceding example.

PERFORMANCE OF AN EXPANSIVE ENGINE. CASE II.

Expansion.	Wetness.	Lbs. of Steam per I.H.P. per hour.	Heat expended per I.H.P. per 1'.	Efficiency.
None.	0	32·5	594	·072
$r = 2$	·04	21·2	385	·112
$r = 5$	·08	15·5	281	·152
$r = 9$	·1	13·8	250	·171
$r = 13$	·12	13·3	243	·176
Complete.	·15	13·2	240	·178

Remarks.—Condensing engine, initial pressure 95 lbs., back pressure 3 lbs. absolute. Jacket just supplies the exhaust waste, which is not included.

It will be seen that the results are nearly the same as in the preceding case: the loss by liquefaction being almost exactly compensated by the circumstance that the steam considered all passes through the cylinder instea[illegible]g partly used

in the jacket, or we may express it roughly by saying that the liquefaction takes place in the cylinder instead of in the steam jacket. But no doubt in the second case the exhaust waste is greater than in the first case, and this has to be provided for by jacket steam.

27. Although the two preceding cases may possibly occur in practice, yet it can hardly be doubted that it is much more common that the exhaust waste is greater than the jacket supply. In this case the difference is abstracted from the steam during the passage from the boiler to the end of the stroke, and it is obvious that, in non-jacketed cylinders, this must always be so.

I shall now consider a particular case of this kind by supposing that the exhaust waste is greater than the jacket supply by 20 per cent. of the whole heat expended in the boiler. This proportion is far from unusual in practice, in non-jacketed cylinders. Then, all that heat is abstracted from the steam during its passage from the boiler to the end of the stroke: a circumstance which is expressed algebraically by the equation,

$$\tfrac{4}{5}(H_1 - h_0) = Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) x_2 v_2,$$

the notation being as in the preceding article. Whence we get an equation for x_2, replacing as before $(P_m - P_2) v_2$ by $P_2 v_2 \log._e r$; namely,

$$x_2 = \frac{\tfrac{4}{5}(H_1 - h_0) - (h_2 - h_0)}{L_2 + P_2 v_2 . \log._e r},$$

from which may be found the amount of water mixed with the steam at the end of the stroke, and the useful work done may be calculated as before. The results are given in the annexed table for the same data as in the two preceding cases. The steam is now very wet at the end of the stroke, as is shown in column 2, and the performance is much inferior on account of the exhaust waste being inclu

PERFORMANCE OF AN EXPANSIVE ENGINE. CASE III.

Expansion.	Wetness.	Lbs. of Steam per I.H.P. per hour.	Heat expended per I.H.P. per 1′.	Efficiency.
None.	·243	42·9	782	·055
$r = 2$	·264	27·6	503	·085
$r = 5$	·285	19·9	362	·118
$r = 9$	·291	17·4	319	·134
$r = 13$	·312	16·9	308	·139
Complete.	·323	16·7	303	·141

Remarks.—Condensing engine, initial pressure 95 lbs., back pressure 3 lbs. absolute. Not jacketed: exhaust waste assumed 20 per cent.

The assumption of a waste of 20 per cent. is of course an arbitrary one introduced to fix our ideas. As a matter of fact, the waste is in general greater at high rates of expansion than at low ones, for reasons which will appear hereafter. So also the expansion curve is not always an hyperbola; but at low expansion, the pressure falls quicker, and at high expansion slower, than is indicated by a hyperbolic curve.

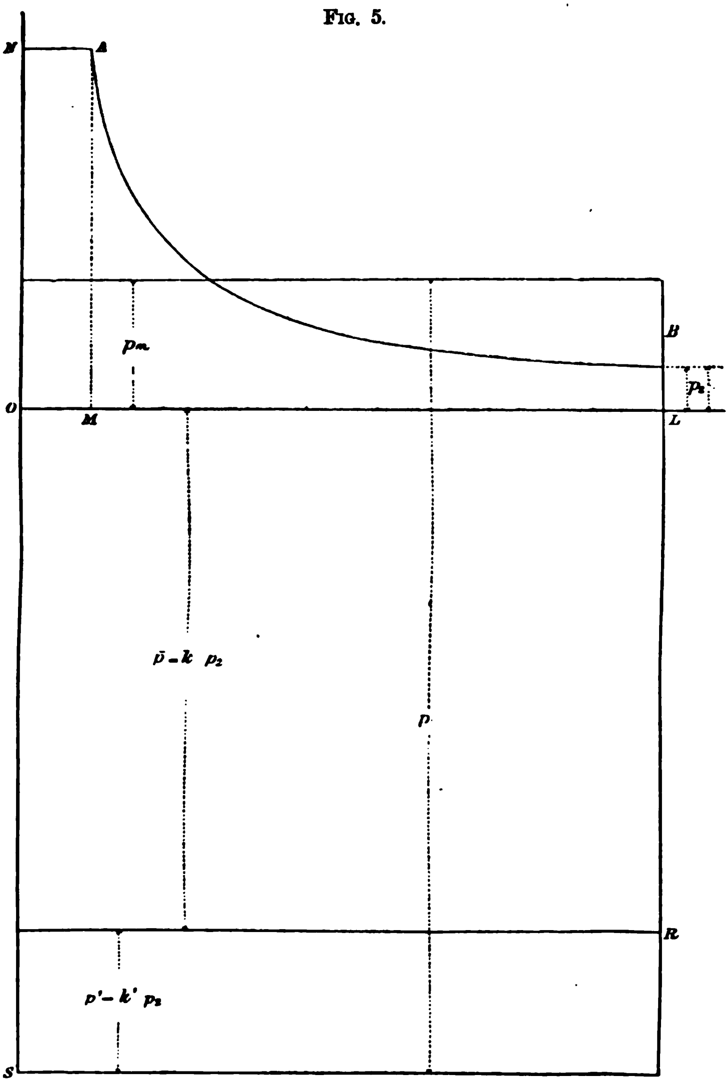
In the present case, the consequence of a uniform curve and a uniform percentage of waste being supposed at all rates of expansion, has been that the consumption of steam in Case III. has been increased, in the fixed proportion of 25 per cent., approximately, as compared with Case II., the advantage of expansion remaining the same, but this will of course not hold good in practice.

Graphical Representation of the Heat Expended. Equivalent Pressure.

28. The graphical method, explained in the last chapter, of representing the internal work done during the evaporation of water under constant pressure, may with great advantage be extended to the present case.

In the figure (Fig. 5), N A B L O represents the expansion diagram, showing, as in Art. 21, the energy exerted by the

Fig. 5.



steam in driving the piston, during admission, and expansion according to any law from volume O M to volume

pressure at the same time falling from A M to B L. Then it was shown that the internal work done during the formation of dry steam, at the terminal pressure p_2, could be represented by the area of a pair of rectangles, the base of which is O L the volume of the steam, and the heights $k p_2$, $k' p_2$ respectively, k k' being numbers calculated for any pressure and temperature of feed by easy rules. (See Arts. 9 and 10.) Or the heights of the rectangles themselves may be found without a previous determination of k k', as explained in detail in the same articles. In the figure these rectangles are O R and R S respectively.

Now, as has been sufficiently explained, the energy expended in internal changes, in forming steam of given quality, is always the same, and to find the total heat of formation we have only to add the energy exerted on external bodies during the process of formation. But this energy is represented by the area of the expansion diagram whatever the form of the curve may be, and thus it is clear that the heat expended must be represented by the area of the whole figure N A B L S, made up by the rectangles and the curve.

Moreover, it was shown that the internal work might be represented to our minds as equivalent to overcoming an ideal pressure on the piston, which we called the "internal-work-pressure," the said pressure being represented by $(k + k') p_2$. Now the energy exerted is equivalent to overcoming a mean pressure p_m on the piston where p_m as usual is the mean forward pressure, and consequently the heat expended must be represented by a pressure p_h on the piston given by

$$p_h = p_m + (k + k')p_2 .$$

This pressure is what Rankine called the "pressure equivalent to the expenditure of heat," but for brevity it may be also called the "heat-pressure"; it may always be calculated by the preceding formula whatever be the treat-

[redacted]

ment of the steam on its way from the boiler to the cylinder, provided the steam be dry at the end of the stroke. If it be not dry, then the same formula serves with the same value of k, but a modified value of k' as explained in Art. 12 of the last chapter. Two examples will now be given of the calculation of p.

First let us suppose

$$p_2 = 5 \quad : \quad t_0 = 90^\circ,$$

being data which might frequently occur in condensing engines.

Here the temperature corresponding to p is found by Table Ia to be 162°, and we must calculate

$$k = \frac{\rho}{\mathrm{P}u},$$

taking the values of ρ and Pu from Table IVa, whence we have

$$\rho = 933 \qquad\qquad \mathrm{P}u = \cdot 67 \cdot 7$$
$$\therefore k = 13 \cdot 78.$$

Next, to find k' we take the formula

$$k' = \frac{t - t_0}{\mathrm{P}u} = \frac{162 - 90}{67 \cdot 7} = \frac{72}{67 \cdot 7} = 1 \cdot 06$$
$$\therefore k + k' = 14 \cdot 84.$$

Again, let

$$p_2 = 25 \quad : \quad t_0 = 62^\circ,$$

being data which might occur in a non-condensing engine in which no special provision was made for heating the feed. Then

$$t_2 = 240^\circ \; : \; \rho = 871 \; : \; \mathrm{P}u = 74 \cdot 6 \; : \; t_2 - t_0 = 178,$$
$$\text{hence} \quad k = 11 \cdot 68 \; : \; k' = 2 \cdot 37,$$
$$\therefore k + k' = 14 \cdot 05.$$

These examples show that $k + k'$ does not vary very much under a great variety of circumstances; hence Rankine recommended the formulæ

$$p_k = p_m + 15\,p_2 \text{ (condensing)},$$
$$p_k = p_m + 14\,p_2 \text{ (non-condensing)},$$

[redacted]

and no formulæ equally simple and general have in fact hitherto been given. It must always, however, be remembered that the exhaust waste is not included, and that hence the results obtained are too small unless in the (probably) exceptional cases in which the steam is dry at the end of the stroke: also it is better to calculate $k + k'$, or use the direct method of construction, than to trust to the average values given.

If the steam be known to be wet at the end of the stroke, then k' is replaced by $\frac{k'}{x_2}$ where x_2 is the corresponding dryness-fraction, but the modification so introduced is not important. The principal difficulty in the application lies in the determination of p_2 which ought to be known with accuracy; whereas in general it can only be determined to a rough degree of approximation. An example will now be given in which the data are taken from an experiment on the *Dexter*, a non-jacketed engine indicating about 220 horsepower with speed of piston of 366 feet per minute as follows:

Terminal Pressure	= 16·87,
Temperature of Feed	= 114°,
Mean Effective Pressure	= 37·54,
Mean Back Pressure	= 3·65.

The value of $k + \frac{k'}{x_2}$ here is 14·2, the value of x_2 being about ·72, whence

$$p_1 = 41\cdot2 + 14\cdot2 \times 16\cdot87 = 281 \text{ nearly.}$$

Dividing the mean effective pressure by 281, we get for the efficiency,

$$\text{Efficiency} = \cdot134.$$

The thermal equivalent of 1 horse-power is 42·75, whence dividing by ·134 we get 319 thermal units per I.H.P. per 1′, approximately, as the expenditure of heat. This result requires a small correction for clearance, but is enough

to show a very considerable exhaust waste, the actual expenditure of heat being probably more than 400 thermal units per indicated horse-power per 1′. (See Chapter XI.)

The method followed in the present article is capable of great extension, but I must postpone further consideration of the numerous and important questions relating to expansion till the chapter specially devoted to that purpose.

The other calculations relating to an expansive engine are just the same as in a non-expansive engine working at the same terminal pressure. (See Arts. 18 and 20.)

Expansion in General.

29. The work done by steam during expansion has been already considered in Art. 21, so far as concerns the simple case of a single cylinder within which the mass of steam is wholly contained. It is, however, essential to consider the question from a more general point of view, in order to be able to deal with cases of a more complicated character.

Instead of supposing the steam to be shut up in a cylinder behind a piston, the motion of which causes changes of volume, let us imagine the steam or other elastic fluid to be contained in a bag of any size or shape which expands, retaining or not its original form as the case may be. Then considering, first of all, a very small increase of size and change of form, it is clear that the surface of the bag may be supposed divided into an indefinite number of small portions, each of which may be conceived to be the base of a small cylinder, in which the corresponding portion of the surface moves as a piston when the bag becomes a little larger. Hence, whatever the size or shape of the bag, if P be the pressure, V the volume through which the surface sweeps, or in other words the increase of volume of the bag, P V will be the work done in overcoming a constant pressure P. If P be variable, then if we set off on a horizontal line abscissæ to represent volumes, and ordinates to

represent the corresponding pressures, it appears by reasoning similar to that of Art. 21, that the area of the expansion curve represents the work done by the expanding fluid, just as in the particular example of a cylinder and piston. Accordingly that work depends solely on the changes of volume and pressure, and not at all on the changes of shape which the bag may have undergone. Again, when a fluid expands, the internal changes it undergoes are sometimes a source of energy, by means of which the whole or a part of the external work is done, and sometimes require a supply of energy in order to produce them. In the first case, the difference between the energy and the work is abstracted from, or supplied to, the expanding fluid in the shape of heat, according as the energy or the work is the greater; while, in the second case, heat enough must be supplied from without to do both the internal and the external work. In neither case is there any reason to believe that change of shape (unless made with very extreme rapidity) can have any sensible influence on the internal work done or the intrinsic energy exerted during a given change of pressure and volume, and consequently it follows that the form of the expansion curve depends solely on the nature of the fluid and the amount of heat (if any) which it receives during expansion. This will be illustrated fully in the case of air and steam in a later chapter; for the present I content myself with the very important conclusion that—

The work done by a given quantity of expanding fluid does not depend in any way on the particular machinery by means of which the expansive energy is utilized.

Thus when 1 lb. of water is forced into the boiler and evaporated, the resulting steam, expanded, exhausted, and finally condensed, the work done by it does not depend on the number of cylinders through which it passes during the series of changes it undergoes, but simply upon the pressure

of admission, the ratio of expansion, and the amount of heat it receives during the process.

In a compound engine it frequently happens, that the series of changes undergone is very complex; the steam is admitted to the high-pressure cylinder during a part of the stroke, is then cut off, and expands till the stroke is completed; on release, instead of passing at once to the condenser, it is exhausted into a second cylinder, either at once or through an intermediate reservoir, and only reaches the condenser after a complicated series of changes of pressure. Nevertheless, if our object is merely to find the power of the engine, we have no occasion to consider these changes at all: we have merely to discover how much steam is used and how many times it expands.

Let the large cylinder or cylinders, if there be more than one, be n times the small cylinder, and let the steam be cut off in the small cylinder at $\frac{1}{r}$th part of the stroke, then it is clear that the volume finally occupied by the steam, immediately before exhaust, is to the original volume, in the ratio $n\,r : 1$; therefore if R be the number of times the steam expands,

$$R = n\,r,$$

and all the calculations for work done, and heat expended, per lb. of steam, are to be conducted just as if it were used in a simple engine with ratio of expansion R.

The weight of steam used depends upon the size of the large cylinder or cylinders alone, because at every stroke the volume of steam discharged is that of the large cylinder, and hence *the power and efficiency of a compound engine are (other things being equal) the same as if the steam were used in the large cylinder alone with the same total expansion.*

The addition of a high-pressure cylinder, then, has, in itself, no influence on the power or the efficiency of the

engine; it is merely a device for partially overcoming some of the difficulties which attend the use of high grades of expansion.

The foregoing statements, however, must be understood with certain qualifications as expressed by the saving clause, "other things being equal." When steam is "wire-drawn," or when communication is opened between vessels containing steam of different pressures, a part of the expansive energy of the steam is employed in generating violent motions in the particles of the steam itself; which motions are ultimately destroyed by fluid friction, and the corresponding kinetic energy transformed into heat. This energy is nearly all lost, so far as any useful purpose is concerned, and consequently the steam does not do all the work which it might have been made to do by a different arrangement. Since unbalanced expansion occurs to a greater extent in a compound engine than in a simple engine, the compound engine is at a disadvantage in this respect; the actual increase of efficiency generally produced by compounding being chiefly due to a diminution in the prejudicial influence of the sides of the cylinder, so that the exhaust waste is greatly diminished.

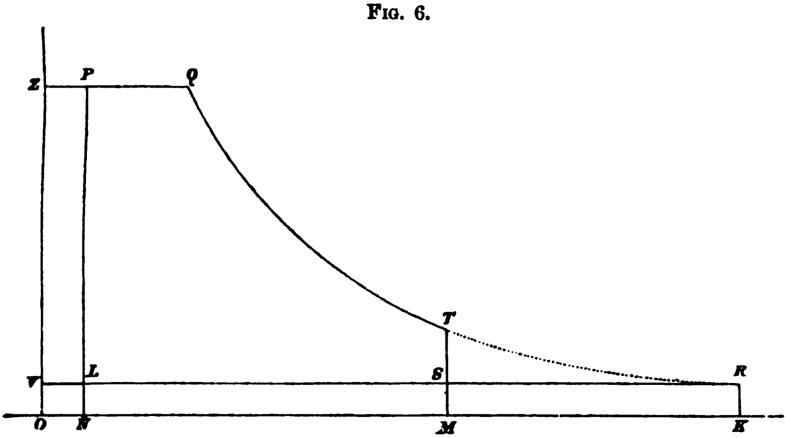
30. The construction of an indicator, and the general nature of the diagram obtained by its use, are supposed to be already familiar to every reader of this work. Suffice it to say that, when properly set, the motion of the drum precisely corresponds with the motion of the piston, so that the diagram drawn on the card shows the pressure of the steam at any point of the stroke: during the forward stroke in front of the piston, and during the backward stroke at the back of the piston. In practice, the diagram is generally considered as showing the total effective pressure urging the piston forwards at each point of the stroke; it being usual to take a diagram from each end of the cylinder and combine the upper half of each with the lower half of the other; thus

obtaining figures often called "true" indicator diagrams, which show the true driving force on the piston.

No doubt, if the object is to draw a true curve of crank effort when studying the variation of the twisting moment on the crank shaft, or the fluctuation of energy of the engine, this course must be adopted; but, for the purposes of a theory of the steam engine, the indicator diagram may be looked at from an entirely different point of view; it may, or rather must, be regarded as the graphic representation of the series of changes undergone by the feed water in the course of its transformation into steam, passage through the engine, and subsequent condensation.

In the figure (Fig. 6), suppose O N to represent the volume of a small quantity of water contained in a bag

Fig. 6.



according to the conception of the preceding article, and let P N represent the boiler pressure: then the vertical line P N represents the rise of pressure which takes place as the water is forced into the boiler by the action of the feed pump. Next, neglecting wire-drawing, the horizontal line P Q represents the gradual increase of volume of the bag as the water within it gradually evaporates and finally becomes

all steam. Imagine the steam suddenly cut off when the bag is wholly within the cylinder, then the bag gradually expands, as represented by the curve Q T, till precisely at the end of the stroke (suppose) the exhaust opens.

When the exhaust is opened, the bag undergoes a sudden expansion as the steam rushes into the condenser, followed by an almost coincident contraction as the steam within it is condensed. The process is not exactly the same for all particles of steam, but may be sufficiently nearly represented by the dotted expansion curve T R, and the horizontal line of condenser pressure L S R. During this expansion the only work the bag does is in overcoming the condenser pressure, the excess of expansive energy being employed in generating kinetic energy, ultimately reappearing as heat in the condenser. During condensation, energy is exerted by the steam piston in compressing the bag, which energy, after allowing for the work done by it in the sudden expansion, is represented by the rectangle S N. Hence, referring to Art. 21, the effective energy exerted by the bag, which has now returned to its original state, is represented by the area P Q T S L, being the difference between P Q T M N, the energy exerted *on* the piston, and S N, the energy exerted *by* the piston.

The figure thus drawn, representing the changes of volume and pressure of the bag, may be called the diagram of energy of a lb. of steam. If clearance and wire-drawing be neglected, the figure is identical with that drawn by an actual indicator, except that the actual indicator diagram includes the very small rectangle O P, representing, as previously shown, the work done by the feed pump (Art. 17).

The effect of clearance and wire-drawing is that all particles of steam do not go through the same series of changes; some are never condensed at all; some are more and some less wire-drawn; there are, therefore, many distinct diagrams, each representing its own particle of

steam. The figure drawn by the indicator may be regarded as representing the average changes undergone by the steam, as will be seen more clearly from the chapter (Chapter IX.) devoted to this part of the subject. In a compound engine the figures drawn by the indicator, when combined by the well-known process, show the changes of state of the steam, care being taken that the figures combined are taken from corresponding ends of the two cylinders, so as to show the changes of the same mass of steam; but while the steam is passing into the low-pressure cylinder, before it is cut off in that cylinder, it must be remembered that the weight of steam in the low-pressure cylinder does not remain constant, nor does it usually vary in proportion to the volume swept out as during admission to a simple cylinder. The subject is too large and complicated to be conveniently dealt with in the present chapter.

[redacted]

CHAPTER IV.

PHYSICAL PROPERTIES OF THE PERMANENT GASES.—AIR ENGINES.

31. It has been shown in the last chapter that, in a steam engine, even when working under very favourable conditions, less than one-fifth of the whole heat expended is transformed into mechanical energy, and it is natural to inquire into the causes of this unfavourable result to discover, if possible, whether those causes are removable, either in the case of a steam engine or by the employment of some other kind of heat engine. A full answer to this question will be given in the next chapter, but a preliminary study is necessary of a heat engine of a much simpler character, namely, the simplest kind of air engine. It is not, however, in this work intended to discuss air engines, except in the very simple elementary form, which is all that is needful for our purpose.

The simplicity of air engines, from a theoretical point of view, lies in the fact that the constitution of air and other permanent gases is incomparably simpler than that of any other body in nature; all permanent gases (that is, gases which cannot be liquefied by cold and pressure) being subject approximately to certain very simple laws of which it will be necessary, before proceeding farther, to give a brief summary, referring for fuller information to Professor Clerk Maxwell's 'Theory of Heat.'

*First law.** *The product* P V *of the pressure* P *and volume* V *is a constant quantity when the temperature remains constant.*

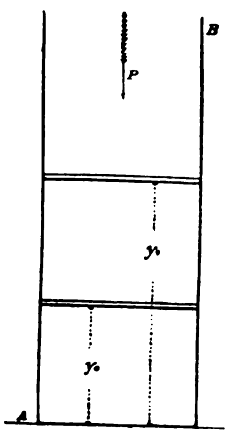
* Le-

The value of this constant is known with great accuracy for different gases by the experiments of Regnault, for certain standard temperatures, and more especially at 32° Fahr. being the temperature of melting ice. For air, if P_0 be the standard pressure of the atmosphere, here taken as 14·7 lbs. per square inch, or 2116·8 lbs. on the square foot, V_0 the volume of 1 lb., found by direct experiment,

$$P_0 V_0 = 26{,}214 \text{ ft. lbs.}$$

For any other value of P the same value of P V should be preserved, according to the first law, so long as the temperature remains that of melting ice, and in fact, though the deviations are perceptible, they are very small except at extreme pressures.

FIG. 7a.



If the temperature be altered, however, the value of this product will alter; two cases of which, amongst others, may be specially noticed.

(1) Let the air be enclosed in a cylinder A B (Fig. 7*a*), beneath a loaded piston which originally, when the air is at 32°, stands at a height y_0 above the bottom. Now let heat be applied, the load on the piston remaining equivalent to a given constant pressure P, then the temperature of the air rises, and at the same time the air expands, causing the piston to rise. Suppose heat to be continually applied till the temperature is that at which water boils under the standard atmospheric pressure, and let the piston then have risen to the height y_1. Then clearly since the pressure is the same, that is $P_1 = P_0$,

$$\frac{P_1 V_1}{P_0 V_0} = \frac{y_1}{y_0}.$$

(2) Let the air be enclosed in a vessel of invariable volume and let heat be applied to it, then the pressure

increases, as heat is applied, and the temperature rises from 32° to 212°. Since the volume is constant we shall have in this case

$$\frac{P_1 V_1}{P_0 V_0} = \frac{P_1}{P_0},$$

so that if P_1 be determined by experiment the ratio of products will be known.

Hence the ratio of products may be determined by observing either the ratio $\frac{P_1}{P_0}$ or the ratio $\frac{y_1}{y_0}$, and if the first law were strictly true the result of the two experiments ought to be identical. This is, in fact, very nearly the case, and, moreover, the result is found to be very approximately the same if any other permanent gas be used in place of atmospheric air—a fact included in the

*Second law.** *Under constant pressure all gases expand alike.*

That is to say, in the first operation supposed above, if the temperatures, initially and finally, be any whatever, instead of 32° and 212°, the ratio $\frac{y_1}{y_0}$, or, by the first law, the ratio of the products P V, will be the same for all gases.

It must be repeated that these two laws are only approximations for actual gases, but the deviations are so small that we are justified in regarding them as essential characteristics of a *perfectly* gaseous body, and the deviations may be considered as caused by actual gases being more or less imperfect. In an absolutely perfect gas, between the temperatures 32° and 212°, the ratio is probably 1·3654, a value which is exceeded in actual gases by quantities, which in permanent gases are very minute, but are more considerable in liquefiable gases, and which become less as the gases are rarefied. (See Appendix.)

Since at different temperatures the product P V has different values, it, evidentl[illegible] as a measure

* Law of Ch[illegible]

of temperature. Temperature is ordinarily measured by a mercurial thermometer, that is, by the expansion of mercury in a glass tube, but there is no reason, except that of mere convenience, why this should be so; a bar of iron might also be used by observing the changes of length which take place when heat is applied. Now the choice of the product P V has this advantage, which is at once obvious—namely, that the measurement is (by the second law) independent of the kind of gas employed; while in other thermometers—unless specially graduated by reference to a standard instrument—the indications are different for each different thermometer; hence, when measured by the expansion of a perfect gas, temperature is said to be "absolute," an expression which hereafter will be justified by much more cogent reasons (see Art. 44). Assuming temperature to be measured in this way, let t be a temperature reckoned on Fahrenheit's scale, then manifestly

$$\begin{aligned} PV &= P_0 V_0 + \frac{\cdot 3654}{180}(t-32) . P_0 V_0 \\ &= P_0 V_0 . \frac{180 + \cdot 3654\,(t-32)}{180} \\ &= \frac{P_0 V_0}{492 \cdot 6}(460 \cdot 6 + t). \end{aligned}$$

We can now make this simpler by measuring temperature, not from the purely arbitrary zero used by Fahrenheit, but from a zero $460^\circ \cdot 6$ lower, so that if T be the absolute temperature,

$$T = 460 \cdot 6 + t$$

$$\therefore\ PV = \frac{P_0 V_0}{492 \cdot 6} . T.$$

The zero in question is called the "absolute zero," and according to the experiments of Joule is $460 \cdot 66$ below the zero of Fahrenheit's scale; in all our subsequent work we shall adopt the nearest whole number—viz. 461, and write

$$PV = \frac{P_0 V_0}{493} . T,$$

[redacted]

where T is measured from a zero 461° below the zero of Fahrenheit's scale. Rankine adopted 461·2, corresponding to 274° centigrade, while continental writers usually adopt 273° centigrade.

A thermometer of this kind gives results not differing materially at ordinary temperatures from those of a mercurial thermometer, a fact usually included, by implication, in statements of the second law. When great accuracy is required in the measurement of temperature, an air thermometer is actually used under one of the two forms mentioned above.

A perfect gas, then, is represented by the formula

$$PV = cT,$$

where c is a number depending on the density of the gas at 32°; for atmospheric air the value of c is about 53·2,

$$\therefore PV = 53{\cdot}2 \, . \, T$$

represents the properties of a perfect gas, the density of which at 32° is the same as that of air, and when in the course of the present and succeeding chapters "air" is mentioned, it is to be understood that such a perfect gas is intended, unless otherwise expressly mentioned. Actual air when perfectly dry differs from such a gas by very small quantities, but when containing moisture, as it always does in practice, the differences are more considerable.

*Third law.** *The specific heat at constant pressure of a gas is the same at all temperatures.*

By "specific heat" is to be understood the amount of heat necessary to raise 1 lb. of the substance through 1°,—temperature being measured by the expansion of the gas itself as explained above,—a quantity which has different values according to the way the change of temperature is accomplished, depending on the amount of external work done. When the gas expands at constant pressure, as in

* Law of Regnault.

the first process mentioned above, the value has been determined by Regnault with great accuracy for various gases, and the result agrees so closely with this third law, that it also may be regarded as an essential characteristic of a perfect gas; the numerical value for air is ·2375 thermal units, or 183·35 ft. lbs.

The heat expended in raising the temperature of air at constant pressure is partly expended in doing external work; for let the original volume of the air be V_1, and the final volume V_2, then the work done in raising the piston is given by

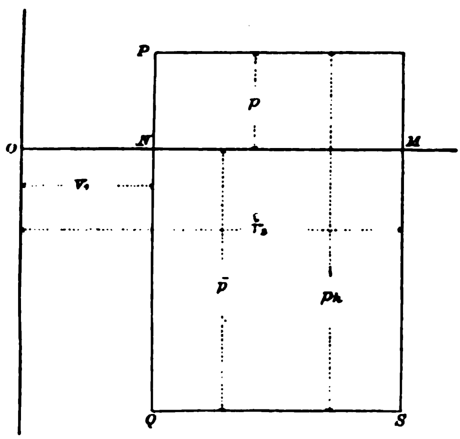
$$\text{Work done} = P(V_2 - V_1) = c(T_2 - T_1).$$

Hence, since the heat expended is $K_p(T_2 - T_1)$ where K_p is the specific heat at constant pressure,

$$\text{Internal Work} = (K_p - c)(T_2 - T_1).$$

Thus the internal work is proportional to the change of temperature whenever the change takes place at constant pressure, and for air is $183 \cdot 35 - 53 \cdot 2 = 130 \cdot 15$ for each degree Fahrenheit.

Fig. 7b.



The process may very conveniently be represented graphically. Let ON (Fig. ... resent the volume before, and

O M the volume after, heat has been applied, while P N represents the constant pressure: then the rectangle P M represents the external work done. Now set downwards N Q, such that

$$\frac{\text{N Q}}{\text{P N}} = \frac{\text{Internal Work}}{\text{External Work}}$$

$$= \frac{\text{K}_p - c}{c} = 2 \cdot 451 \text{ (for air)},$$

then the rectangle N S represents the internal work, and the rectangle P S the whole heat expended. As in previous cases, we may regard the heat expended in internal work as overcoming a pressure on the piston represented by N Q, while N Q and P N together represent the heat-pressure, hence

$$p_h = 3 \cdot 451 p.$$

*Fourth law.** *If a gas expand without doing external work its temperature is unaltered.*

When air expands, overcoming the resistance of a piston, its temperature falls, as appears by experience; the kind of expansion indicated here is, however, different. Let two vessels be provided, one of which is empty, and the other contains air of any pressure: let the vessels be connected by a pipe provided with a stop-cock, which, when opened, allows the air to flow from one vessel to the other; then, at first, the expansive energy of the air is employed in generating violent motions in the air, which, however, quickly subside from fluid friction when equilibrium of pressure has been attained, after which, if the temperature be observed before any heat is lost by radiation, it will be found sensibly the same as before the cock was opened.

Now the difference between this case and the first case supposed is, that no external work of any kind is done, that the air has lost no energy, and the result

shows that the intrinsic energy (Art. 15) possessed by the air is always the same at the same temperature, whatever be its pressure, a fact which enables us at once to show that however air changes its state, the internal work done must be proportional to the change of temperature.

For let 1 lb. of air expand in the way supposed from pressure P_1 volume V_1, till its volume is V_2; its temperature by hypothesis remains the same (T_1, say) and its pressure is therefore $\frac{V_1}{V_2} . P_1 = P_2$. Let it now expand at this constant pressure until it reaches any other volume, and its temperature has risen to T_2; heat must be added to render this possible, and the internal work done has been shown to be $(K - c)(T_2 - T_1)$, but during the free expansion no internal work is done, therefore the air has been changed from pressure P_1, temperature T_1, to pressure P_2, temperature T_2, with an expenditure of heat in internal work of $(K_p - c)(T_2 - T_1)$ which depends solely on the difference of temperature. Hence, by Art. 14, the internal work done in changing the temperature of air is always given by

$$\text{Internal Work} = (K_p - c)(T_2 - T_1).$$

This reasoning applies directly to the case where P_2 is less than P_1, and by a slight modification also when P_1 is less than P_2.

Now, in the second method of changing the product P V mentioned above, the volume of the air remains constant, so that no external work is done; and we learn that when air is heated at constant volume the expenditure of heat for 1° rise of temperature is $K_p - c$, a constant quantity. Hence the specific heat at constant volume is, like the specific heat at constant pressure, a constant quantity, and moreover, if it

$$K_p - K_v = c,$$

a relation which is a necessary consequence of the fourth experimental result. In the case of air from the values given above, we find

$$K_v = 183 \cdot 7 - 53 \cdot 2 = 130 \cdot 2 \text{ foot lbs.}$$

Now, if we could make experiments on the specific heat of air at constant volume, we should be able to verify this result. This has not yet been done directly, but we can obtain a value indirectly by means of the knowledge we possess of the ratio $\frac{K_p}{K_v} = \gamma$, a number on which the velocity of sound depends, irrespectively of any special theory of heat, and which for air and some other simple gases has been shown to be $1 \cdot 408$, whence

$$K_v = \frac{183 \cdot 4}{1 \cdot 408} = 130 \cdot 25,$$

agreeing closely with the previous calculation. Thus we might have assumed as our fourth experimental result the equation

$$K_p - K_v = c,$$

from whence can be shown without difficulty that the temperature is unaltered by free expansion. This equation shows that, for gases which have the same value of γ, the specific heat at constant pressure is inversely proportional to the density; a result which has been experimentally verified.

The fourth experimental result is not to be regarded as exact for actual gases any more than the three others, but merely as an approximation so close that we are justified in regarding it as another essential characteristic of a perfectly gaseous body.

Completely Superheated Steam.

32. Saturated steam is no[illegible] [illegible]ect gas, as is sufficiently shown by the fifth colu[illegible] [illegible]ich shows the differences $\Delta P u$ wh[illegible] [illegible]ly the same

as $\Delta P v$. These differences would be constant if the steam followed the perfectly gaseous laws, whereas they actually diminish very considerably as the temperature rises. There is, however, little doubt that, when sufficiently superheated, steam becomes sensibly a perfect gas, and it may then conveniently be said to be "completely" superheated. It then follows the law expressed by

$$PV = cT,$$

like atmospheric air, but with a different value of the constant c. As to the amount of superheating necessary, much uncertainty exists; but there can be no doubt that the higher the temperature of saturated steam the greater is the rise of temperature needful to produce from it completely superheated steam. In the intermediate state, in which steam is neither saturated nor a perfect gas, its properties are not fully known, but such experiments as exist show that its rate of expansion is much greater than that of a perfect gas near the saturation point, but quickly diminishes as the superheating progresses. In order to deal with the steam in this state, hypotheses not fully warranted by experiment are necessary.

The density of steam as calculated from its chemical composition is ·622, nearly, that is, a cubic foot of steam should weigh this fraction of the weight of a cubic foot of air at the same pressure and temperature (see 'Dixon's Heat,' p. 187), whence

$$c = \frac{53 \cdot 2}{\cdot 622} = 85 \cdot 5,$$

so that the theoretical equation for completely superheated steam is

$$PV = 85 \cdot 5\, T.$$

The density of dry saturated steam at low temperatures has not hitherto been determined in a satisfactory way, but the results usually given indicate that, below 104°,

saturated steam is sensibly a perfect gas of density (·63) rather greater than the theoretical value. The theoretical values of P V are given for every 27° from 104° to 401° in Table IV*a*, for the sake of comparison with the same values for saturated steam, which, it will be seen, are smaller, and at high temperatures considerably smaller.

The value of K_p, according to the latest experiments, is in thermal units ·48, or in foot lbs. 370·56, whence $K_v =$ 285·03 and $\gamma = 1·3$.

Thermodynamics of a Perfect Gas.

33. From the four experimental results expressing the physical properties of a perfect gas, it is possible to give a complete theory of the action of heat in such a body. For in the most general case of the action of heat in a body we found in Chapter II. that

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work};$$

and further it was shown above that the internal work is always $K_v (T_2 - T_1)$ where T_1 T_2 are the temperatures at the beginning and end of the operation.

$$\therefore \text{ Heat Expended} = K_v (T_2 - T_1) + \text{External Work}$$

$$= \frac{1}{\gamma - 1}(P_2 V_2 - P_1 V_1) + \text{External Work},$$

an equation from which all questions can be answered about the action of heat in a perfect gas.

First. Suppose the temperature constant, then the air expands, when heat is added, according to the hyperbolic law, and the internal work done is zero, so that the whole of the heat expended is employed in doing external work, and flows out of the air in this shape as fast as it enters. If the ratio of expansion be *r*, it was shown in Chapter III. that

the work done during hyperbolic expansion is $\mathrm{P\,V} \,.\, \log._{\epsilon} r$; hence if Q be the heat expended,

$$Q = \mathrm{P\,V} \,.\, \log._{\epsilon} r = c\,\mathrm{T} \log._{\epsilon} r,$$

where T is the constant absolute temperature of the air, and the same equation serves for any gas by substitution of the proper value of c. For air $c = 53 \cdot 2$ also $\log._{\epsilon} r = 2 \cdot 302 \log._{10} r$

$$\therefore Q = 122 \cdot 5 \,.\, \mathrm{T} \log. r \text{ foot lbs.}$$
$$= \cdot 1584 \,.\, \mathrm{T} \log. r \text{ thermal units,}$$

where the logarithm is now common.

Expansion at constant temperature is called "isothermal" expansion, and the curve is the isothermal curve, which for perfect gases is therefore an hyperbola.

Secondly. Let the expansion of the air take place according to the law expressed by the equation

$$\mathrm{P\,V}^n = \text{const.}$$

Curves of this kind occur constantly in the theory of the steam engine; thus the relation between the pressure and volume of saturated steam is expressed approximately by such an equation for which $n = \frac{17}{16}$, and we shall have numerous instances hereafter of such curves, with various values of n. Their general appearance is that of an hyperbola, which is indeed a particular case in which $n = 1$. In the Appendix, Note C, it is shown that the area included between two ordinates (Fig. 8, page 88), A N, B M, the curve A P B and the base N M is given by the equation

$$\text{Area} = \frac{P_1 V_1 - P_2 V_2}{n - 1},$$

a rule which fails when $n = 1$, when we must resort to that previously given for the case of the hyperbola. In any other case then we have

$$\text{External Work} = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{c}{n - 1}(T_1 - T_2),$$

so that in this law of expansion the external work done is proportional nge of temperature, which is a rise

when n is less than unity, and a fall when n is greater than unity. Placing this value in the equation for Q the heat expended,

$$Q = K_v (T_2 - T_1) + \frac{c}{n-1}(T_1 - T_2)$$

$$= \left(K_v - \frac{K_p - K_v}{n-1}\right)(T_2 - T_1)$$

$$= \frac{n K_v - K_p}{n-1} . (T_2 - T_1),$$

which shows that the heat expended is also proportional to the change of temperature, or, in other words, that the specific heat (K) is a constant, given by

$$K = \frac{n K_v - K_p}{n-1} = K_v . \frac{n - \gamma}{n-1}.$$

When $n = \gamma$ we have a very important case, for then the heat expended is zero, showing that when a gas expands without either gaining heat or losing heat the expansion curve is given by

$$P V^\gamma = \text{const.},$$

a curve which is called the *adiabatic* curve, and the expansion is said to be *adiabatic* expansion. In this case the external work is done at the expense of the internal energy stored up in the gas, therefore the temperature falls from T_1 to T_2, say, where the ratio $T_2 : T_1$ is readily found from the equations

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad : \quad P_1 V_1 = c T_1 \quad : \quad P_2 V_2 = c T_2,$$

whence if $r = \dfrac{V_2}{V_1}$ be the ratio of expansion,

$$T_2 = T_1 . \left(\frac{1}{r}\right)^{\gamma - 1}.$$

The accompanying table shows how the pressure and temperature fall during an expansion to double its vol
air, the initial pressure of which is 100 lbs. pe
and temperature 539° Fahr.; the correspo
during hyperbolic expansion being given fo

ADIABATIC EXPANSION OF AIR.

Temperature.		Pressure.		Ratio of Expansion.
Ordinary.	Absolute.	Adiabatic.	Isothermal.	
539	1000	100·0	100·0	1·
501	962	87·5	90·9	1·1
467	928	77·3	83·3	1·2
437	898	69·1	76·9	1·3
410	871	62·2	71·4	1·4
386	847	56·5	66·7	1·5
363	824	51·5	62·5	1·6
343	804	47·3	58·8	1·7
326	787	43·7	55·6	1·8
307	768	40·4	52·6	1·9
293	754	37·6	50·	2·0

The table applies to any other initial temperature and pressure by multiplication by the initial absolute temperature, and dividing by 1000 for the absolute temperature and by multiplying by the initial pressure and dividing by 100 for the pressure. To become familiar with the curve the reader should construct it to scale for some convenient initial pressure and temperature, obtaining the initial volume from the formula

$$P_1 V_1 = c T_1 .$$

The table shows in a striking manner the rapid rate at which the temperature falls; the fall being as much as 246° in the moderate expansion of 2 : 1. The work done during the expansion is found by multiplying the fall of temperature by K_v. By use of a table of squares the results given can be extended to higher rates of expansion. The curve can also be constructed graphically, by a modification of the process employed for steam (Art. 69).

34. Returning to the general case where n has any value, have

$$\text{External Work} = \frac{c}{n-1} \cdot (T_1 - T_2) = \frac{K_p - K_v}{1-n} \cdot (T_2 - T_1),$$

$$\text{Internal Work} = K_v (T_2 - T_1).$$

[redacted]

therefore we see that the internal work always bears a fixed proportion to the external work, namely, $\gamma - 1 : 1 - n$, a result which is represented graphically thus:

FIG. 8a.

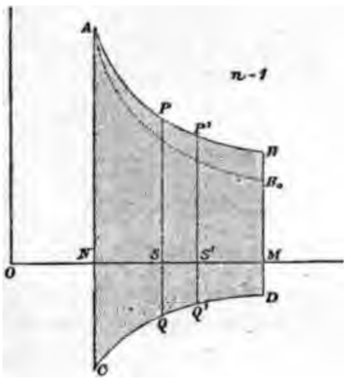
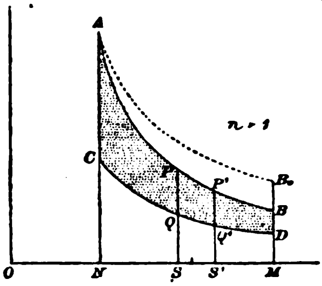


FIG. 8b.



In Figs. 8*a*, 8*b*, A B represents the expansion curve, which when $n > 1$, (Fig. 8*b*) falls below and when $n < 1$, (Fig. 8*a*) falls above, the hyperbola through A, represented in both figures by the dotted line A B_0; P P^1 are two points close together, the ordinates of which are P S, $P^1 S^1$; then, as has been previously shown, the external work done during expansion from S to S^1 is represented by the area of the strip P S^1.

Taking first the case (Fig. 8*a*) in which the curve falls above the hyperbola, it is clear that in this case the temperature rises as the expansion proceeds, and internal work must be done in order to produce this rise of temperature. Set, therefore, downwards S Q such that

$$\frac{\text{S Q}}{\text{S P}} = \frac{\text{Internal Work}}{\text{External Work}} = \frac{1 - n}{\gamma - 1},$$

and carry out the same construction at every step of the expansion, a curve C Q D is thus obtained, the area of a strip S Q^1 of which represents the internal work done during sion from S to S^1 on the same scale that the strip S P^1

of the expansion curve represents the external work, and the internal work done may be represented as overcoming at every instant a pressure on the piston, represented by S Q, which may be called the internal-work-pressure. The heat expended is the sum of the internal work and the external work, and is represented by the area of both curves shaded in the figures; the pressure (p_h) equivalent to it is clearly represented by S Q + S P, and we have, if the external pressure be p,

$$p_h = p + p \cdot \frac{1-n}{\gamma - 1} = p \cdot \frac{\gamma - n}{\gamma - 1}.$$

In the second case when $n > 1$ (Fig. 8*b*) the temperature falls and part of the external work is done at the expense of the intrinsic energy of the expanding air; we must then set off S Q upwards instead of downwards, and the heat expended is shown by the difference of area of the curves shaded in the figure. If $n = \gamma$ the two curves coincide, no heat being added or taken away; if $n > \gamma$, the expansion curve falls below the adiabatic curve, in which case heat must be taken away from the air as it expands, and this would be shown on the figure by the curve of internal work C Q D lying above the expansion curve A B.

I have given full details in this particular case not so much from its intrinsic importance as because it shows, very clearly and simply, how the expansion of an elastic fluid is affected by the heat it receives from without, during the process of expansion. In other cases also the graphical method may be employed with great advantage; as, however, I shall have occasion to explain these methods in detail for the case of steam in Chapter VII. (See Art. 69) I shall not dwell on them here.

35. When a gas is compressed, its pressure increases, and the relation between volume and pressure, represented graphically by a compression curve, depends, as in the case of expansion, solely on the mode in which heat is added or

subtracted. If the circumstances of the compression are the same as those of the expansion, the compression curve will be identical with the expansion curve, but otherwise not. Thus, when a gas is compressed without the addition or subtraction of heat, the pressure and temperature rise according to the law expressed by the table given above for adiabatic expansion. The work done in compressing the gas is then proportional to the rise of temperature. But if the rise of temperature be prevented by the continual abstraction of heat as the compression progresses, the compression curve will be an hyperbola, and the energy exerted in compressing the gas will be equivalent to the heat abstracted, and will be given by the same formulæ as in isothermal expansion.

Theory of a Heat Engine working with a Perfect Gas. Conditions of Maximum Efficiency.

36. The simplest kind of heat engine is constructed as follows:

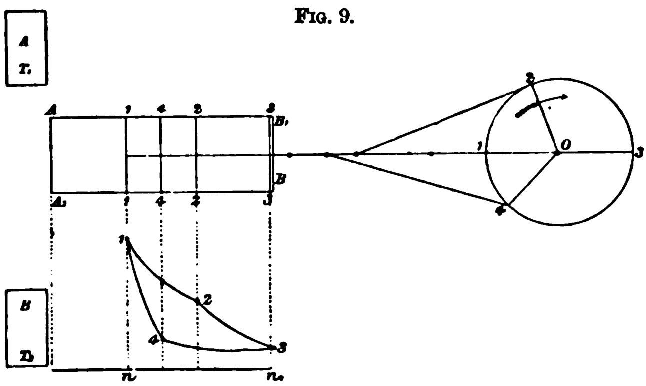
A B is a working cylinder containing a given quantity, say 1 lb., of a perfect gas, always included between the cylinder cover $A A^1$ and a piston, successive positions of which are represented in the figure by 1 1, 2 2, 3 3, 4 4, and which may be supposed connected in the usual way with a crank, corresponding positions of which are O 1, O 2, O 3, O 4. The right-hand portion of the cylinder between the cover $B B^1$ and the piston is empty.

A and B are two bodies, of temperatures (absolute) T_1 and T_3, capable of communicating to, or abstracting from, any body placed in contact with them, indefinite amounts of heat.

Suppose the crank moving in the direction of the arrow, and initially let it be in the position O 1, and let the pressure, volume and temperature of the gas be then p_1 v_1 T_1 respectively. Then, as the crank moves on, the volume of the

gas increases, and if no heat were applied to it the temperature would fall: but this is prevented by placing the body A in contact with the cylinder, which is to be supposed

FIG. 9.



a perfect conductor, so that the slightest depression of the temperature of the gas below T_1 causes heat to flow from A into the gas, and thus the temperature of the gas is maintained constantly at T_1. During this first operation, then, the gas expands at constant temperature, and the expansion curve 1, 2, in the indicator diagram above, is a common hyperbola.

The expansion having reached some convenient point 2, the body A is to be removed from the cylinder, so that no more heat is received by the gas; its temperature then falls instead of remaining constant, and the expansion curve 2, 3 on the indicator diagram above is now an adiabatic curve given by $P V^\gamma$ = constant. This goes on until the piston has reached the end of its stroke and begins to return so as to compress the air again, and raise its temperature according to the same law by which it fell during the expansion. But this rise of temperature is prevented by the application of the body B, the temperature of which is T_3, the temperature of the gas at the end of the stroke, and which abstracts heat from the gas the instant its temperature rises above T_3.

[redacted]

Thus the gas is compressed at constant temperature T_3, and the compression curve 3, 4 on the indicator diagram given on page 91, is a common hyperbola.

This compression goes on till the piston reaches a point 4, the position of which will presently be determined, when the body B is removed and the temperature of the gas allowed to rise. The gas is then compressed without gain or loss of heat, so that the compression curve 4, 1 on the indicator diagram given on page 91, is an adiabatic curve. If now the point 4 has been properly taken, the temperature of the gas at the end of the stroke will be T_1, and the gas having returned exactly to its initial state, the process may be repeated as many times as we please.

The required point 4 is easily found thus: let $p_2\,v_2$, $p_3\,v_3$, $p_4\,v_4$ be the pressure and volume at the points 2, 3, 4, then since 1, 2, and 3, 4 are common hyperbolas,

$$\frac{p_1}{p_2} = \frac{v_2}{v_1} \text{ and } \frac{p_3}{p_4} = \frac{v_4}{v_3},$$

and since 2, 3, and 4, 1 are adiabatic curves,

$$\frac{p_2}{p_3} = \left(\frac{v_3}{v_2}\right)^\gamma \text{ and } \frac{p_4}{p_1} = \left(\frac{v_1}{v_4}\right)^\gamma$$

Hence multiplying all four equations together,

$$v_1\,v_3 = v_2\,v_4 \text{ or } \frac{v_2}{v_1} = \frac{v_3}{v_4},$$

that is to say—*the ratio of expansion during the reception of heat must be equal to the ratio of compression during the rejection of heat*, or since we equally have

$$\frac{v_3}{v_2} = \frac{v_4}{v_1}$$

—*the ratio of adiabatic expansion must be equal to the ratio of adiabatic compression.* It is easily seen that each of these last ratios must be equal to $\left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}}$ which determines their value when the temperatures T_1 T_3 of the bodies A and B are supposed given.

The first ratios mentioned, or, as we may call them, the ratios of isothermal expansion and compression, are each given by

$$r^1 = \frac{v_2}{v_1} = \frac{v_2}{v_3}\,\frac{v_3}{v_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma - 1}} \cdot \left(\frac{T_2}{T_1}\right) \cdot \frac{p_1}{p_2}$$

$$= \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} \cdot \frac{p_1}{p_2},$$

and therefore depend on the ratio of greatest and least pressure employed as well as on the temperatures.

Let us next examine how much work is done by this engine and at what expenditure of heat.

During the operation 1, 2, the gas is receiving heat from A, and the quantity of heat it receives, according to the last article, is

$$Q = c\,T_1 \log. r,$$

where r is the ratio of expansion. During the operation 2, 3 which completes the forward stroke, the gas receives no heat, therefore Q is the heat expended in the forward stroke. At the same time the energy exerted on the piston by the gas is represented by the area 1, 2, 3, $n^1\,n$.

In the backward stroke the gas rejects the heat into B, and the heat so rejected is

$$R = c\,T_2 \log. r.$$

At the same time the piston compresses the gas and does work upon it represented by the area 1, 4, 3 $n^1 n$; the engine, in fact, is single-acting, and a fly-wheel will be required to carry it through the whole backward stroke.

The difference between these areas, namely, the area of the indicator diagram 1, 2, 3, 4, represents as usual the work done by the engine. We might calculate this area, but it is simpler to apply the principle of the cycle of operations, for since the gas returns exactly to its original pressure, volume, and temperature, we must have

$$\text{Work done} = Q - R = c\,(T_1 - T_2) \log. r,$$

The efficiency is now found from the consideration that the heat expended is Q,

$$\text{Efficiency} = \frac{\text{Work done}}{\text{Heat expended}} = \frac{T_1 - T_2}{T_1}.$$

Thus the efficiency of this engine is calculated by a very simple rule, which, moreover, is not only true for the simple construction, incapable of being practically worked, which we have chosen for consideration precisely on account of its simplicity, but likewise for any air engine, however complex, which receives and rejects heat in the same way. For it was shown in Art. 29 that the work done by a given quantity of elastic fluid is not dependent on the particular kind of machinery, by means of which its expansive energy is utilized, but solely on its law of expansion, and the grade of expansion, while, as has been illustrated in the last article, these depend only on the mode in which the fluid receives heat. In the present case the fluid expands partly at constant temperature, and partly without gain or loss of heat, the ratio of expansion depending in the first case on the ratio of absolute temperatures of the hot and cold body, and in the second case also on the ratio of pressures admissible. If these two ratios are given, the power and efficiency of the engine will be just the same whatever its construction, if no part of the expansion takes place according to any other law. But, as before, this statement is subject to some qualification when a part of the expansion is wholly or partly unbalanced.

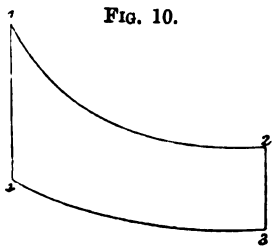
As an example, let us suppose that the temperature of the hot body is 660° Fahr., and that of the cold body 32° Fahr., then the efficiency is $\dfrac{660 - 32}{1121} = \cdot 56$, so that in this engine 56 per cent. of the heat expended is transformed into mechanical energy. Yet this result is only attained at a heavy sacrifice, for the high temperature of 660° Fahr. is destructive to the working [illegible] of 32° is hardly attainable ir[illegible] ound

of air with pressures that are at all convenient in practice is so small that the bulk of the engine is excessive. This maximum result of 56 per cent., though much greater than that found in the case of a steam engine, is nevertheless so low that we are led once more to the question as to the cause and as to the possibility of removing it.

37. The simple heat engine just considered may be arranged to work in an indefinite number of other ways, one of which will now be considered as an example.

Let us suppose that the body A remains in contact with the cylinder throughout the forward stroke, and the body B throughout the backward stroke, the indicator diagram will now be as shown in the figure. 1 2 is an hyperbola as before, but now extends through the whole forward stroke, and 3 4 is also an hyperbola as before, but extending through the whole backward stroke; but 1, 4, and 2, 3, are vertical straight lines representing the sudden rise of pressure on contact with the body A, and fall of pressure on contact with the body B, at the commencement of the forward and backward strokes respectively.

FIG. 10.



To find the efficiency of this arrangement we have, as before,

$$Q = c T_1 \log. r = \text{heat expended in forward stroke,}$$
$$R = c T_3 \log. r = \text{heat rejected in backward stroke,}$$

where the logarithms are supposed hyperbolic, and, as shown above, these quantities are also the work done upon the piston by the gas in the forward stroke, and upon the gas by the piston in the backward stroke,

$$\therefore \text{Work done by engine} = c\,(T_1 - T_3) \log. r, \text{ as before,}$$

but the whole heat taken away from A, that is, the whole heat expended, is now no longer Q, because at the commencement of the stroke a quantity of heat $K_v (T_1 - T_3)$ has

to be expended in raising the temperature of the gas from T_1 to T_3, and we consequently have

$$\begin{aligned}\text{Heat expended} &= Q + K_v(T_1 - T_3)\\ &= c\,T_1 \log. r + K_v(T_1 - T_3)\\ \text{Efficiency} &= \frac{c(T_1 - T_3)\log. r}{c\,T_1\log. r + K_v(T_1 - T_3)}\\ &= \frac{T_1 - T_3}{T_1}\cdot\frac{1}{1 + \dfrac{K_v}{c\log. r}\cdot\dfrac{T_1 - T_3}{T_1}}.\end{aligned}$$

Thus we see that the efficiency of this arrangement is less than that of the other, and we likewise see why it is so, namely, because at certain points in the process the gas has its temperature raised and lowered by contact with the bodies A and B, that is, *that it receives and rejects heat at temperatures sensibly different from the temperatures of A and B.*

There are two conceivable ways of lessening this loss of efficiency; the first is by storing up the heat rejected during the cooling represented by 2, 3, and employing it to produce the rise of temperature represented by 4, 1. This has been done in actual air engines by the contrivance called the regenerator, a device, which, but for radiation, would be a perfect remedy. The second is by inserting an auxiliary heat engine, receiving heat from the hot gas of the original heat engine after it has finished its work in that engine; the work of this auxiliary engine will be so much additional work done without any additional expenditure of heat.

The remarks made in the former case apply also to this, the work done by 1 lb. of air and the efficiency of the engine do not depend upon the mechanical means by which the expansive energy of the fluid is utilized, but solely on the manner and degree in which the fluid receives heat. Hence in no engine of this kind can the efficiency exceed that of the engine previously considered. To prove this with respect to any possible engine will be the principal object of the next chapter.

NOTE.—On the subject of air engines, the reader is referred to Rankine's 'Steam Engine,' chap. iii., and [illegible] in 'Engineering,' vol. xix.

CHAPTER V.

PERFECT HEAT ENGINES.

38. Mechanical power is produced from heat through the agency of an elastic fluid, such as steam or air, capable of assuming different volumes under the action of heat and cold, and exerting mechanical energy on external bodies during such changes of volume.

Observation of the action of heat engines, combined with reasoning of the same character as that already given in the case of steam and air, leads us to certain general conclusions as follows:

(1) The changes consist in a continual repetition of operations of the same kind, whether on the same mass of fluid or on a continual succession of exactly similar masses.

(2) Each repetition includes, *first*, a period during which the fluid on the whole increases in volume, and receives heat while exerting energy on a working piston; *secondly*, a period during which the fluid, on the whole, contracts in volume and rejects heat, while work is done upon it either by the working piston overcoming a back pressure, or by some special compressing apparatus, or by both these causes combined. The final result is that the fluid returns to the same state as before; that is to say, the changes constitute a cycle of operations or circular processes.

(3) The energy exerted on the working piston, and the work done during compression, do not at all depend on the particular mechanism by means of which the changes of volume of the fluid are carried out, but solely on the quantity of fluid, and the nature of the changes it undergoes.

Hence a heat engine implies, (1) a source or sources from which the fluid is supplied with heat; (2) a working piston or other means of utilizing the expansive energy of the fluid; (3) a refrigerator capable of abstracting heat from the fluid; and (4) a compressing apparatus, in consequence of which the fluid returns to its original state; and it is to be especially remarked that the contraction of volume and rejection of heat is just as indispensable as the enlargement of volume and reception of heat.

The useful work done by the engine is the difference between the energy exerted by the fluid during expansion and the work done upon it during compression; in some engines, as for instance the ideal air engines of Chapter IV., the work done in compression is large, so that the useful work is a small fraction of the energy exerted; in others, as for instance the steam engine, the work done in compression is comparatively small; in all cases, if U be the useful work done, E the energy exerted, C the work done in compression,

$$U = E - C,$$

where the ratio E : C may have any value according to the nature of the fluid.

But, further, the useful work done is also the work-equivalent of heat which disappears during the process; that is to say, the heat supplied by the source or sources is greater than the heat abstracted by the refrigerator exactly by the equivalent of the useful work done, so that, if Q be the heat expended, R the heat rejected, we have necessarily,

$$U = Q - R.$$

The ratio Q : R, however, is not capable of variation in the way that the ratio E : C is, according to the nature of the fluid; on the contrary, it will be shown that, whatever the fluid be, that ratio will alwa[illegible] same value provided the changes of state [illegible] a certain prescribed law. Hence the [illegible] ngine

is to a great extent independent of the nature of the fluid, just as much as of the mechanism of the engine (see Art. 29), being chiefly dependent on the way in which the fluid is supplied with heat, and whatever the engine be, a large amount of heat always passes into the refrigerator, being merely conveyed there from the source by the agency of the fluid.

In the present chapter I shall suppose that the supply of heat proceeds from a single source of given fixed temperature.

39. Returning to the first arrangement of air engine (Fig. 9) explained in the last chapter, and supposing the crank to be initially in the position O 1, let the engine turn in the opposite direction to that indicated by the arrow, neither A nor B being in contact, the air will expand without gain or loss of heat, and the adiabatic curve 1, 4 will be described on the diagram. As soon as 4 is reached, let the body B be applied to prevent the temperature from falling below T_3; heat is continually abstracted from B, the hyperbola 4, 3 is described on the diagram. Now removing the body A, the piston returns, and the adiabatic curve 3, 2 is described; and, finally, applying A to prevent the temperature from rising above T_1, the hyperbola 2, 1 is described, heat all the while passing from the air into A.

The whole process, and every step of it, is now precisely the reverse of what it was; a heat R is abstracted from B, and a heat Q passes into A, while, instead of the engine doing work on external bodies, some force must be applied to the crank which in each revolution will do the work Q−R. And thus, instead of heat passing from A to B, and during its passage a part of it being converted into mechanical energy, we have, conversely, by application of mechanical energy, heat passing from B to A, and during its passage mechanical energy converted into heat.

In short, the process is *reversible*, not only in its final result, but in each of its successive steps.

The characteristic of complete reversibility, possessed by certain kinds of heat engines, is of the highest importance, as will be seen presently; in order that it may exist, two principal conditions are necessary, and probably sufficient.

In the first place, the reception of heat from the source, and the rejection of heat into the refrigerator, must take place at temperatures differing insensibly from those of the bodies themselves. For example, take the case of the second arrangement of air engine described in Chapter IV., and suppose that engine worked backwards, then, instead of the heat $c\ T_3 \log. r + K_v (T_1 - T_3)$ being taken from B, and the heat $c\ T_1 \log. r + K_v (T_1 - T_3)$ being added to A, as would be the case were the process to which the air is subjected exactly reversed; it will be found that the heat taken away from B will be $c\ T_3 \log. r - K_v (T_1 - T_3)$, and the heat added to A, $c\ T_1 \log. r - K_v (T_1 - T_3)$, so that although the same amount of energy is applied to work the engine, as was given out when the engine worked forwards, a less amount of heat flows from B to A than originally passed from A to B.

In the second place, in order to secure reversibility, it is necessary that the expansive force of the fluid should be exactly balanced by the resistance which is being overcome. If it should be greater than the resistance, then the excess takes effect by generating kinetic energy in the particles of the fluid which are thrown into violent motion. In order exactly to reverse such a process, it would be needful to direct the motion of the particles so as to compress the fluid again without the application of any other external force than that originally overcome when the fluid expanded. Such direction is obviously impossible, nor is it less impossible to set the particles of fluid in motion by the direct action of heat reversing the process by which kinetic

energy is transformed into heat by fluid friction; hence unbalanced expansion is necessarily irreversible. When, for instance, the exhaust is opened at the end of the stroke in a steam cylinder, the steam rushes violently into the condenser, and the greater part of its expansive energy is employed in generating kinetic energy, which is afterwards changed into heat by fluid friction; to take hold of the particles of steam, and direct their motions so as to cause them to enter the cylinder again without the application of pressure, is impracticable, even were it possible to set them in motion by the direct action of heat, and the ordinary steam engine in which the expansion is incomplete is consequently irreversible.

40. Whether precisely reversible or not, however, a heat engine may be conceived to work backwards, as has just been illustrated in the simple cases of Chapter IV., and, when it does so, the original source of heat plays the part of a receiver of heat, and the original refrigerator becomes a source from which heat is derived. Hence, by a proper application of mechanical energy, heat can be taken away from a body of low temperature, and supplied to a body of high temperature.

Now this is essentially an artificial or non-natural effect; when we speak of one body as hot, and another as cold, no other meaning can be ascribed to these words than that heat tends to flow from the hot body to the cold one, and will certainly do so if no external cause prevent; much more then are we justified in saying that in the natural order of things heat will not pass from a cold body to a hot one, but only under the influence of some external agency. This is expressed in formal terms by the annexed statement of the SECOND LAW of thermodynamics.

Heat cannot pass from a cold body to a hot one by a purely self-acting process.

It is ea[illegible]hat enormous consequences the denial

of this principle would involve in the theory of the steam engine, for all the heat expended in the boiler which is not transformed into mechanical energy—that is to say, at least five-sixths of the whole amount—appears in the condenser being employed in heating the condensation water, and if it were possible by some self-acting contrivance to cause that heat to flow from the condenser into the boiler, it is manifest that the said five-sixths of the consumption of heat might be saved. It is certain, however, that this is impossible, but that to cause the heat to flow from the condenser into the boiler we must have recourse to some artificial process which, like working a heat engine backwards, involves in some way or other, directly or indirectly, the expenditure of energy to as great or greater amount than we can recover by utilizing that heat in the boiler; and the second law of thermodynamics merely amounts to a statement of this impossibility.

By aid of this law we are able to prove an extremely important theorem, known as "Carnot's Principle," which may be thus enunciated:

The efficiency of all reversible engines, working between given limits of temperature, is the same.

For, let us imagine two engines, A and B, of which in the first instance we suppose B reversible in the sense explained above, and let the power of these two engines be the same, then the engine A may be employed to drive the engine B backwards, and the combination of the two engines will be self-acting, requiring no energy derived from without to drive them, but continuing in motion (neglecting friction) for ever when once set going. Let Q_A, R_A be the heat expended and rejected respectively by the engine A, and let Q_B R_B be corresponding quantities for the engine B, so that by A the heat Q_A is taken from the hot body, and the heat R_A added to the cold body, while by B the heat Q_B is added to the hot body, and the heat R_B taken away from the cold

body. Then the final result of the working of the combination is that $Q_A - Q_B$ has been taken away from the hot body, and $R_A - R_B$ has been added to the cold body. But since the power of the engines is the same,

$$Q_A - R_A = Q_B - R_B,$$

$$\therefore Q_A - Q_B = R_A - R_B;$$

so that the result of the working of the combination is that an amount of heat $Q_A - Q_B$ has passed from the hot body to the cold one. Now, since the combination is self-acting, the second law of thermodynamics tells us that Q_A cannot in any case be less than Q_B; but must be either equal or greater, for if Q_B were the greater the heat $Q_B - Q_A$ would pass from a cold body to a hot one through the agency of a self-acting machine. But, the efficiencies of the engines are $\dfrac{Q_A - R_A}{Q_A}$ and $\dfrac{Q_B - R_B}{Q_B}$, of which fractions the numerators are equal, and hence we learn that the efficiency of the engine B cannot be less though it may be greater than that of the engine A.

Next imagine not only the engine B, but also the engine A to be reversible, and suppose the direction of the combination reversed, so that B works forward and A works backward, then manifestly the same reasoning shows that the efficiency of A cannot be less but may be equal to that of B, and consequently when both A and B are reversible we must conclude that their efficiencies must be equal. Moreover, we conclude that the efficiency of an irreversible engine cannot be greater, so that an engine which is reversible is also an engine of maximum efficiency. In fact, an engine which is not reversible has probably always a lower efficiency, as is illustrated by the example of the last chapter, and shown more fully presently.

Now, in the last chapter, we considered the case of an air engine which was shown above to possess the special charac-

teristic of reversibility, and we found that its efficiency was $\frac{T_1 - T_3}{T_1}$ where T_1 T_3 are the absolute temperatures of the hot and cold body respectively; and we, therefore, conclude that every engine which is reversible must have an efficiency expressed by this formula, and, moreover, that no non-reversible engine can have a greater efficiency.

The same result may be expressed somewhat differently thus: since

$$Q - R = U = Q \cdot \frac{T_1 - T_3}{T_1},$$

$$\therefore \frac{R}{T_3} = \frac{Q}{T_1}, \text{ or } \frac{R}{Q} = \frac{T_3}{T_1};$$

that is to say, when heat is expended in working a reversible engine, the fraction $\frac{T_3}{T_1}$ merely passes through the engine from the source to the refrigerator, while, if the engine be non-reversible, the fraction in question is still greater.

41. We can now state certain general conclusions respecting the action of heat engines of maximum efficiency, or, as we may express it, of PERFECT heat engines.

Each repetition of the series of changes which the fluid undergoes includes four periods.

(1) Expansion at constant temperature accompanied by a reception of heat from the source.

(2) Expansion at falling temperature without gain or loss of heat.

(3) Compression at constant temperature accompanied by abstraction of heat.

(4) Compression at rising temperature without gain or loss of heat.

In the steam engine, as will be explained farther on more fully, the first period is that of the evaporation of the water the boiler, the second is that of expansion in the cylinder, and that of condensation in the condenser, while the

fourth is that of forcing the water into the boiler; but inasmuch as the ordinary steam engine is not a perfect engine, but can only be made so by certain ideal arrangements to be explained farther on: none but the first satisfy the needful conditions for maximum efficiency, and even the first is usually imperfect in practical cases.

Let now I_1 be the internal work done during the first period, and E_1 the energy exerted on the working piston, then

$$Q = I_1 + E_1;$$

so, if I_3 be the internal energy given out during compression in the third period, and C_3 the corresponding work done upon the fluid,

$$R = I_3 + C_3.$$

In the second period a portion of the internal energy I_1 stored up in the fluid in the first period is utilized in performing external work by expansion: while, in the fourth period, energy derived from external sources is employed in compressing the fluid and increasing its store of internal energy.

The result obtained at the end of the last article may be written thus:

$$\frac{I_1 + E_1}{T_1} = \frac{I_3 + C_3}{T_3};$$

that is to say, although the internal and external work may vary greatly according to the nature of the fluid, yet they are not altogether independent of each other, but must satisfy the above equation which furnishes a restriction on the possible variation of constitution of fluids.

In the case of air the quantities I_1 I_3 are zero, since no change of internal energy takes place when perfect gases expand or contract at constant temperature, all the heat received or rejected being represented by the energy exert or work done; hence,

$$\frac{E^1}{T_1} = \frac{C_3}{T_3};$$

[redacted]

that is to say, is liable as mentioned, although all the heat expended is transformed into mechanical energy in the first instance, yet the power required to work the indispensable compressing apparatus is so great as to absorb the greater part of it, leaving only a fraction available for useful purposes. On the other hand, if instead of air we employ steam, then the power required to work the compressing apparatus, that is to say, to overcome back pressure, is comparatively small; but then the quantities I, I, representing heat expended in internal changes, so far from being zero, are very large, so that in this case also but a small fraction of the heat expended is available for useful purposes.

Our general reasoning shows that this difficulty can never be overcome, and that if the heat be used in any other way than that just described, the result will be still more unfavourable.

Analogy between a Heat Engine and a Water-power Engine. Imaginary Heat Engine.

92. A reader who for the first time is introduced to the reasoning just now given, will complain, and not altogether without reason, of its abstract and difficult character. This is in part due to the great generality, for all reasoning is more difficult, to grasp the greater the number of cases it covers; but it is also due to the novelty of the ideas involved, which are in many respects different from anything to be found in any other branch of science. It may be then of some service, if I draw a parallel between the case of a heat engine and that of an engine driven by falling water, which although far from perfect, is nevertheless so far true as to be some help in the comprehension of the ideas in question.

In order to produce mechanical energy by means of heat, we must in the first place have bodies of different temperatures, since all changes produced by heat are due to the passage of heat from one body to another, for which passage difference

of temperature is an indispensable condition. In the second place, in addition to the hot body or source of heat, and the cold body or receiver of heat, we must have a third intermediate body, by the agency of which heat is transferred from the hot body to the cold body, and through the changes of volume of which mechanical energy is exerted. If no such third body exist, the heat simply passes from the hot to the cold body without doing any work, and, when once it has done so, the second law just enunciated tells us that the opportunity of doing work which might have been utilized has been irretrievably lost.

Now, when work is done by means of falling water, it is in the first place necessary to have a fall, that is, a passage of water from a high level to a low level, and in the second, a suitable machine to receive the falling water, and transfer it mechanically from the high level to the low level; if no such intermediate agency exist, the water descends indeed, just as the heat flows from the hot body to the cold one, but none of the energy of the falling water is changed into useful work.

Hence heat may be described as descending from a high temperature to a low one and doing work by the agency of steam or air in the course of its descent, while the power of doing work by such descent, if not properly utilized, is lost in the water-power engine, just as in the heat engine. Hence also the conditions of perfect efficiency may be described in terms to a great extent identical. For in the water-power engine, in order that every particle of the energy of the falling water may be employed in useful work, the transfer of the water from a high level to a low level must be wholly produced by the artificial agency of the water wheel; if the water pour from a certain height on to the wheel, or pour off the wheel on to the lower level, the difference of level thus represented is wholly or partially wasted. So in the heat engine, if the heat descend from the source to the steam or

air, or from the steam or air to the cold body, through a sensible interval of temperature, then the difference of temperature in question, which might have been utilized, is wasted, or, in other words, for maximum efficiency the steam or air must receive heat at the constant temperature of the source of heat, and reject it at the constant temperature of the cold body or receiver of heat. And still further the condition of maximum efficiency of a water machine may be described as consisting in the machine being *reversible*, for that condition consists in the water entering the wheel without shock, and leaving it without velocity, a condition which, if exactly satisfied, would enable us by reversing the motion of the water wheel exactly to reverse the process to which the water is subjected, raising it without loss of energy from the low level to the high level by means of energy drawn from external sources. An imperfect water wheel may in like manner be described as non-reversible.

The parallel here drawn is due to the celebrated Carnot, to whom we also owe the conception of a reversible heat engine: but it applies more completely to Carnot's conception of the action of a heat engine than to its true action as now understood. Carnot was a believer (like most persons of his time) in the material nature of heat, and hence, when he speaks of heat descending from one level to another, none of it is regarded as disappearing in the process, but the work done is conceived as done at the expense of the difference of temperature: heat of high temperature possessing more energy (to use the language of modern science), than heat of low temperature. We now know that this is not so, the mechanical equivalent of the heat in the condenser of an engine is just the same as that of the heat in the boiler, and hence difference of temperature does not in itself constitute energy, but is merely an essential condition, that heat may be changed into work. But although temperature is not in itself energy, yet it is temperature which gives to heat its value for all useful

purposes, so that the parallel in question still to a great extent holds good. The parallel is by some writers made closer by the introduction of certain ideal quantities called "heat-weights," found by dividing quantities of heat by the corresponding absolute temperatures, but the "heat weight" appears to me such a purely artificial conception, that I have not thought it desirable to introduce it here. (See Chapter VIII.)

43. An additional help towards the comprehension of the principles I have been trying to explain occurs in considering the action of the steam and ether engine actually used for marine propulsion, and only abandoned on account of the practical difficulties involved.

Ether is a fluid which evaporates and produces vapour of considerable pressure at temperatures not exceeding the temperature of an ordinary condenser. Accordingly, the surface condenser of a steam engine may be used as the boiler of an ether engine, which will do work without the expenditure of any heat, except that furnished by the exhaust steam of the steam engine; the work so done will be so much clear gain, and the efficiency of the combined steam and ether engine will be greater than that of the steam engine alone. Now the reason of this is clearly that we make use of the difference of temperature, otherwise wasted, between the condenser of the steam engine and surrounding bodies.

So again the hot gases of a furnace have a very high temperature, far higher than the temperature of the steam boiler by which the heat is utilized. If then we imagine a fluid to exist which, evaporated at or near that temperature, produces vapour of considerable pressure, then that fluid might be used in an auxiliary engine, which received heat from the hot gases, and rejected heat into the steam boiler, which would serve as its condens In such a case the work done by the combination and the h expended would be increased by *equal* quantities, and

efficiency would consequently be greater than that of the simple steam engine.

Such auxiliary engines may involve practical difficulties, rendering them incapable of being brought into practical use, but their conception alone is sufficient to enable us to see how completely the power of a heat engine is dependent on difference of temperature, and how certain it is, that the greater the difference of temperature the more efficient the engine must be, other things being equal. Moreover, we can see that every time difference of temperature is wasted efficiency is wasted, or, in other words, that among the conditions of maximum efficiency must be those already stated, namely, that the steam, air, or other fluid *must receive heat at the constant temperature of the source of heat, and reject heat at the constant temperature of the receiver of heat.* To put the same thing in other words: we must utilize, as far as possible, every available difference of temperature, just as in the case of an hydraulic machine we carefully utilize every part of the available fall.

The other condition, previously stated to be included in reversibility, has not perhaps been absolutely proved to be in all cases indispensable to maximum efficiency, but no doubt in all practical cases it is necessary that no part of the expansive energy of the fluid should be converted into kinetic energy by wholly or partially unbalanced expansion.

Every possible condition of whatever kind, however, is always included in the one word *reversibility;* every heat engine, which is truly reversible, will be of maximum efficiency, though it may be difficult to prove conversely, that the efficiency of every conceivable non-reversible engine is necessarily less.

Much difference of opinion has existed, and still does exist, as to the best mode of statement of the second [illegible] thermodynamics and its consequences, but the [illegible] now be considered beyond cont[illegible]

Clerk Maxwell remarks, '*Theory of Heat*,' p. 153, those facts are of far more importance than the particular form of words in which they are embodied. The results will be further developed in a later chapter (Chapter VIII.), but for further information on questions of principle I must refer to Professor Maxwell's work just cited.

44. Let us now imagine the temperature T_1 of the hot body, or source of heat, to be divided into n equal parts, and let us imagine a quantity of heat Q to flow from that body to a second body, the temperature of which is $T_1\left(1 - \frac{1}{n}\right)$, then our results show that a quantity $\frac{Q}{n}$ of mechanical work is capable of being produced, and that consequently, if such conversion be effected, the quantity of heat $\frac{n-1}{n} \cdot Q$ will pass into the second body. Now imagine a third body, the temperature of which is $T_1\left(1 - \frac{2}{n}\right)$, and let this heat pass from the second body to the third body; then the heat capable of being turned into work is

$$\frac{n-1}{n} \cdot Q \cdot \frac{T_1}{n} \cdot \frac{1}{T_1\left(1 - \frac{1}{n}\right)};$$

that is $\frac{Q}{n}$ as before. This process may be continued indefinitely, and we thus see that—*If the temperature of a source of heat be divided into any number of equal parts, then the effect of each of these parts in causing work to be performed is the same.*

It is in this form that Rankine enunciates the second law of thermodynamics, and his view may be illustrated by the analogy just considered between the difference of level, which causes work to be performed by an hydraulic machine, a[nd] the difference of temperature which causes work to be pe[r]formed by a heat engine. Each foot of fall in a water whe[el]

is equally effective in doing work by means of the water wheel, and just so each degree of temperature passed through during the passage of heat from a hot body to a cold one is equally effective in causing work to be done by a heat engine working by means of this heat.

The temperatures in question are, as our investigation shows, to be measured on the perfect gas thermometer, which is therefore entitled to be considered as a definite measure of temperature. Temperature, being by its nature incapable of direct measurement, can only be measured by considering some physical effect which difference of temperature produces: thus the ordinary mode of measurement is, by observing the expansion which bodies undergo when their temperature is raised. This, however, is inconvenient for scientific purposes, since no two thermometers give exactly the same results; for instance, the mercurial and air thermometers, if graduated to indicate correctly the temperatures of melting ice and of water boiling under a given pressure, will be found to differ at intermediate temperatures. We must, therefore, consider some other physical effect due to difference of temperature, and the only one known to be independent of the particular body operated on is the power, of which we have just been speaking, which difference of temperature possesses of converting heat into work. If equal intervals of temperature be understood to mean equal capability of converting heat into work, we get a scale of temperature, which may, with propriety, be called *absolute*, and which, as our investigation shows, coincides with that of the perfect gas thermometer, that is (sensibly) with the air thermometer; and the term "absolute," already applied to temperature measured in this way, is hereby justified.*

* It may here be noticed that it was not absolutely necessary to have stated in Chapter IV. the second characteristic of a perfectly gaseous body, namely, that all such bodies expand alike; it would have been sufficient for the purposes of the argument to state the three other laws to which they are subject. The second law then becomes an experimental verification of Carnot's principle.

Performance of a Perfect Heat Engine. Application to the Steam Engine.

45. Thus we see that if a heat engine be perfect, its efficiency is $\frac{T_1 - T_2}{T_1 + 461}$ where T_1 T_2 are the temperatures Fahrenheit, between which the engine works; or if U be the work done in a given time, Q the heat expended in the same time,

$$Q = \frac{T_1 + 461}{T_1 - T_2} \times U,$$

hence the expenditure of heat per H.P. per minute is given by

$$Q = 42{\cdot}75 \,.\, \frac{T_1 + 461}{T_1 - T_2} \,.\, \text{(thermal units)}, \qquad (1)$$

a formula which gives the least amount of heat necessary to produce the given power, so long as we are restricted to work within the given limits of temperature. Let us now consider what those limits of temperature are in the case of the steam engine.

The inferior limit T_2 can in no case be less than the temperature of the atmosphere, and in the case of the condensing steam engine may be taken as 100° F., which is about the temperature of the condenser. Du Tremblay, indeed, has virtually lowered this temperature to 60° F., or thereabouts, by the addition of an ether engine, working between the temperature of the exhaust steam and the temperature of condensation of ether; but on account of the practical difficulties attending the use of ether, this plan is not likely to come into ordinary use. In the non-condensing steam engine, T_2 is the temperature corresponding to the atmospheric pressure, that is to say, 212° F.

The superior limit T_1 in a simple steam engine, is the temperature of the boiler; for although the hot gases of the furnace have a vastly higher temperature (say t), yet power of turning heat into work, due to the differenc

temperature $t - T_1$, cannot be realized by the arrangements of an ordinary steam engine. In order to realize it, it would be necessary to have a fluid which evaporates at a temperature t of say 1000° F. and condenses at a temperature a little above that of an ordinary steam boiler. Such temperatures are impracticable in practice, and hence the superior limit must be as stated the temperature of the boiler.

The annexed table shows the performance of various descriptions of perfect heat engines working under various circumstances, and in the case of the steam engine also the minimum consumption of steam needful per I.H.P. per hour calculated from the formula

$$N = \frac{60\,Q}{H_1 - h_0}, \qquad (2)$$

where Q is the expenditure of heat per minute found by (1) and $H_1 - h_0$ is the total heat of evaporation of water *from* the temperature of the feed, *at* the temperature of evaporation.

The consumption of coal will of course depend on the quality of the coal and the efficiency of the boiler. In seeking a theoretical limit to the amount of power which can be produced from a pound of coal, it is perhaps proper to consider the efficiency of the boiler unity, in which case if we adopt pure carbon as the standard quality of coal, the consumption will be given by

$$C = \frac{60\,Q}{14{,}500}. \qquad (3)$$

The total heat of combustion of actual coal is sometimes nearly 10 per cent. greater than that of carbon, but is more often less.

The table, then, shows the consumption of a perfect engine and boiler under the circumstances indicated: to provide for losses connected with the boiler from 30 to 50 per cent. must be added to the numbers given.

Performance of a Perfect Heat Engine.

Description of Engine.	Superior Temperature.	Boiler Pressure absolute.	Thermal Units per I.H.P. per Min.	Pounds of Steam per I.H.P. per Hour.	Pounds of Carbon per I.H.P. per Hour.	Efficiency.	Heat rejected per I.H.P. per Min.
Non-condensing steam engine. Inferior temperature, 212°.	401°	250	195	11·4	·806	·219	152
	363	160	233	13·8	·964	·183	190
	341	120	266	15·8	1·10	·161	223
	312	80	329	19·9	1·36	·130	286
	287	55	427	26·0	1·77	·100	384
Condensing steam engine. Inferior temperature, 100°.	341	120	143	7·5	·592	·299	100
	324	95	150	8·1	·621	·285	107
	293	60	167	9·0	·691	·256	124
	250	30	203	11·2	·840	·211	160
	228	20	230	12·8	·952	·186	187
Steam and ether engine. Inferior temperature, 60°.	341	120	122	6·2	·505	·351	79·3
	293	60	138	7·3	·571	·309	95·3
Air engine. Inferior temperature, 60°.	660	..	79·8	..	·33	·536	37·1

When a superheater is used, the superior temperature will of course be that of the superheater, which will not then correspond to the boiler pressure.

The sixth column shows the efficiency, from which it appears that, in the best possible steam engine, unless the steam be superheated considerably, at least two-thirds of the whole heat expended is wasted, the waste arising from no fault in the construction or nature of the engine, but solely from the narrow limits of temperature within which we are restricted to work. To obtain a better result it will be indispensable to overcome in some way or other the practical difficulties which exist in employing unusual temperatures.

Again, the consumption of steam in a perfect steam engine. as shown by the fourth column, is much less than that (actual engine under the same circumstances, showing

faults in the construction of the engine or the treatment of the steam must exist, which, at least theoretically, are remediable. This is shown in a striking manner by comparing the performance of a perfect engine working at 95 lbs. per square inch absolute, with that of the same engine working as in Chapter III. (page 58). Here the boiler pressure is the same and also the condenser temperature, yet the consumption of steam is even at the greatest expansion theoretically available 13 lbs. of steam per I.H.P. per hour instead of 8·1 as in the perfect engine. The exhaust waste is here not included, and the loss arises from improper application of heat and excess back pressure, as will be explained fully in a later chapter.

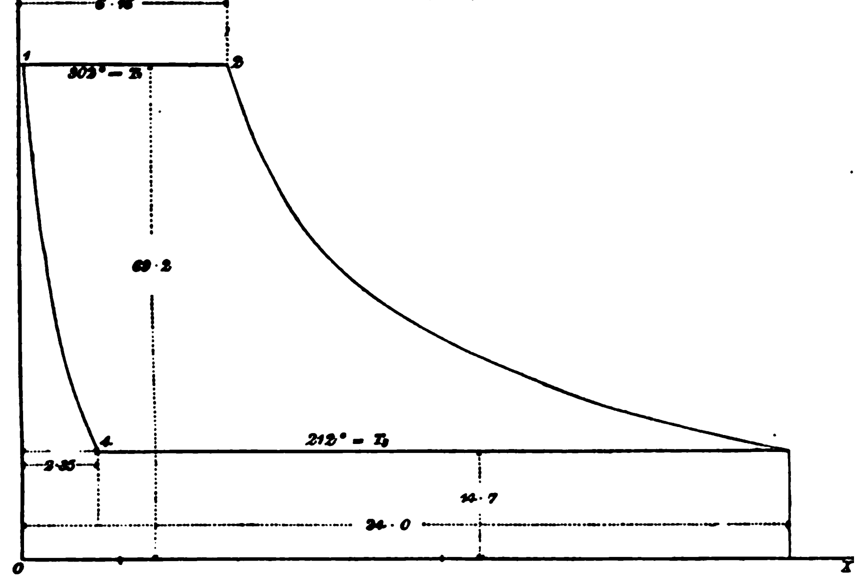
The table further shows that the gain by the use of steam of high pressure is not very great, because the temperature of steam at high pressure increases but slowly with the pressure. The theoretical gain by the addition of a condenser on the other hand is very large, but it will be seen hereafter that the condensing engine is a much more imperfect machine than the non-condensing, so that much of that gain cannot be realized in practice. Indeed it may be stated generally that the wider the limits of temperature the more difficult will it be to approach in practice the theoretical efficiency.

46. We will now consider the case of a steam engine working under conditions of maximum efficiency.

The figure (Fig. 11) shows the indicator diagram of an engine of maximum efficiency, O X being the line from which pressures are measured, and O Y the line from which volumes are measured. At the point 1 the pressure and volume of 1 lb. of water in the boiler are represented, which water during admission is evaporated at constant pressure, as represented by the straight line 1 2, the other extremity 2 of which represents the pressure and volume of the st
produced. The steam is then cut off, and expansion

place without gain or loss of heat till the pressure has fallen to 3, which must be supposed the pressure in a surface condenser. At the end of the stroke the piston returns, and condensation takes place under that same constant pressure.

FIG. 11.



So far the diagram is exactly the same as an ordinary indicator diagram, in which expansion has been carried to its extreme limit, namely, till the pressure has fallen to the pressure of the condenser. But now, instead of the condensation being complete, we must imagine it stopped at a suitable point 4, and the mixture of steam and water compressed without gain or loss of heat until it becomes once more water of the pressure and temperature of the water in the boiler. Then, if we suppose the condensed steam returned into the boiler, the process may be repeated indefinitely. The diagram is drawn to suit the particular case in which the engine is working between the temperatures 302° and 212°: the possibility of the supposed operations and

11

[illegible]

Calculation of the Density of Steam.

47. A detailed comparison between the operation of a perfect steam engine and the steam engine as it actually exists will be instituted in a later chapter: for the present I postpone it, and shall conclude this chapter by showing how the formula just given is applied to calculate the density of steam.

In Table I*a* the pressure in lbs. per square inch is given for any temperature, and, besides, the rise of pressure for 1° rise of temperature. By multiplication by 144 we get Δ P, when Δ T = 1°, and by the addition of two consecutive results the value of Δ P is obtained for 2°, corresponding approximately to the intermediate temperature: but L is known from Regnault's experiments, hence $v - s$ is determined.

For example, to calculate the volume of 1 lb. of steam at a pressure of 25 lbs. on the square inch. Here the corresponding temperature is 240° Fahr., and the value of Δ P for 2° is

$$\Delta \mathrm{P} = (\cdot 458 + \cdot 452)\ 144 = 131 \text{ lbs. on the square foot};$$

also by Table II*b* the latent heat of evaporation per lb. is

$$\mathrm{L} = 730{,}700 - 550 = 730{,}150;$$

therefore, using the formula found above,

$$131\ (v - s) = \frac{730{,}150 \times 2}{461 + 240};$$

$$\therefore\ v - s = \frac{1{,}460{,}300}{701 \times 131} = 15 \cdot 902,$$

whence putting $s = \cdot 016$ we get to two places of decimals,

$$v = 15 \cdot 92,$$

closely agreeing with the result given in the density table. To obtain very accurate results by this method it is necessary, in order to obtain a more approximate value of Δ P, to use a formula derived from one of the formulæ representing t[illegible] between pressure and temperature, instead of

resorting to the table, because the differences are not given by the table with sufficient accuracy and (especially at high temperature) the interval of 2° is not sufficiently small. This is further explained in the Appendix.

It is by this method that the density of steam is found by calculation as mentioned in Chapter I. The possible errors in the calculation are as follows:—

(1) The mechanical equivalent of heat is taken as 772. As stated in Chapter II. this value is certainly very approximate, and there is good reason to believe that the maximum possible error here is less than one-half per cent. From the formula it is clear that the calculated values of the density of steam are all in the same proportion subject to any error occasioned in this way.

(2) The temperatures were measured by Regnault in his experiments on a thermometer of real air, whereas in the formula temperatures are measured by a thermometer in which a perfectly gaseous body is used. The error here cannot be estimated precisely, and will be different at different parts of the scale: but there is reason to believe that such error is very small. Also the position assumed for the absolute zero is open to possible error not exceeding a degree. (See Appendix.)

Subject to these observations the density of steam is calculated with the same degree of accuracy that Regnault's experiments were made; and although no calorimetrical experiments can be expected to be free from minute errors, yet it is certain that the accuracy actually attained was very great: so that the density of steam determined in this way must be within (probably we may say) one per cent. of the truth for steam of the same degree of dryness as that experimented on by Regnault. The precautions taken by Regnault to secure dry steam were explained in a former chapter, and it is probable that the steam from an ordinary boiler is usually not free from suspended moisture, in

which case its latent heat will be less and its density greater as before stated.

The densities of steam at different pressures are tabulated by Rankine in his work on the Steam Engine; they differ by minute quantities from the results given by continental writers who mostly base their work on Clausius' investigations made independently at about the same time. The densities given in Table III., and certain quantities in Table IV. dependent on them, are derived from Rankine's results, as more exact values are not attainable in the present state of our knowledge. It is much to be wished that a further direct experimental investigation should be made of the density of steam.

In Chapter II. the internal work done during evaporation at constant temperature was expressed by means of an equivalent pressure on the piston, called, for brevity, the "internal-work-pressure." The same equation which furnishes the density of steam likewise furnishes the value of this pressure. For let $\bar{P}$ be this pressure, then

$$\text{Internal Work} = \bar{P}\,(v - s);$$

$$\text{External Work} = P\,(v - s);$$

$$\text{Heat Expended} = L;$$

$$\therefore\ L = (\bar{P} + P)\,(v - s) = (\bar{P} + P)\,.\,\frac{L}{T}\cdot\frac{\Delta T}{\Delta P},$$

whence

$$\bar{P} = \frac{T\,\Delta P}{\Delta T} - P,$$

an equation which furnishes an easy means of calculating $\bar{P}$, which, it will be observed, is just the same whether the evaporation be partial or whether it be complete: that is to say, the values of $\bar{P}$ are not subject to any such uncertainty as may be considered to exist respecting the density of steam, ‑cept that the scale of Regnault's air thermometer is sup‑ ‑l identical with that of a perfect gas thermometer.

‑le V. gives the internal-work-pressure during evapora

tion at various constant pressures in lbs. per square foot and lbs. per square inch, together with the differences needful for interpolation. It has been calculated from a formula derived from Rankine's formula for the pressure of steam (see Appendix), but a result in close agreement may be obtained by the use of Table I*a* together with the formula just now given.

The ratio $\overline{\mathrm{P}} : \mathrm{P}$ is given in another column of the same table, it is the number denoted by k in Chapters II. and III., and given in Table III., for various temperatures. This ratio may likewise be obtained direct from Table I*a*, for from the above equation

$$k = \frac{\overline{\mathrm{P}}}{\mathrm{P}} = \frac{\mathrm{T}}{\mathrm{P}} \cdot \frac{\Delta \mathrm{P}}{\Delta \mathrm{T}} - 1,$$

which may also be written

$$k = \frac{\Delta (\log. \mathrm{P})}{\Delta (\log. \mathrm{T})} - 1.$$

Numerical examples will be found at the end of the table.

[redacted]

CHAPTER VI.

GENERATION OF STEAM IN A CLOSED BOILER.

48. THE question of the evaporation of water in a closed vessel has been already considered in the simple case in which the quantity of water is so small in proportion to the size of the vessel that all the water can be converted into steam without producing excessive pressure. It was then shown that the heat required completely to evaporate the water could be found without difficulty; but it was not shown how to find the heat required partially to evaporate the water, a case of some interest, involving as it does the expenditure of heat in getting up steam to a given pressure in a steam boiler, and the rate at which the pressure will rise when the safety valves are fastened down and the engine is standing.

Let the water-room in a boiler be m times the steam-room, then if s be as usual the volume of a pound of water, it is clear that for each cubic foot of water in the boiler before sensible evaporation commences there will be $1 + \dfrac{1}{m}$ cubic feet of total room for water and steam together, and the volume of 1 lb. of water and steam together must therefore be

$$V = \frac{m+1}{m} \cdot s,$$

which volume remains constantly the same during the whole operation. Now it has been already shown, in Art. 12, that when water is partially evaporated at *constant* pressure, the internal work done, reckoned from water at 32°, is

$$I = h + \overline{P}(V - s),$$

where $\overline{P}$ is the internal-work-pressure in lbs. per square foot, V is the volume of 1 lb. of the mixture of steam and water, and h has the usual meaning. Substituting for V

$$I = h + \frac{s}{m} . \overline{P},$$

a formula which gives the internal work done in producing 1 lb. of the mixture from water at 32°, when the process takes place at constant pressure. But it has been repeatedly explained that the same amount of internal work is done, however the process of evaporation is conducted, and therefore, since in the present case no external work is done, this formula gives for each lb. of weight of the contents of the boiler the heat expended in getting up steam from water at 32°. If the water originally have any other temperature t_0, then the corresponding value of h must be subtracted, so that the heat expended (Q) is

$$Q = h - h_0 + \frac{s}{m} . \overline{P} \text{ foot lbs.}$$

It will be convenient to have the result in thermal units, which is easily obtained by division by 772, whence we have

$$Q = h - h_0 + \frac{s}{772\, m} . \overline{P} \text{ thermal units,}$$

in which h, h_0 must now of course be taken from the thermal unit table. Finally, by writing

$$\overline{P} = 144 \overline{p} \text{ and } s = \cdot 016$$

[illegible] formula

$$Q = h - h_0 + \frac{\overline{p}}{336\, m}$$

[illegible] expenditure of heat in getting up steam [illegible] water at temperature t_0. The first two [illegible] represent the heat which would have [illegible] formation of steam had been com[illegible] application of sufficient pressure to

the surface of the water; the second term (always relatively small) is the correction necessary to provide for the formation of steam.

In steam boilers it appears that m is seldom, if ever, less than unity: that is to say, that the boiler is rarely less than half full, and is usually much more. Putting then $m = 1$ as the case in which the correction is greatest, and taking the values of $\bar{p}$ from Table V. at the end of the book, we obtain the results shown in the annexed table, assuming the boiler when cold to be at temperature 60°.

p. (Absolute.)	Q. (Thermal Units.)	$t - t_0$.	Δ Q. From 50 lbs. pressure.	θ'.
250	353·7	341	129	98
140	301·2	293	76·5	60
90	265·8	260	41·1	31
50	224·7	221		
25	182·2	180		

If the temperature of the boiler when cold is not 60°, but a different temperature, then the results of the table are to be modified by the addition of the difference when below, or the subtraction when above, 60°. The third column of the table shows the value of $t - t_0$, which is less, partly on account of the specific heat of water being greater than unity, so that $h - h_0$ is greater than $t - t_0$, and partly on account of the correction spoken of above. It will be seen that the increase of Q due to these two causes is not of great importance, and hence for practical purposes may often be neglected.

The heat necessary to raise steam at one given pressure to steam at another pressure is found by taking the difference of the corresponding values of Q. Thus, for example, let us suppose a boiler working at 50 lbs. pressure absolute, and let it be asked what amount of heat is required to rais[illegible] pressure by a given amount; then we have only to sul[illegible]

the value of Q for 50 lbs. from its value for the new pressure. The fourth column of the table shows the result for the pressures indicated, from which we see that a comparatively small amount of heat is required to produce a great increase of pressure.

The time occupied in raising the pressure can be found when we know the amount of heat furnished by the furnace, and the cubic contents of the boiler. The proportion which the water contained in a boiler bears to the evaporation in a given time appears to vary a good deal, according to the size and type of boiler, and even in boilers of the same size and type. According to Armstrong's rule for flue boilers, the water-room of a boiler should be 13½ times the volume of water evaporated per hour, or 810 times the volume evaporated per minute, in which case the weight of water in the boiler would also be 810 times the weight evaporated per minute. Now, the total heat of evaporation of water from 100° at the temperature corresponding to 50 lbs. pressure is 1068 thermal units; and therefore, assuming the evaporation equally active and efficient when the stop valves and safety valves are closed, the time in minutes necessary to raise the pressure to p lbs. on the square inch will be found by multiplying by 810 and dividing by 1068, the results of which operation appear in the fifth column of the table, headed θ^1; from which it appears that in half an hour the pressure will have risen to almost 90 lbs. per square inch; in an hour, to more than 140 lbs. per square inch; and in an hour and forty minutes, to over 250 lbs. per square inch.

The last result shows the rapidity with which the pressure rises when we have to do with high-pressure steam, the increase of *temperature* being approximately proportional to the time, but the increase of *pressure* far more rapid.* The

* An experiment made by the late Sir W. Fairbairn in 1853 on a locomotive agrees closely with this theoretical conclusion. See 'Useful Information for Engineers,' 2nd edition, p. 324.

time occupied in getting up steam to 50 lbs. pressure from water at 60° is, on the same principle, about 2¾ hours. These results are confirmed by experience for stationary boilers of the type indicated.

In tubular boilers, the water-room is generally much less than that given by Armstrong's rule, and the times required are consequently less than those given. The principles explained are, however, sufficient to enable any particular case to be calculated at pleasure.

49. If in the formula for Q we consider a rise of temperature of 1°, we then get the "specific heat at constant volume" of a mixture of steam and water. The formula may then be written

$$\Delta Q = \Delta h + \frac{\Delta \bar{p}}{336\,m},$$

in which the value of Δh is given in the table for h, being the specific heat of water at the temperature considered, and $\Delta \bar{p}$, which for an increase of 1 lb. is given in Table V., is easily found by multiplying the tabular result by the value of Δp given in Table I*a* for the temperature indicated. Thus, for example, to find the specific heat at constant volume of the contents of a boiler at pressure 100 lbs. per square inch. Here, from the tables,

$$\Delta h = 1 \cdot 03, \qquad \Delta p = 1 \cdot 39,$$

and the tabular value of $\Delta \bar{p}$ is $8 \cdot 66$,

$$\therefore\ \Delta Q = 1 \cdot 03 + \frac{1 \cdot 39 \times 8 \cdot 66}{336\,m} = 1 \cdot 03 + \frac{\cdot 036}{m};$$

or if the boiler be half full as before,

$$\Delta Q = 1 \cdot 066 \text{ nearly.}$$

50. Another question closely connected with the subject of the present chapter is to find the amount of heat necessary to dry a given volume of moist steam. Let $1 - x$ be the weight of suspended moisture in a lb. of steam of pres

P^1, then, assuming the hyperbolic law as sufficiently approximate,

$$P^1 v^1 = P v \quad : \quad v = V^1,$$

where P is the pressure when the steam has become dry, v the corresponding volume, $V^1 = v^1 x$ the original volume, hence

$$P = \frac{P^1}{x}$$

$$\text{and } \Delta P = P^1 . \frac{1 - x}{x}$$

gives the increase of pressure, very approximately, necessary to dry the steam at constant volume.

To obtain the heat needful we have

$$\begin{aligned} \Delta Q &= \Delta h + V_1 \, \Delta \overline{P} \\ &= \Delta h + x v^1 \, \Delta \overline{P} \end{aligned}$$

from which numerical results can readily be computed by aid of the tables.

For example, suppose steam of 60 lbs. pressure to contain 10 per cent. of suspended moisture, how much heat is required to dry it at constant volume? Here $x = \cdot 9$ and hence

$$\Delta p = \frac{60}{9} = \frac{20}{3} = 6\tfrac{2}{3},$$

so that the pressure rises to $66\frac{2}{3}$ approximately. The corresponding rise of temperature is 7° nearly, whence, referring to Table II*b*, the value of Δh is

$$\Delta h = 7 \times 793 = 5551 \text{ foot lbs.}$$

Also referring to Table V., we find for $\Delta \overline{P}$

$$\Delta \overline{P} = \frac{20}{3} \times 1327 = 8847 \text{ lbs. on the sq. ft.}$$

$$\therefore \ \Delta Q = 5551 + \cdot 9 \, v^1 . \, 8847.$$

Also v^1 at the pressure of 60 lbs. is 7 cubic feet nearly.

$$\begin{aligned} \therefore\ \Delta Q &= 5551 + 6 \cdot 3 \times 8847 \\ &= 5551 + 55{,}736 = 61{,}287 \text{ foot lbs.} \\ &= 79 \cdot 4 \text{ thermal units nearly,} \end{aligned}$$

which determines the required amount of heat. It will be seen that the term representing the evaporation of the water is much greater than that representing the elevation of temperature.

CHAPTER VII.

EXPANSION OF STEAM.

51. As explained in Chapter III. and confirmed by universal experience, the expansion curve of steam is approximately a common hyperbola; but I now propose to consider this question more fully by investigating the law of expansion under given circumstances, and conversely the circumstances under which steam will expand according to a given law.

In the case of air, it has been already shown that the law of expansion depends on the amount of heat (if any) received or lost by the air at each step of the expansion. Precisely the same thing takes place in the case of steam, and the question reduces itself to this: to find the law of expansion when the heat received or lost is given, and conversely, to find the heat which must be supplied or abstracted in order that the expansion curve may be of given form. Of this question I shall now consider certain ideal cases, commencing as usual with the simplest; the cases most often occurring in practice being too complex to be discussed at the commencement of the subject.

Dry and Saturated Steam.

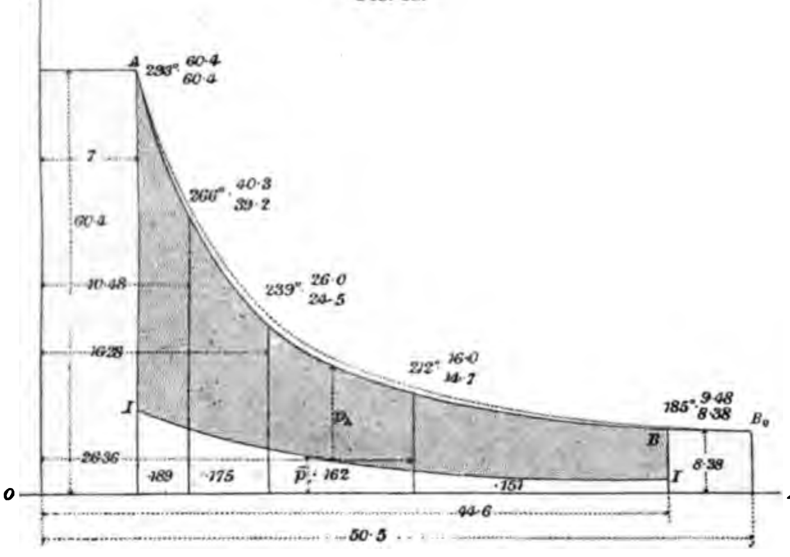
52. First in order of simplicity is the case in which the steam is supposed always dry and saturated during the whole progress of the expansion; and the question to be answered will be as to the amount of heat, to be supplied from without, to keep the steam in the state supposed.

Here the law of expansion is expressed approximately by the equation

$$p\, v^{\frac{17}{16}} = \text{constant},$$

being the approximate relation which according to Art. 5 always connects the pressure and volume of saturated steam so long as it remains dry. Probably the easiest way of constructing the curve is to take the volumes corresponding to given pressures from Table III., and setting them off along the volume axis O X (Fig. 12), to set up the corresponding pressures as ordinates; then a curve drawn through

Fig. 12.



the extremities of the ordinates will be the expansion curve, which, as the form of the equation shows, does not differ greatly from an hyperbola: though other methods may also be adopted, one of which will be mentioned presently. In drawing the curve for a very great range of pressure, it is advisable to represent the upper part and lower part on different scales for the sake of distinctness. Figure represents the part of the curve which lies between

pressures 60·4 and 8·38 lbs. on the square inch absolute; in this case the volume increases from 7 cubic feet to 44·6 cubic feet, showing a ratio of expansion of about 6·37. The dotted curve A B_0 lying above the expansion curve A B represents an hyperbola drawn through A, and shows that the pressure of the steam is always less than if it followed the hyperbolic law, the diminution being represented at each point by the vertical distance between the two curves. As the pressure falls, the temperature falls too, according to the law expressed by Table I., and this is shown on the figure at various points of the expansion by giving the values of the temperature attached; the other numbers giving the pressures for the hyperbola and the actual curve respectively. Thus when the volume has increased from 7 cubic feet to 26·36 cubic feet, that is when the steam has expanded 3·77 times, the temperature has fallen to 212° and the pressure to 14·7, while the pressure according to the hyperbolic law would have been 16, showing a diminution of pressure of 1·3 lb. Hence if the steam really expand according to the hyperbolic law from the dry and saturated state at A, it must become superheated, and that the more the greater the expansion; also the ratio of expansion in the hyperbola in order that the pressure may fall by a given amount must be greater, thus in the figure the steam must expand till its volume is 50·5 in place of 44·6 before its pressure can fall to 8·38.

Hyperbolic expansion will be discussed in the next section; at present we have to do with dry steam, and the question is to find what heat, if any, is to be supplied in order that it may always be dry and yet not superheated. For this purpose it is only necessary to apply the equation,

Heat Expended = Internal Work + External Work,

by considering how much work of either kind is done during each step of the expansion.

First, as to the external work: it is shown in the Appendix that the area of any curve of the form

$$P\,V^{n} = \text{constant},$$

included between two ordinates and the base, is

$$\text{Area} = \frac{P_1 V_1 - P_2 V_2}{n - 1};$$

but in the present case the value of n is approximately $\frac{17}{16}$, and for V we write v, because the volumes are those of dry saturated steam: hence

$$\text{Area} = 16\,(P_1\,v_1 - P_2\,v_2),$$

and thus, to find the work done on the piston as the steam expands from volume v_1 to volume v_2, we have only to take the difference of the values of P v at the beginning and end of the portion of the expansion considered and multiply by 16. But the values of P v are given in Table IV*a*, and thus the external work done is found without difficulty. It will be convenient to divide the expansion into stages during each of which the fall of temperature is 27°; the figure represents four of these stages, namely, from 293° to 185°: the annexed table, in the column headed $16\,.\,\Delta\,P\,v$, shows the external work done during each stage, not merely for these four, but for the whole range of pressure from 250 lbs. to 4½ lbs. on the square inch: the absolute values are divided by 27 to obtain the mean value per 1°.

Next, for the internal work it is only necessary to consider the values explained in Chapter II., Art. 11, and tabulated in Table IV*c*, for the total internal work (I) done in changing water at 32° into dry steam of any pressure; for, since the amount of internal work done during any change does not depend on the way in which the change is produced, it is clear that to change dry steam at one pressure into ' steam at another pressure, we have only to take the diffe of the corresponding values of I. Inspection of the

shows that I increases slowly with the temperature: thus, to form dry steam at 212° from water at 32° requires 1074·4 thermal units, while if the steam be formed at 401°, 1119·2 thermal units are necessary, whence it follows that to change dry steam at 212° into dry steam at 401° we must expend in internal work 1119·2 – 1074·4, say 44·8 thermal units. Conversely, when steam expands always remaining dry a part of the external work done is done at the expense of the internal energy of the steam, that is to say so much as is equivalent to the corresponding diminution of I, so that we have only to take the work-equivalent of I to find out how much that is.

EXPANSION OF DRY STEAM.

$t°$.	$16.\Delta P v$ per 1°.	Δ I per 1°. Foot lbs.	$\frac{\bar{p}}{p}$	Δ Q. Foot lbs. per 1°.	Δ Q. Thermal units per 27°.
401°					
	717	197	·275	520	18·2
374°					
	776	192	·247	584	20·4
347°					
	836	187	·224	649	22·7
320°					
	889	182	·205	707	24·7
293°					
	942	178	·189	764	26·7
266°					
	996	174	·175	822	28·8
239°					
	1049	170	·162	879	30·8
212°					
	1102	166	·151	936	32·8
185°					
	1155	162	·140	993	34·7
158°					

The column headed Δ I shows the result of this calculation for each stage of the expansion, from which it appears, taking the third stage for example, in which the temperature

falls from 347° to 320°, corresponding to a fall of pressure from 130 lbs. to 90 lbs., that the external work per 1° is on the average 836 foot lbs. and the corresponding diminution of the work-equivalent of I is 187 foot lbs., the difference of 649 foot lbs. is the external work done by the agency of heat supplied from without, and if that heat be not supplied the steam will not remain dry, but some of it will be condensed. The columns headed $\Delta\dot{Q}$ are thus calculated, and show, the first in foot lbs. per 1°, and the second in thermal units for the whole stage of 27° the heat which must be supplied from without during each stage of the expansion to keep the steam dry.

The column headed $\frac{\bar{p}}{p}$ shows the proportion which the internal work bears to the external work and enables us to exhibit the whole process graphically by constructing a curve of internal work in the same manner as was explained in detail in Chapter IV. for the case of air (see Art. 32). Let $\bar{p}$ be the internal-work-pressure, $\bar{p}$ will be to the external pressure p in the same proportion that the internal work during a very small part of the expansion bears to the external work, and the numbers given in that column will therefore be the average values of $\frac{\bar{p}}{p}$ during the stage of expansion indicated. Hence the curve of internal work showing the internal-work-pressure at each point is readily drawn: in the figure that curve is represented by I I, its area gives the internal work just as the area of the expansion curve gives the external work, and the difference of areas (shaded in the figure) shows the heat supplied during expansion.

Conversely, if steam be compressed, in order that it may remain in a saturated condition, heat must be taken away from it at each step of the compression as its temp and pressure rise, otherwise it will become superheate

heat so taken away per 1° of rise of temperature is called the "specific heat" of steam when that term is used without qualification: it is said to be negative because heat is subtracted, not added, as the temperature rises. The column headed Δ Q shows the work-equivalent of the mean specific heat of steam during the interval of temperature in which it occurs. The second column headed Δ Q shows in thermal units per 27° the same amount of heat; thus in the case illustrated by the figure where the steam expands from 60·4 to 8·38, by adding the numbers given for each stage we find, for the four stages, 119 thermal units as the heat required to keep the steam dry. The numerical results here found are only rough approximations, for any minute error in the determination of the density of steam by the empirical formula $p\, v^{\frac{17}{16}} = \text{const.}$ and of the products P v by the tables is multiplied many times by the process of calculation; but they are without doubt a tolerable approximation, and they show that the heat required to keep steam dry varies from four-fifths to five-sixths of the external work done during expansion.

A somewhat more exact result is obtained by the use of Zeuner's index 1·0646 in place of $\frac{17}{16}$ths, but when numerical exactitude is required a method described in the Appendix, Note D, is far preferable. By this method the exact value of the specific heat of steam can be found at any temperature, but it requires more mathematical knowledge, and does not show so clearly the nature of the process as the method given in the text.

Another formula (Fairbairn and Tate's) was given in Art. 5, namely,

$$v = \cdot 41 + \frac{389}{p + \cdot 35},$$

which suggests a method of drawing the curve in [illegible] way without reference to the tables. Draw [illegible] reference for measurement of abscissæ and ordi[illegible]

to the right of and below the old ones by ·41 cubic foot and ·35 pound on the square inch respectively, and with these new axes describe an hyperbola, then the form of the formula shows that this hyperbola considered relatively to the old axes will be the required curve, for the formula may be written

$$(v - \cdot 41)(p + \cdot 35) = 389 = \text{constant}.$$

The curve may likewise be drawn, and the mean pressure calculated or graphically constructed by the general method applicable to all curves of the form $P\ V^n =$ const., explained in the Appendix.

The curve being constantly used in our subsequent work, will for the sake of a name be called the "saturation curve."

Hyperbolic Expansion.

53. The next case to be considered is that in which the expansion curve is an hyperbola, that is to say where

$$p\,V = \text{constant} = p_1\,V_1,$$

in which, as usual, p is the pressure and V the volume of a lb. of steam. In this case the steam becomes drier and drier as it expands if it be originally moist, and becomes superheated if it be originally dry, for by Art. 6

$$V = u\,x + s = v\,x \text{ (approximately)},$$

hence since

$$p\,v^{\frac{17}{16}} = p_1\,v_1^{\frac{17}{16}}$$

$$x = x_1\left(\frac{v}{v_1}\right)^{\frac{1}{16}} = x_1\left(\frac{p_1}{p}\right)^{\frac{1}{17}};$$

but if r be the ratio of expansion,

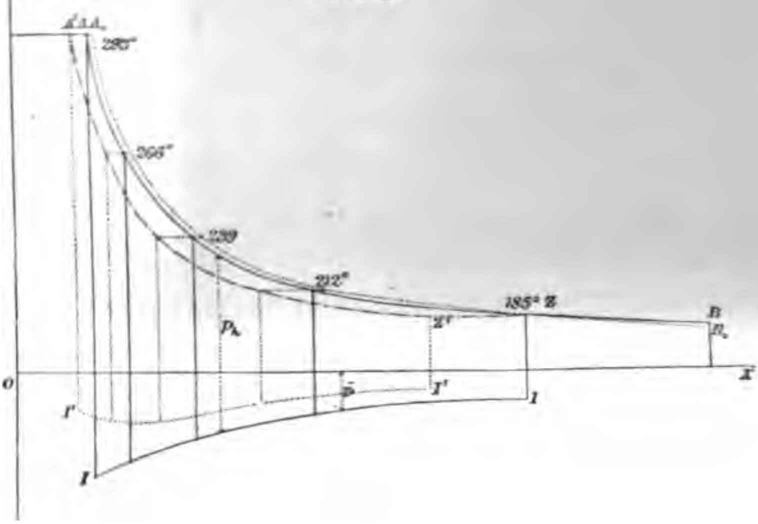
$$r = \frac{V}{V_1} = \frac{p_1}{p},$$

$$\therefore\ x = x_1\,.\,r^{\frac{1}{17}};$$

this shows that, if the expansion starts from a point indicated by the suffix 1, x is greater than x_1, so that if the expansion be c[illegible] enough, the steam becomes dry, and if still f[illegible]ed, a result which agrees with the last article.

In the figure (Fig. 13), if A Z B be the hyperbolic expansion curve, $A_1 Z B_1$ the saturation curve considered in the last article: these curves cross one another at a point Z, easily determined if the amount of moisture in the steam at any given pressure is known. Above this point the steam is wet, below, it is superheated.

FIG. 13.



Taking first the expansion above Z, the amount of moisture at any point 1 is given by putting $x = 1$, then

$$x_1 = r^{-\frac{1}{17}}.$$

Thus, for example, let steam of pressure 60·4 lbs. expand till its pressure has fallen to 8·38 lbs., that is to say, with a ratio of expansion 7·22, and let it be dry at that pressure, then

$$x_1 = (7\cdot 22)^{-\frac{1}{17}} = \cdot 89,$$

so that the steam must contain originally 11 per cent. of moisture.

The same result may be obtained graphically if the figure be accurately drawn on a large scale, for the horizontal

distance between the curves always represents the volume of steam existing in a state of moisture.

The temperature always falls with the pressure, according to the same invariable law, until superheating commences, and it is convenient, as before, to split up the expansion into stages during each of which the fall of temperature is 27°. The figure shows four of these stages from 293° to 185°, just as in the previous case, and we have to find the external and internal work done during each stage. First, as to the external work: let p be the pressure at the commencement, and p' at the conclusion of any stage, then by Art. 23

$$\text{External Work} = \mathrm{P\,V} \,.\, \log._e \frac{p}{p_1},$$

where P V is the constant product (in foot lbs.) of the pressure and volume. At Z the volume V is that of dry steam at the corresponding pressure; so that P V is the same as P v, which by Table IV*a* we find to be at 185°, 53,900 foot lbs.

$$\therefore \ \text{External Work} = 53{,}900 \log._e \frac{p}{p^1},$$

from which formula, by division by 27 and substitution of the pressures, the external work per 1° is found for each of the four stages, and tabulated in the second column of the annexed table. To find the internal work, the formula

$$\mathrm{I} = h + \overline{\mathrm{P}}\,(\mathrm{V} - s) \qquad (\text{Art. } 12)$$

is to be used, which shows the internal work done in forming V cubic feet of steam at any pressure by any process, or neglecting s as usual

$$\mathrm{I} = h + \overline{\mathrm{P}}\,\mathrm{V} = h + k\,\mathrm{P\,V},$$

in which last form k P is written for $\overline{\mathrm{P}}$.

Then P V is constant as above, and k is known from th tables, so that the change of internal work is found by

$$\Delta\,\mathrm{I} = \mathrm{P\,V} \,.\, \Delta\,k - \Delta\,h,$$

where $\Delta\,k$, $\Delta\,h$ are the differences of k and h for e

It is to be observed that Δh has a negative sign, because h diminishes as the temperature falls. Writing for P V its value 53,900 foot lbs. we get from Tables II*a* and IV*a* in the example considered the values of Δ I shown in the third column of the table in foot lbs. per 1°; from which it appears that when steam expands according to the hyperbolic law, not only is heat required to do the external work, but also to produce internal changes in the steam itself.

HYPERBOLIC EXPANSION.

Remarks.	$t°$.	External work per 1°.	Internal work per 1°.	$\frac{\bar{p}}{p}$	Value of Δ Q. Ft. lbs. per 1°.	Value of Δ Q. Thermal units per 27°.
Steam dry at 185°.	293°					
		860	310	·360	1170	41
	266°					
		940	434	·462	1370	48
	239°					
		1020	577	·565	1600	56
	212°					
		1120	660	·589	1780	62
	185°					
Steam contains 20 per cent. moisture at 185°.	293°					
		688	89	·130	777	27
	266°					
		752	188	·250	940	33
	239°					
		816	300	·368	1116	39
	212°					
		896	365	·407	1261	44
	185°					

This heat, which must be supplied from without, in order that the steam may expand exactly in accordance with the hyperbolic law, is shown in the columns headed Δ Q, in the first column in foot lbs. per 1°, in the second in thermal its for the whole stage of 27°. As before, the numbers

obtained are only approximations, but are undoubtedly close approximations, to the actual facts.

The value of $\frac{\bar{p}}{p}$ is given in another column, and the whole process exhibited graphically in Fig. 13, as in the previous case.

If the expansion be carried beyond Z the steam becomes superheated, and from the existing deficiency in experimental data we cannot say with exactness what takes place; it is, however, clear that the temperature will go on falling till it reaches a limit given by the equation

$$85{\cdot}5\,(t + 461) = \mathrm{P\,V} = 53{,}900 \quad . \quad \text{(Art. 32)},$$

whence

$$t = 170^\circ,$$

that is to say, till the temperature has fallen 15° more; the steam will then be completely superheated, and the curve of internal work will reach the axis of volumes (see Art. 34). But how far expansion must proceed to realize this, cannot at present be determined with any certainty. If B be the point where the steam becomes completely superheated, the internal-work-curve will reach the axis at B if not before; according to the not improbable supposition of Hirn that the isodynamic curve (Art. 68) is an hyperbola not only when the steam is completely superheated but even when the superheating is only partial, it would follow that the internal-work-curve reaches the axis immediately superheating commences, and in that case the heat supplied from without during that part of the expansion in which the steam is superheated is simply the heat-equivalent of the external work done during that part.

54. If, instead of supposing the steam dry at the end of the expansion, as in the preceding example, it be supposed wet, then the results obtained will be somewhat modified. For example, imagine the steam to contain 20 p

moisture at 185°, then to find the amount of moisture initially,

$$x_1 = \cdot 8 \,.\, r^{-\frac{1}{16}},$$

or, since the ratio of expansion will be unaltered,

$$x_1 = \cdot 8 \times \cdot 89 = \cdot 712,$$

that is, the initial amount of moisture is 28·8 per cent., showing that 8·8 per cent. is evaporated during expansion instead of 11 per cent.

The value of P V is now ·8 × 53,900, or 43,120, and the external work during each stage is diminished in like proportion. For the internal work

$$\Delta \mathrm{I} = 43{,}120 \,.\, \Delta k - \Delta h,$$

the results of which formula are shown in the table.

In Fig. 13 the process is graphically represented: $A^1 Z^1$ is the expansion curve, which is an hyperbola four-fifths the size of the original. The horizontal distance between the expansion curve and the saturation curve still represents the volume of steam existing as moisture, which of course now is everywhere much greater than before. The dotted curve $I^1 I^1$ below is the internal-work-curve, which now is no longer always convex towards the axis, but attains a maximum distance and then approaches the axis again. This always happens when the steam contains much water.

55. If, now, we compare the results obtained, when the steam expands according to the hyperbolic law, and when it remains always dry and saturated, it appears that an apparently trifling difference in the law of expansion makes a great difference in the heat needful to produce it. Thus it was found above that in expansion from 293° to 185°, while remaining dry, the heat supplied is 119 thermal units; but in hyperbolic expansion, on adding the numbers in the last column of the last table it will be found to be in the first case 207 thermal units, and in the second case 143 thermal units. Hence, conversely, a great change in the heat suppl

produces a small change in the law of expansion; and if it is further considered that the indicator tells us nothing about the absolute size of the expansion curve, so that the two cases of hyperbolic expansion just considered would appear identical, it will not be surprising that the expansion curve of steam does not appear to vary much in practice in the most various circumstances.

It is sometimes supposed that hyperbolic expansion in steam is the same as hyperbolic expansion in a perfect gas; in fact, however, the two cases are very different. When a perfect gas expands according to the hyperbolic law, its temperature remains constant, and the heat supply is just that needful to perform the external work; whereas when steam so expands, its temperature falls rapidly, and the heat supply must be from one-fourth to one-half greater than that equivalent to the external work done.

56. In the last articles it has been found convenient, as in the case of air (Art. 34), to represent the internal work which is being done during expansion, by means of an ideal pressure on the piston, just as the external work is represented by the real pressure; yet it must always be remembered that this ideal pressure depends not merely on the actual state of things at the instant considered, but also on the law of expansion, that is, on the way in which that state varies from instant to instant.*

Thus in the two kinds of expansion just considered the same actual pressure on the piston corresponds to two different internal pressures; in the first, a pressure in the same direction as, and forming part of, the actual steam pressure; while in the second it forms a resistance to be overcome, or, so to speak, a back pressure. It usually varies from point to point of the expansion, as shown by the curve of internal

* For this reason in seeking an abbreviation for the phrase "pressure equivalent to internal work," I have preferred the rather cumbrous expression "internal-work-pressure" to the briefer term "internal pressure" which might prove misleading.

work, and its mean value may be found just in the same way as the mean actual pressure on a piston, and may be expressed either with reference to the whole stroke, including both admission and expansion, as is usually done, or with reference to the expansion alone. Examples will occur hereafter of both ways of expressing it.

When steam is formed by evaporation under constant pressure, the internal-work-pressure is constant, and is given for each external pressure by Table V. When steam is formed in any way from water of the same temperature the internal-work-pressure is not generally constant, but its mean value is the same as if the steam were formed under constant pressure.

Any Given Expansion Curve.

57. I shall next show that, if a perfectly accurate indicator diagram be given together with the weight of steam used per stroke, it will be possible to determine the curve of internal work, and to deduce the heat supplied to the steam at each step of the expansion.

In the first place, let it be supposed that the pressures at the beginning and end of the expansion are exactly known. Then, by Table I., the temperatures are likewise known, and the corresponding values of h and $\overline{P}$ in the formula

$$I = h + \overline{P}(V - s)$$

can be found by Tables II. and V. Moreover, dividing the volume of the cylinder by the weight of steam used per stroke, the terminal value of V is known, and the initial value can then be found by division by the ratio of expansion. Thus, the internal work reckoned from water at 32° is determined both for the beginning and end of the stroke. Subtraction now furnishes the internal work done during expansion, and by adding the heat-equivalent of the area of the expansion curve which represents the external work, the supply of heat is obtained.

The effects (often very important) of clearance and wire-drawing are throughout this chapter wholly neglected, being reserved for discussion at a later period.

58. The graphical method of Chapter II. not only enables us to represent the process on the diagram, but also to obtain readily definite results.

Fig. 14 represents the admission line S A, and the expansion curve A B, neglecting all effects of wire-drawing: to fix our ideas I take as data the results of one of the experiments on the *Bache* (Chapter XI.), and suppose the initial pressure 90·14 and the terminal pressure 11·73 lbs. on the square inch; further, I suppose the ratio of expansion 8·57. Also, I shall suppose the terminal volume of 1 lb. of the steam two-thirds the volume of dry steam at the terminal pressure, as was probably approximately the case in the experiment in question. Now, the volume of dry steam at 11·73 lbs. per square inch is 32·5 cubic feet nearly, taking two-thirds of which, 21·67 is obtained for the actual volume at the end of the stroke. In the figure then, O N, the base of the diagram, represents 21·67 cubic feet, and O M represents $\frac{21 \cdot 67}{8 \cdot 57}$ or 2·53 cubic feet, which is the initial volume of the steam. The accuracy of these data is not here the question. I have merely to show the results deducible when accurate data are attainable.

First, to find the state of the steam at any point of the expansion, lay off O H_0 = 32·5 cubic feet, and trace the saturation curve A_0 B_0 (see last articles), thus showing the volume of dry steam at any pressure, then the horizontal distance between this curve and the expansion curve shows the amount of water in the steam at any point of the expansion. In particular, the volume S A_0 of dry steam at the initial pressure is 4·8 cubic feet, therefore, A A_0 = 2·27 cubic feet, showing that 47·4 per cent. of steam was in the state [illegible] the commencement of the expansion. The

[illegible]

[illegible]

[illegible]
[illegible]

[illegible]
[illegible]

form of the expansion curve shown in the figure is ideal, the diagram not being given, but only the mean pressure, so that the wetness of the steam cannot be determined at any other point of the stroke.

Next, to find the internal work according to the graphical process of Arts. 9, 12, it is only necessary to refer to Table V., from which it appears that the internal-work-pressure corresponding to 90·14 is 904·5 nearly. Then, constructing the rectangle O Z on the base O M, which represents the actual volume of the steam initially, the internal work done in evaporation at 90·14 is represented by the area of this rectangle; this gives the initial value of the internal work reckoned from water at the initial temperature of 320° corresponding to 90·14. In the same way the terminal value of the internal work is represented by the area of the rectangle O R on the base O N, and of height N R = 149 corresponding by Table V. to 11·73 the terminal pressure, this work being reckoned from 201° the temperature corresponding to 11·73. We have now to take the difference, but before doing so, it is necessary that the initial and terminal values of the internal work should be reckoned from water at the *same* temperature. The most convenient temperature to choose is the initial temperature of 320°; hence we take the difference 320° – 201° or 119° and perform the construction of Fig. 3, Arts. 10, 12, as indicated in Fig. 14, remembering that the difference of 119° must be reckoned negative, because we are now reckoning from the higher temperature. Thus the rectangle O T′ is obtained, which represents the terminal value of the internal work reckoned from water at 320°.

We have now only to find the difference of these values, and for this a simple construction suffices. Complete rectangle O K; join K T, and produce it to meet the pres axis in L; then draw the horizontal line L I_m I_m, as sh in the figure; the required difference is simply the rectan

... proposition of Euclid the rect- ... and consequently the difference ... the same as the difference of L N, L M. ... that the horizontal line $I_1 I_1$ is the line ... during the expansion. In ... the line lies below the volume axis, show- ... greater at the end of the stroke ... the internal-work-pressure ... per square inch.

... represented by the ... and consequently the ... area shaded in the figure, ... and the line $I_1 I_1$. ... the curve, pro- ... the beginning

... example,

... 31,

... Art 90.

... cylinder

clearance to be explained hereafter. Also the influence of wire-drawing—in some cases considerable—is neglected, but in the present case the result is probably substantially correct.

If now, we ask, from whence the cylinder derived so much heat, our first idea would, probably, be, that it was derived from the steam jacket which was in operation during this experiment; if, however, the observed quantity of steam liquefied in the jacket be considered, it will be found that only 16 thermal units were so derived, and the remainder must, therefore, have been obtained from some other source, and that source can only be the steam during admission. Thus at least 106 thermal units were so derived, but in fact much more steam must have been liquefied during admission than is represented by this quantity of heat; for although there are no data from which the state of the steam can be determined as it left the boiler, yet there is no reason to suppose it contained any considerable amount of water; let us assume it dry, then complete the rectangle A Z_0 in Fig. 14, then that rectangle represents the amount of heat abstracted from the steam during admission. To make comparison more easy, reduce this rectangle by the constructions indicated in the figure to the base I_m I_m, then we obtain the rectangle E I_m, which is greater than the heat supplied during expansion by the area between E E and the expansion curve A B. This last area represents the whole heat abstracted by the cylinder from the steam during its passage from the boiler to the end of the stroke, which heat, in addition to that furnished by the jacket, is chiefly transmitted to the exhaust during the return stroke (compare Art. 24). This is true, whatever be the form of the curve in the present case

$$\begin{aligned} \text{Heat abstracted} &= 421 - 126 = 295 \\ \text{Jacket supply} \quad .. \quad .. \quad .. &= 16 \end{aligned}$$

$\therefore$ Exhaust waste = 311 thermal units.

If the result be compared with the whole heat expended as shown by the quantity of water evaporated, it is found that the exhaust waste in the present case was 28 per cent. of the whole; and though the effect of clearance modifies this conclusion to some extent, as will be seen hereafter (Chapter IX.), yet there is little doubt of its substantial accuracy, if the data are correct. I shall return to this subject in a later chapter.

59. So far we have only considered the initial and terminal state of the steam on which alone the position of the line $I_m I_m$ of mean internal-work-pressure depends; if, however, it be desired to know, not merely how much heat on the whole is supplied to the steam, but according to what law it is supplied, it will then be necessary to have an exact expansion curve, which will give the exact value of the pressure at every point, then dividing the expansion into small portions, and going through the construction of the last article for each portion, a series of lines $I_m I_m$ will be obtained, each of which refers to its own part of the expansion. Trace now a curve through the middle points of all these short lines, and the result will be an internal-work-curve, such as was found in the two kinds of expansion first considered in this chapter. The reader will find it a useful exercise to construct in this way the diagrams for the two cases in question.

To each particular expansion curve corresponds its own curve of internal work, showing the law of supply of heat, but, as has been shown in the preceding cases, a very small change in the expansion curve causes a very great change in the position of this curve, so that it will rarely happen that the curve drawn by an indicator can be relied on sufficiently for the purpose.

60. As another example of the method just explained of finding the heat supplied during expansion, when the expansion curve is given, I take as data one of the *Bache*

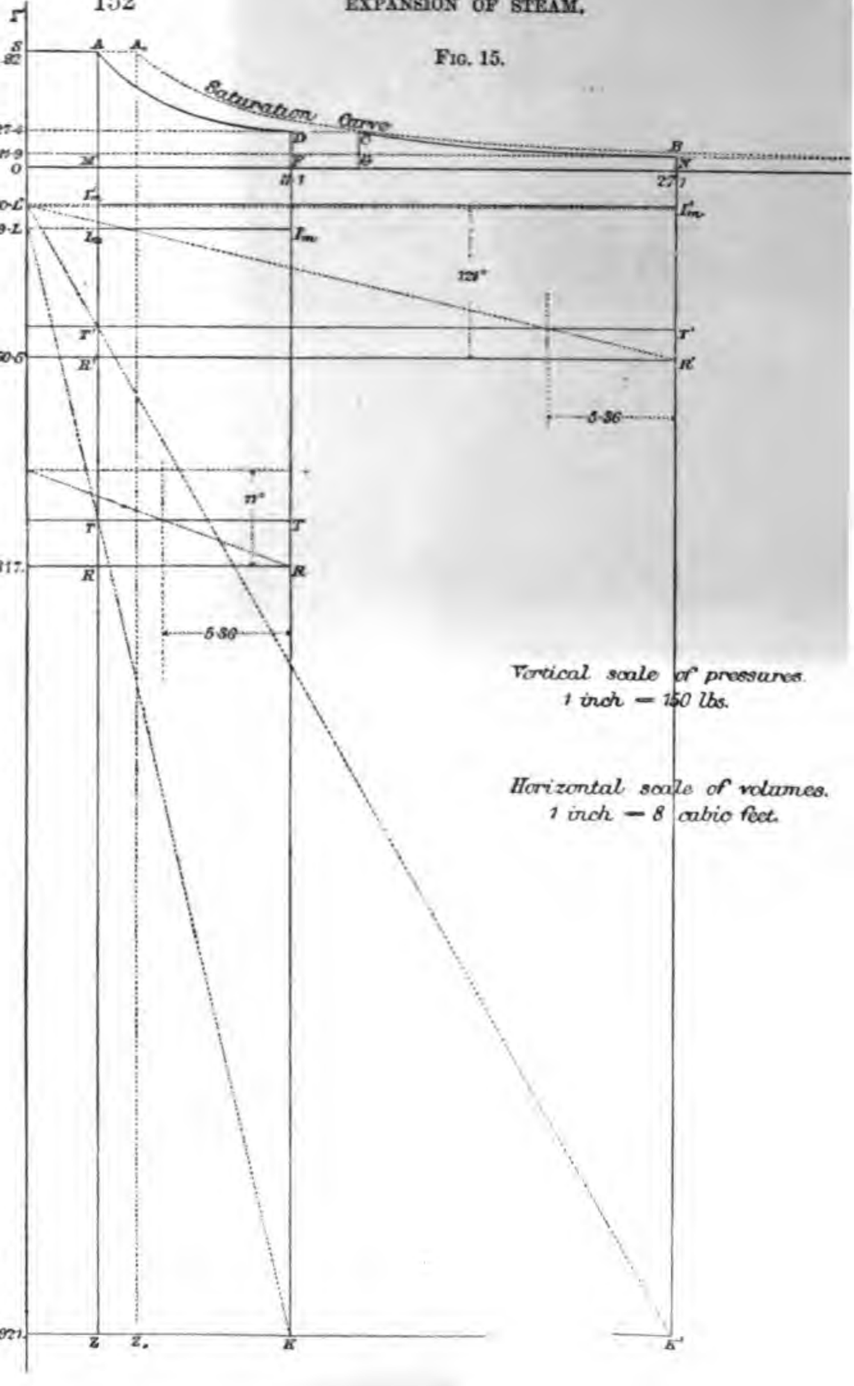
experiments (Chapter XI.), when the engine was tried as a compound engine, as follows:

Initial pressure (small cylinder)	= 92·
Terminal " " "	= 27·4
" " (large cylinder)	= 11·9
Total expansion	= 9·19
Ratio of cylinders	= 2·44
Water in steam at end of stroke (large cylinder)	= ·156

In Fig. 15 lay off O N_0 to represent the volume (32·1) of dry steam at the end of the stroke in the large cylinder, and trace the saturation curve A_0 B_0 as before, which shows the volume of dry steam at any pressure; then, multiplying by ·844 the proportion of dry steam, we get for the volume of the actual steam 27·1 cubic feet; this is represented on the diagram by O N, and the true expansion curve therefore starts from B. Divide now 27·1 by 2·44 the ratio of the cylinders, and 11·1 is obtained for the volume of 1 lb. of the steam at the end of the stroke in the high-pressure cylinder; this is represented by O F in the figure, while D F represents the terminal pressure. Again, dividing 27·1 by 9·19, we obtain 2·95, which is represented in the figure by O M, while A M represents the corresponding pressure of 92.

These points being determined by the data of the question, expansion curves are now to be drawn, which, as before, are ideal, except that the mean pressures are given, as will be explained presently. The full curve A D represents the expansion in the high-pressure cylinder, while D C B represents the expansion in passing from the high-pressure cylinder to the end of the stroke of the low-pressure cylinder. For simplicity it is supposed that the reservoir is very large, and wire-drawing is neglected in drawing the ideal curve D C B; although in the subsequent calculation of areas the actual mean pressures are used as found by experiment: thus C D is a straight line representing an increase of volume at the constant pressure of the reservoir consequent on the

FIG. 15.


Saturation Curve
82
27·4
11·9
0
30·L
48·L
150·5
317.
971
5·36
5·36
Vertical scale of pressures.
1 inch = 150 lbs.
Horizontal scale of volumes.
1 inch = 8 cubic feet.

being partially dried by the exhaust heat of the high-pressure cylinder, while C B is the expansion in the low-pressure cylinder. Then, as before, the horizontal distance between the whole expansion curve A D C B and the saturation curve represents the amount of water in the steam at each point of its passage through the engine from the beginning of the high-pressure stroke to the end of the low-pressure stroke.

Next, the heat supplied to the steam during the expansion, is found by performing the construction of Art. 58 with reference, first, to the high-pressure expansion, secondly, to the total expansion: this is shown in Fig. 15, in detail with the same letters attached as in Fig. 14, so that it is unnecessary to describe the process further. The calculations are now conducted as follows, commencing with the high-pressure cylinder:

$$\text{Internal work area } F I_m = 49 \times (11 \cdot 1 - 2 \cdot 95)$$
$$= 399 \cdot 3.$$

For the external work it is necessary to estimate the area A D F M, which cannot be done exactly, because the mean forward pressure is not given, but only the mean effective pressure for the high-pressure cylinder: it can, however, be approximately estimated without fear of serious error, by taking the mean of the values of P V at the beginning and end of the stroke, and multiplying by the value of $\log._\epsilon r$,

$$\text{Initial value of } P V = 92 \times 2 \cdot 95 = 271 \cdot 4$$
$$\text{Terminal} \quad ,, \quad P V = 27 \cdot 4 \times 11 \cdot 1 = 301 \cdot 4$$
$$\therefore \text{ Mean value of } P V = 286 \cdot 4.$$

The ratio of expansion in the high-pressure cylinder is $9 \cdot 19 \div 2 \cdot 44 = 3 \cdot 76$, the hyperbolic logarithm of which is $1 \cdot 32$, hence we obtain

$$\text{Expansion area} = 377 \cdot 5 \text{ nearly.}$$

Adding which to the internal work area found above, we obtain

$$\text{Total area} = 776 \cdot 8,$$

whence by division by 5·36

$$\text{Heat supplied} = 145 \text{ thermal units approximately.}$$

Assuming now that the boiler supplied dry steam, then, as before, the rectangle A Z^0 is the heat abstracted from the steam during admission:

$$\begin{aligned} \text{Area A } Z_0 &= (4 \cdot 70 - 2 \cdot 95) \times 1013 \\ &= 1773, \end{aligned}$$

whence by division by 5·36

$$\text{Heat abstracted} = 331 \text{ thermal units.}$$

The cylinder was not jacketed, hence neglecting radiation and conduction 331 – 145 or 186 thermal units must have been transmitted to the exhaust steam on its passage out of the high-pressure cylinder.

Passing on to consider the total expansion, the internal work done, while the steam passes from its initial pressure in the high-pressure cylinder to its terminal pressure in the low, is represented by the rectangle M I'_m, and this is true, whatever amount of wire-drawing exists between the cylinders, and whatever be the treatment of the steam in the reservoir. The external work is equal to the whole area of the diagram, known from the mean pressures given by the experiment, diminished by the admission area S M.

$$\begin{aligned} \text{Total mean effective pressure} &= 27 \cdot 52 \text{ (reduced to L.P.)} \\ \text{Back pressure} \;\ldots\;\ldots\;\ldots &= \;\; 3 \cdot 05 \text{ approximately} \\ \text{Mean forward pressure} \;\ldots &= 30 \cdot 57 \end{aligned}$$

$$\begin{aligned} \therefore \text{ Area of diagram} &= 30 \cdot 6 \times 27 \cdot 1 \\ &= 829 \cdot 3; \end{aligned}$$

but

$$\text{Admission area} = 271 \cdot 4;$$

$$\begin{aligned} \therefore \text{ Total expansion area} &= 558 \text{ nearly} \\ \text{Internal work area} &= 30\,(27 \cdot 1 - 2 \cdot 95) \\ &= 724 \\ \text{Total area} = 724 + 558 &= 1282 \end{aligned}$$

$$\therefore \text{ Heat supplied during total expansion} = 239 \text{ thermal units.}$$

Let us now trace the process from the instant when the steam is cut off in the high-pressure cylinder to the instant when it exhausts from the low-pressure cylinder into the condenser.

Heat received during high-pressure expansion = 145
" " " " exhaust = 186

∴ Total heat received before finally leaving the high-pressure cylinder = 331 thermal units;

but

Total heat received during total expansion = 239;

∴ Heat abstracted in passage through reservoir and low-pressure cylinder = 92 thermal units.

This heat is partly abstracted in the reservoir, but probably the greater part is abstracted in the low-pressure cylinder by the action of its sides. If we add the heat supplied by the steam jacket of the low-pressure cylinder, we shall obtain the exhaust waste.

The steam jacket in the present case supplied far more heat than in the case previously described in Art. 58. It appears from the experiment that about 7·8 per cent. of the working steam was the additional supply required to replace the liquefaction in the jacket: a result which shows that about 67 thermal units per lb. of working steam was supplied to the low-pressure cylinder; hence

Exhaust waste = 159 thermal units,

which is about 13½ per cent. of the whole heat expended as shown by the total amount of water evaporated. As before, the numbers given require certain corrections on account of clearance, and are not intended as precisely accurate results, but they are probably substantially correct unless the data are rejected as wholly unreliable, a point which is not in question in the present chapter.

61. It has been already stated that complete and exact results can only be obtained when the weight of steam passing through the engine per stroke is precisely know

tion to the data furnished by an indicator diagram. The determination of this quantity is by no means easy, and therefore in most cases this essential datum is wanting, so that it will be desirable to examine what results can be obtained in its absence.

As an example I shall take a diagram from a Corliss engine, working at *Saltaire*, given in 'Naval Science,' vol. iii. p. 160.

FIG. 16.

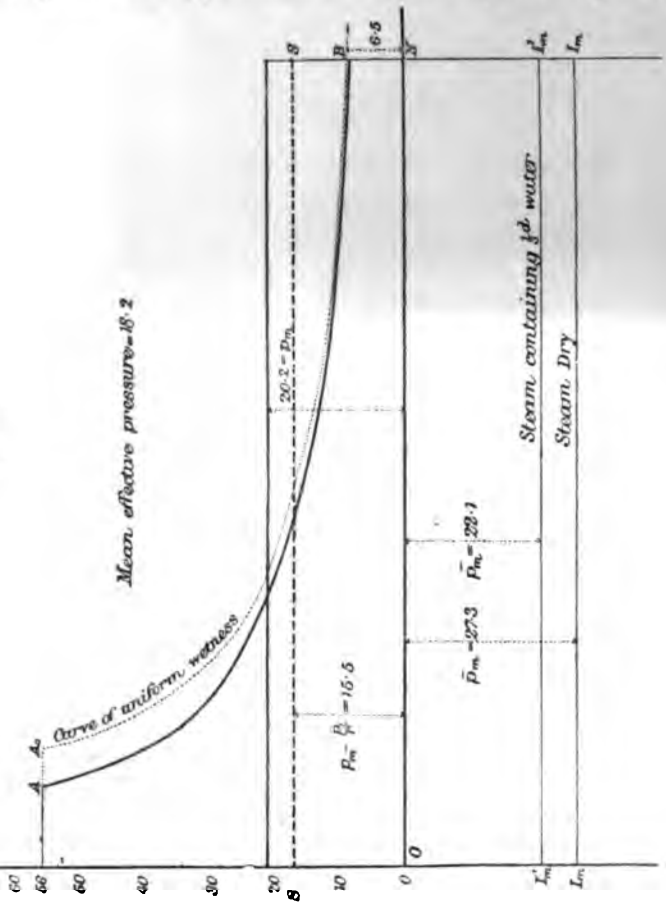


Fig. 16 shows this diagram so far as the admission line and expansion line are concerned, from which it appears that

the engine was working with initial pressure 56 and ratio of expansion 12; the terminal pressure is not easy to estimate exactly, but is taken at 6·5 lbs. per square inch. The consequence of an error of $\frac{1}{2}$ lb. will be mentioned presently. Clearance and wire-drawing (Chapter IX.) are throughout neglected except in measuring the volumes.

The expansion curve A B now gives the pressure at each point of the stroke, but since the weight of steam is not known, the volume per lb. cannot be found. The volume of dry steam at 6·5 lbs. per square inch is 56·6 cubic feet; let us take the base O N of the diagram to represent it, then if the steam were dry, the diagram would represent the changes gone through by 1 lb. of steam, and hence, if we imagine the steam to contain at the end of the stroke $1 - x_2$ lbs. of water, it is clear that the diagram will actually represent the changes undergone by $\frac{1}{x_2}$ lbs. of steam.

From Table III. set off the volumes of 1 lb. of dry steam at various pressures, then if the steam were everywhere equally wet, the resulting curve would be the actual expansion curve A B. Instead of this, however, the dotted curve $A_0 B$ is obtained, which may be called a curve of uniform wetness; were the steam dry at the end of the stroke, it would be the saturation curve drawn in preceding examples. This curve falls above the actual expansion curve A B, and the horizontal distance between the two curves represents the excess water in $\frac{1}{x_2}$ lbs. of steam at the point considered; hence these distances multiplied by x_2 represent the excess water in 1 lb. of steam. For example, at the beginning of the stroke, the volume of the steam considered is $\frac{56 \cdot 6}{12}$, or 4·72 while the volume of steam in the terminal state of wetness is 7·51 cubic feet, therefore in the $\frac{1}{x_2}$ lbs. of steam 2·79

cubic feet exist as water in addition to the water at the end of the stroke, if any. Let then $1 - x_1$ be the amount of water initially in 1 lb. of the steam, then

$$x_2 - x_1 = \frac{2\cdot 79}{7\cdot 51} \cdot x_2 = \cdot 345 \cdot x_2.$$

Hence if the steam be dry at the end of the stroke, $34\frac{1}{2}$ per cent. of moisture must have been evaporated during expansion, or, if it then contain one-third water, 23 per cent. This calculation, of course, presumes that there is no valve leakage, which might be suspected if this were a solitary instance of the kind.

Next, to find the heat supplied during expansion, we might resort to the graphical construction of the preceding articles, but, for the sake of variety, I will proceed differently.

Taking the difference of the two values of I found by the formula of Art. 12, the change of internal work will be

$$I_2 - I_1 = h_2 - h_1 + \overline{P}_2 V_2 - \overline{P}_1 V_1,$$

where the suffix 2 refers to the end, and the suffix 1 to the beginning of the stroke, while $s = \cdot 016$ is omitted, and V_2 V_1 are the actual volumes of 1 lb. of steam:

$$\therefore \frac{I_2 - I_1}{V_2} = \frac{h_2 - h_1}{V_2} + \overline{P}_2 - \frac{\overline{P}_1}{r},$$

since $V_2 \div V_1$ is the ratio of expansion r.

Now let $\bar{p}_m$ be the mean pressure which working throughout the stroke would do the same work, then:

$$\bar{p}_m \cdot 144 \cdot V_2 = I_2 - I_1,$$

and

$$h_2 - h_1 = -772\,(t_1 - t_2) \text{ nearly};$$

thus

$$\bar{p}_m = \bar{p}_2 - \frac{\bar{p}_1}{r} - \frac{5\cdot 36\,(t_1 - t_2)}{V_2}.$$

In the present example if we seek t_1 t_2 from Table I., the difference will be found to be 119°, also by Table V.

$$\bar{p}_2 = 87\cdot 4 : \bar{p}_1 = 596;$$

moreover,

$$V_2 = x_2 v_2 = 56 \cdot 6 x_2 : r = 12,$$

hence we find

$$\bar{p}_m = 37 \cdot 7 - \frac{10 \cdot 4}{x_2}$$

$$= 27 \cdot 3 \quad (x_2 = 1)$$

$$= 22 \cdot 1 \quad (x_2 = \tfrac{2}{3}).$$

These results give the line of mean internal-work-pressure I_m I_m for two cases which may be regarded as extremes. These lines differ from the corresponding lines in the preceding diagrams only in the circumstance that the pressures are supposed reduced to the whole stroke instead of, as before, referring to the expansion alone. Should an error of $\frac{1}{2}$ lb. have been made in estimating the terminal pressure, the effect would be to shift the lines up or down by about 6 lbs. pressure.

Now the external work is represented on the same scale by the mean forward pressure p_m which may be taken at about 20·2 lbs. per square inch, diminished by $\frac{p_1}{r}$, which represents the admission work. This difference is 15·5, and is shown by the dotted line S S.

The heat expended is the sum of the internal work and the external work, and is represented by a pressure on the piston of 42·8 if the steam be dry at the end of the stroke, or 37·6 if it contain one-third water. Now, the mean effective pressure was 18·2, and hence we learn that the heat supplied during expansion must have been 2·35, or 2·07 times the heat-equivalent of useful work done, and hence must have amounted to 100, or 88 thermal units per I.H.P. per 1′.

Again, as in previous examples, the heat abstracted by the cylinder during liquefaction at admission is represented by a rectangle the height of which is the internal-work-pressure + the actual pressure, and the base is the volume of s condensed. This must be reduced to a rectangle, th

of which is O N, then the height of that rectangle will be the equivalent pressure on the piston.

First let the steam be dry at the end of the stroke, then the volume of steam condensed is represented by A A_0 on the same scale that O N represents the terminal volume; thus since it was shown above that A A_0 is 2·79 cubic feet when O N is 56·6 cubic feet, it appears that in the reduction we must multiply by ·0493. Referring to Table V. the internal-work-pressure is found to be 596, and hence

$$\text{Required pressure} = (596 + 56) \times \cdot 0493 = 32 \cdot 14.$$

This represents an abstraction of heat equivalent to 1·76 times the useful work, or about 75 thermal units per I.H.P. per 1′. The difference between this and the heat supplied during expansion is 25 thermal units per I.H.P. per 1′ which must have been supplied by the steam jacket in addition to the exhaust waste, which, if the steam were really dry at the end of the stroke, may be supposed small.

Secondly, if the steam be supposed to contain one-third water at the end of the stroke, no doubt an extreme supposition under the circumstances, the volume of steam condensed in admission, assuming as in previous cases that the boiler supplied dry steam, is found from

$$x_2 - x_1 = \cdot 345\, x_2 \text{ (see above, p. 158)};$$

$$\therefore\ x_1 = \cdot 655\, x_2 = \cdot 43, \quad \text{or} \quad 1 - x_1 = \cdot 57.$$

$$\begin{aligned}
\text{Volume of steam condensed} &= \cdot 57 \times 7 \cdot 51 \\
&= 4 \cdot 28 \text{ cubic feet per lb.} \\
\text{Terminal volume} \quad .. \quad .. &= 56 \cdot 6 \times x_2 = 56 \cdot 6 \times \cdot 666 \\
&= 37 \cdot 7 \text{ cubic feet per lb.} \\
\therefore \text{Required pressure} \quad .. \quad .. &= 652 \times 4 \cdot 28 + 37 \cdot 7 \\
&= 73 \cdot 8.
\end{aligned}$$

Thus the heat abstracted during admission is 4·06 times the useful work done, or 174 thermal units per I.H.P. per 1′. Subtracting the heat supply, we obtain 86 thermal units per

I.H.P. per 1′, which together with the jacket heat would form the exhaust waste.

These results will show how far it is possible to go without further information than is furnished by an accurate indicator diagram. The accuracy of the diagram is not discussed here, and the complicated effect of clearance and wire-drawing here neglected may have considerable influence on the results.

62. Before leaving this part of the subject, another formula may be mentioned which is of great use in theoretical questions.

In Chapter III., Art. 25, it was shown that the total heat of formation of steam at the end of the stroke of an engine is given by

$$Q = H_2 - h_0 + (P_m - P_2) v_2;$$

if the steam be then dry, or if the steam be then wet,

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) x_2 v_2 \quad \text{(Art. 26)},$$

the notation being as explained in the articles cited.

Now there is no reason to restrict this formula to the end of the stroke, the reasoning used being applicable in any case, only P_m must now mean the mean pressure exerted on the piston during the part of the stroke considered, while P_2 becomes the pressure at the end of that part. Let the part considered then be the admission, then $P_m - P_2$ vanishes and $Q^1 = h_1 - h_0 + x_1 L_1$ is the total heat of formation initially, where the suffix 1 corresponds to the commencement of the stroke. This may also be seen by considering that the total heat of formation is in this case identical with the total heat of evaporation.

If now S be the heat supplied during expansion, S must be identical with $Q - Q^1$,

$$\therefore S = h_2 - h_1 + L_2 x_2 - L_1 x_1 + (P_m - P_2) x_2 v_2,$$

which determines the heat supplied to 1 lb. of steam du expansion, when the initial and terminal state of the st

is known together with the mean forward pressure exerted on the piston during admission and expansion. The results of the last articles can also be obtained by use of this formula.

63. The examples given in the present section show sufficiently how to deal with cases in which the law of expansion is known, by experiment or otherwise, and it is required to find the law of supply, and at the same time furnish materials for subsequent consideration. I next go on to the converse question, where the supply of heat is supposed known, and it is required to find the law of expansion.

Expansion of Steam under a Given Supply of Heat.

64. When the supply of heat is given for each step of the expansion, it is then possible, at least theoretically, to construct the expansion curve step by step, so that the given supply may be equal to the internal work + the external work. The data of the question would be, either the heat supplied during each degree of the fall of temperature which always takes place whatever the law of expansion be, or else the supply of heat as the volume increases by a given amount: it is the first case which is the most simple, and at the same time generally the most important, and to that I shall confine myself in the present chapter.

Case I.—First, suppose that no heat is supplied to the steam during its expansion, then the expansion curve is, as in the case of air (Chapter IV.), called the "adiabatic" curve, the form of which it is our object to investigate. Adiabatic expansion does not occur in practice, for it presupposes a perfectly non-conducting cylinder, but its consideration is nevertheless indispensable in any theory of the steam engine, both as an interesting ideal case, and because, although the whole mass of steam cannot expand adiabatically, yet the central portion, not in immediate contact with the sides, probably does so. The general nature of the curve can be foreseen from what was said at the commencement

of this chapter, when we considered the expansion of dry and saturated steam; for it was there shown that, to keep steam dry, heat must be supplied from without, and the necessary inference is that, if the heat be not supplied, the steam will condense, and hence the adiabatic curve must fall below the saturation curve. On the other hand, it cannot fall much below, for it has been already seen what a small difference in the expansion curve corresponds to a great difference in the heat supply.

To construct the curve it is only necessary to remember that the whole external work done in expansion must now be derived from the internal energy stored up in the steam itself, that is to say, it must be equal to a diminution of internal energy which must take place during the expansion. Thus, if the steam expand from a point 1 to a point 2,

$$I_1 - I_2 = \text{Area of curve};$$

or with the previous notation (Art. 61),

$$5 \cdot 36\,(t_1 - t_2) + \bar{p}_1 V_1 - \bar{p}_2 V_2 = \text{Area of curve};$$

where areas are supposed expressed by the product of a pressure in lbs. per square inch and a volume in cubic feet. The construction of the curve is now to be carried out, so as to satisfy this equation.

In Fig. 17 O X is as usual the volume axis and O Y the pressure axis from which lines are drawn with ordinates representing the pressures 15, 20, 25, 30, 40, 50, 70 lbs. per square inch, and any other pressures which may be required; for convenience, such pressures are chosen as occur in Tables III. and V. Corresponding lines in the lower part of the figure show the internal-work-pressures taken from Table V.; these lines are distinguished by the numerical values of the pressures in question being written against them.

Imagine, for example, that we have dry steam at 70 lbs. pressure, and take A on the corresponding pressu

as to represent its volume, that is to say, 6·09 cubic feet. Let that steam expand, according to the curve A B, till its pressure has fallen to 50 lbs. per square inch without gain or loss of heat, it is required to find the corresponding volume, which we already know to be less than 8·35 cubic feet, the volume of dry steam at that pressure.

Complete the internal-work-rectangle corresponding to A precisely as in previous questions, but let the internal work be reckoned from water at the lower temperature of B instead of from the upper temperature of A; the construction is shown in the figure, resulting in the line S S which forms the base of A's rectangle; while the base of B's rectangle is simply the line of internal-work-pressure corresponding to B, which in the figure meets the vertical through A in G. Now draw the line Z I_m I_m midway between A and B, the ordinate of this line must represent the internal-work-pressure corresponding to the expansion from A to B wherever B is, exactly if the expansion curve were a straight line between A and B, and very approximately if, as is actually the case, the line be curved. Hence, by the same reasoning as in previous questions, it is clear that the other side of B's rectangle will be determined by joining Z G and producing it to meet S S produced in K, then a vertical through K must determine B; the difference of internal work at A and B being then equal to a rectangle, the area of which is very approximately the same as the area of the expansion curve A B.

The construction can now be repeated as often as desired, and the adiabatic curve is thus determined. The figure shows expansion by successive stages, from 70 lbs. pressure to 15 lbs. pressure; the numbers written below the axis give the volumes of the expanding steam at the pressures indicated by the ordinates, while the numbers written above the expansion curve on the level of the line of 70 lbs. pressure represent the corresponding volumes of dry steam, as shown


6·87

by the saturation curve A D^1 dotted in the diagram; thus, the volume of steam condensed at each step of the expansion is the difference of these numbers.

The process here described requires considerable care to obtain accurate results; hence, when great exactness is desired in the determination of the condensation, numerical calculation is preferable, according to a formula to be explained in the next chapter, or by the method followed in Case II. immediately following; the numbers given in the figure were obtained in this way.* The results show that in the whole expansion the ratio of expansion is 3·88, and the steam condensed 2·3 cubic feet; that is to say, when steam expands adiabatically 3·88 times from a pressure of 70 lbs. per square inch, the terminal pressure, instead of being 18·1, as would be the case if the expansion were hyperbolic, is nearly 3 lbs. less, and about 9 per cent. of the steam is liquefied. Inspection of the figure gives a clear idea of the gradually increasing liquefaction as the pressure falls. The mean pressure in adiabatic expansion can always be found when the terminal volume and pressure are known by inverting the construction of the present article and applying it to the total expansion. This is illustrated in the Appendix, Note C.

65. *Case II.*—Secondly, as before, let the expansion be adiabatic, but let the steam be initially wet, then the construction is in general identical with that just given, and the general results can be foreseen. The position of the lines of internal-work-pressure is unaltered, but the rectangle representing the effect of difference of temperature is of smaller breadth, and consequently greater height; thus, the lines S S are all shifted downwards, and consequently the points K (for the same initial volume) shifted to the right; that is to say, the volume of the steam is greater (relatively to the

* Another formula is given in the Appendix, Note C, by means of which the condensation can be found approximately with great facility.

initial volume) than if the steam had been initially dry. Thus wet steam does not condensė so fast as dry steam when expanding adiabatically.

The extreme case of wet steam is when there is no steam, but only water initially, and then it is obvious that water must be evaporated, not steam condensed, on diminution of pressure. In Fig. 18 let O N represent ·016, the volume of 1 lb. of water enclosed in a cylinder behind a piston, and let the piston be loaded with 70 lbs. per square inch (say); take A N to represent that pressure, and draw horizontal lines to represent various pressures as in the previous case. Now imagine the temperature of the water to be 303°, corresponding to 70 lbs. per square inch, and then suppose that pressure gradually to diminish; the water will gradually evaporate and its temperature fall, it is required to find the expansion curve. We might employ the purely graphical method of Case I., introducing a suitable modification, but instead of this, I shall proceed differently by a method likewise applicable in Case I. and the following Case III. Let the point of departure A be denoted by 1, and let the next point corresponding to a pressure of 60 lbs. be denoted by 2, as shown on the diagram, then if, as usual,

$$\mathrm{I} = h + \overline{\mathrm{P}}\,(\mathrm{V} - s),$$

we can now no longer neglect s, because V is a small quantity; hence, since $\mathrm{V}_1 = s$, the formula employed in the last article becomes

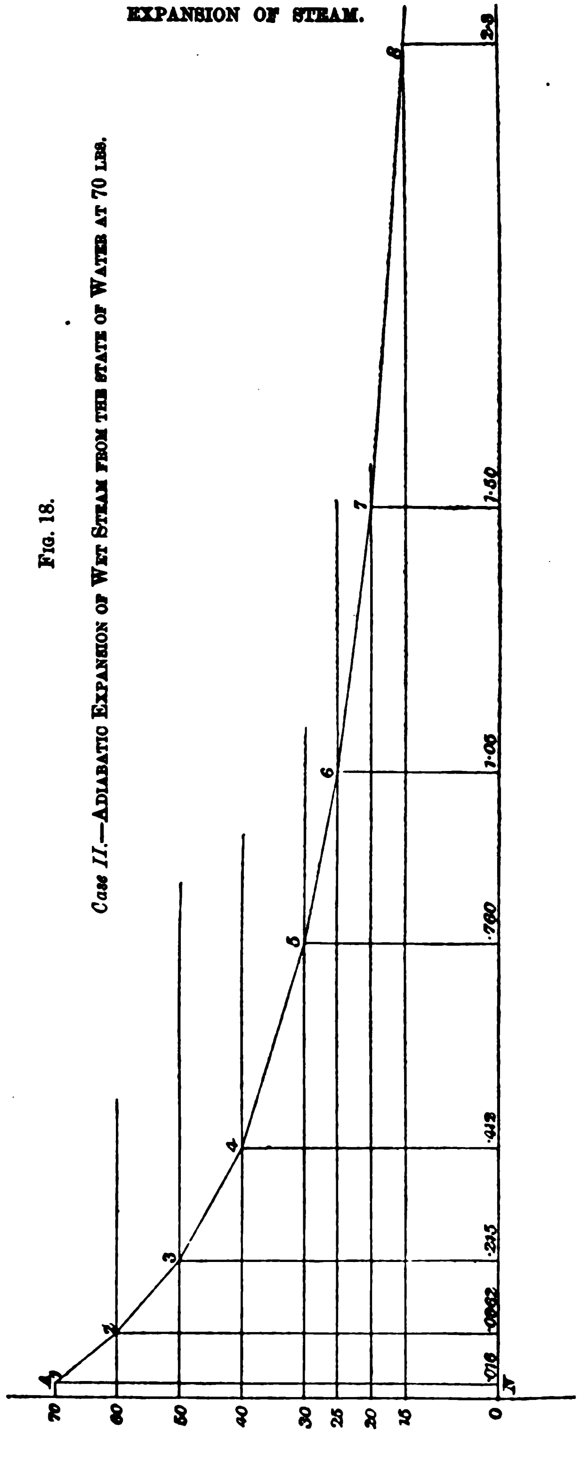
$$5{\cdot}36\,(h_1 - h_2) - \overline{p}_2\,.\,(\mathrm{V}_2 - s) = \text{Area of curve},$$

in which $h_1 - h_2$ is not replaced by its approximate value $t_1 - t_2$, because the error thus introduced is much greater than in previous cases. Now, for the area of the curve between 1 and 2, it is sufficient to consider the curve as a straight line between these points, then

$$\text{Area} = \frac{p_1 + p_2}{2}\,.\,(\mathrm{V}_2 - s) = 65\,(\mathrm{V}_2 - s)\,;$$

Fig. 18.

Case II.—Adiabatic Expansion of Wet Steam from the state of Water at 70 lbs.



introducing which into the foregoing equation, and replacing $\bar{p}_2$ by its value, namely, 633·4 lbs. on the square inch,

$$(V_2 - s)(633 \cdot 4 + 65) = 5 \cdot 36\,(h_1 - h_2).$$

On referring to Table II*a* it will be found that the specific heat of water between 303° and 293° is about 1·025, hence

$$h_1 - h_2 = 1 \cdot 025\,(t_1 - t_2) = 10 \cdot 25 \text{ nearly;}$$

substituting and performing the numerical calculations,

$$V_2 - s = \cdot 0802, \qquad V_2 = \cdot 0962,$$

so that the volume of the mixture of steam and water is about ·0962 cubic foot. The point 2 can now be laid off on the diagram, and shows the volume at a pressure of 60 lbs. per square inch. Now pass on to the point 3 on the line of 50 lbs. pressure, then considering that area as a pair of trapezoids,

$$\text{Area of curve up to } 3 = 65 \times \cdot 0802 + (V_3 - V_2)\,55\,;$$

$$\therefore \text{ Area} = 5 \cdot 2 + 55\,(V_3 - V_2),$$

and the equation becomes

$$5 \cdot 2 + 55\,(V_3 - V_2) = 5 \cdot 36\,(h_1 - h_3) - \bar{p}_3\,(V_3 - s),$$

or

$$5 \cdot 2 + (V_3 - V_2)(\bar{p}_3 + 55) = 5 \cdot 36\,(h_1 - h_3) - \bar{p}_3\,(V_2 - s).$$

Replacing $\bar{p}_3$ by its value, namely, 540 lbs. per square inch, and $V_2 - s$ by its value just found,

$$V_3 - V_2 = \frac{5 \cdot 36\,(h_1 - h_3) - 540 \times \cdot 0802 - 5 \cdot 2}{595}.$$

Assume the specific heat of water by Table II*a* as 1·02, then

$$h_1 - h_3 = 1 \cdot 02\,(t_1 - t_3) = 1 \cdot 02\,(303° - 281°)\,;$$

substituting which and performing the arithmetical operations,

$$V_3 - V_2 = \cdot 119,$$

whence

$$V_3 = \cdot 119 +$$

This process can be repeated indefinitely, and hence the volume of the steam is determined after a fall of pressure of any extent; the results are shown so far as 15 lbs. in the diagram by the numbers placed against the ordinates above the volume axis; whence it appears that the volume of the mixture, after the pressure has fallen to 15 lbs., is about 2·3 cubic feet.

Thus it appears that when steam is very wet, instead of condensation taking place during expansion, just the reverse is true, some of the water being evaporated. Of course it follows that some proportion of steam to water must exist for which neither evaporation nor condensation takes place. This proportion can be found either by graphic construction or by an approximate calculation of the kind just made, or by a formula which will be given in the next chapter; it varies for each particular case, but never differs greatly from half.

66. If, instead of supposing steam, or a mixture of steam and water, to expand, we imagine conversely that it is compressed by a gradual increase of the pressure on the piston; then the effect produced is exactly reversed. In the case of moderately moist steam, the moisture is gradually evaporated as the compression proceeds, and when sufficiently compressed the steam becomes superheated. In the case of very wet steam containing more than half its weight of water, condensation takes place gradually as the compression goes on, and that the more rapidly the wetter the steam; thus, if the compression be sufficient, the steam is wholly condensed, and this is what was supposed in Art. 46, Chap. V., when considering the action of a perfect steam engine.

67. *Case III.*—Next, instead of supposing, as in the two preceding cases, that no heat is supplied or abstracted during expansion, let us imagine that heat is added as the temperature falls by equal quantities for each degree. This will be realized if the steam be supposed to expand in a non-

conducting cylinder, but with a thin metallic plate attached to the piston; then the temperature of the plate will closely follow that of the steam, and if its specific heat be supposed constant, it will supply the steam with equal quantities of heat as the temperature of the steam falls through each degree. Let the weight of the plate be m times that of the steam and its specific heat c, then the heat supplied per lb. of steam will be $m\,c$.

The supposition of such a metallic plate is very interesting theoretically, because it imitates more closely than any other case, sufficiently simple to be thoroughly investigated, the real action of the sides of the cylinder.

Let t' be the temperature of the exhaust steam, and t be the temperature initially in the cylinder; then on admission the plate has to be heated from t' to t by the fresh steam from the boiler, whereby $m\,c\,(t - t')$ thermal units are subtracted from that steam, and $1 - x$ lbs. are liquefied, given by

$$1 - x = \frac{m\,c\,(t - t')}{\mathrm{L}}.$$

This liquefied steam is deposited as a film of moisture on the surface of the plate, which is afterwards re-evaporated, partly during expansion and partly during exhaust. This process is highly instructive, and will be carefully examined hereafter; for the present we are only concerned with its effect on the expansion curve.

Let Q be the amount of heat furnished by the plate as the expansion proceeds from a point 1 to a point 2, then by the general principle

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work},$$

$$\mathrm{Q} = \mathrm{I}_2 - \mathrm{I}_1 + \text{Area of expansion curve};$$

or using the same formula as in Art. 61,

$$\mathrm{Q} = h_2 - h_1 + \overline{\mathrm{P}}_2 \,.\, (\mathrm{V}_2 - s) - \overline{\mathrm{P}}_1\,(\mathrm{V}_1 - s) + \text{Area of curve}.$$

Omitting s and writing as usual

$$h_1 - h_2 = 5 \cdot 36\,(t_1 - t_2),$$

to correspond with pressures in lbs. per square inch,

$$Q = \bar{p}_2 V_2 - \bar{p}_1 V_1 - 5{\cdot}36\,(t_1 - t_2) + \text{Area of curve};$$

but on the same scale

$$Q = m\,c\,(t_1 - t_2) \times 5{\cdot}36,$$

and therefore

$$5{\cdot}36\,(1 + m\,c)\,(t_1 - t_2) + \bar{p}_1 V_1 - \bar{p}_2 V_2 = \text{Area of curve}.$$

The construction of the curve is now to be carried out so as to satisfy this equation. On comparing the equation from which the adiabatic curve was constructed in Art. 64, it will be seen to differ solely in $t_1 - t_2$ being replaced by $(1 + m\,c)$ $(t_1 - t_2)$, and the construction must therefore be the same save a slight modification. To fix our ideas, let us imagine the plate to be of iron, the specific heat of which is ·12, and let the weight of the plate be 8·33 times that of the expanding steam; then $m\,c = 1$, and in the construction we have only to use $2\,(t_1 - t_2)$ in place of $t_1 - t_2$.

This has been done in Fig. 17, which shows the construction on the further supposition that the steam contains initially one-third water; the inner strongly dotted curve *a b d* is the expansion curve. As before, to obtain very exact numerical results, a calculation method is preferable, such as the process adopted in Case II., or the formula given in a subsequent chapter (Chapter VIII.). The numbers given in the diagram at the various points of the curve *a b d* were obtained in this way; those immediately above the volume axis representing the actual volumes of the expanding steam, and those in the upper part of the figure giving the volumes of steam containing one-third water, as shown by the faintly dotted curve of uniform wetness $a\,d^1$. The first set of numbers are the greater, showing that the action of th plate is sufficient, not merely to prevent the condensat which would otherwise take place, but make the steam c siderably drier. Thus, at the end of the stroke the stea occupies 18·8 cubic feet, instead of 17·27 cubic feet, as i

[illegible] at the end it still contain as much as one-third water. The ratio of expansion is in this case 4·63, and the curve [illegible] nearly bulked up to the hyperbola.

The steam contains, on the whole, at the end of the stroke [illegible] per cent. of water; but it is highly probable that not all of this is deposited on the plate, but that the central mass of the steam condenses as it would do if the plate were not there. If this be the case then by Case I., 8·9 per cent. of [illegible] the whole mass is distributed uniformly throughout the mass, while the rest represents the film still existing on the plate at the end of the expansion. This will be discussed further hereafter.

The curve of internal work may be easily derived by graphic construction for

$$Q = mc(t_1 - t_2),$$

and hence, since the area included between the curves of internal work and external work represents the heat expended, all that is necessary is to reduce a rectangle representing Q to a base representing the corresponding increase of volume. Or, by calculation, let $V_1 V_2$ be two volumes corresponding to the temperatures $t_1 t_2$, then if p_h be the height of such a rectangle in lbs. per square inch,

$$p_h = \frac{mc \cdot 5{\cdot}36 \cdot (t_1 - t_2)}{V_2 - V_1}.$$

In the example $mc = 1$ then, commencing with the first stage of the expansion from 70 to 50 lbs., it has been already shown that

$$V_2 - V_1 = 5{\cdot}71 - 4{\cdot}06 = 1{\cdot}65;$$

$$\therefore p_h = 5{\cdot}36 \cdot \frac{303^\circ - 281^\circ}{1{\cdot}65} = \frac{5{\cdot}36 \times 22}{1{\cdot}65},$$

or

$$p_h = 71{\cdot}5.$$

A similar calculation is made for each of the five other stages into which the expansion is divided, whence is obtained

$$71{\cdot}5 : 51{\cdot}4 : 38{\cdot}8 : 28{\cdot}5 : 23{\cdot}8 : 17{\cdot}1.$$

numbers which show the mean pressures equivalent to the heat expended during each of the six stages of the expansion, from which the curve is readily constructed; it evidently falls a little below the axis at the higher pressures and almost coincides with it at the lower: to avoid confusion it is not represented in the diagram. In adiabatic expansion the curves of internal work and external work obviously coincide.

The expansion of steam in contact with a thin plate does not differ materially from the expansion of wet steam; indeed the cases would be identical if the specific heat of the metal varied according to precisely the same law as the specific heat of water. For it is clear that the material of the plate does not influence the result in any other way, and consequently we may just as well suppose a mass of water as a mass of metal. The water, however, will follow the temperature of the steam more readily than the metal when the expansion is rapid; an important consideration, as will be seen hereafter.

Isodynamic Expansion. General Remarks.

68. One other case of expansion remains to be mentioned; namely, that in which the supply of heat is just equivalent to the external work done, so that the internal energy of the expanding steam suffers no change. This is called isodynamic expansion, and in perfect gases is the same as isothermal expansion (Art. 33); so that the expansion curve, or, as it is called, the isodynamic curve, is a common hyperbola. The curve of internal work then coincides with the volume axis, and, consequently, on comparing Articles 52, of the present chapter, it appears that the isodynamic cu must lie between the saturation curve and the comm hyperbola, and hence differs very little from either; mor over, it follows that in this kind of expansior am

[redacted]

becomes drier as it expands, though not so rapidly as in hyperbolic expansion. This curve likewise represents the relation between the volume and the pressure of saturated steam when expanding without doing any external work. (See page 80.)

The isodynamic curve is graphically constructed by an easy modification of the process adopted for the adiabatic curve (Art. 64): we have only to draw the radiating lines (Fig. 17) through the fixed point O instead of through the middle points of the corresponding pressure intervals; then a curve will be determined for which the internal-work-rectangles are constant, and this curve, by the definition, is the isodynamic curve.

69. The principal object of the present chapter has been to point out the connection between the expansion curve of steam and the supply of heat during the expansion, as had been already done in the case of air in Chapter IV.; and we see clearly that in both steam and air the law of expansion depends solely on the treatment of the fluid as regards the reception or rejection of heat, as has been already shown in general terms in Art. 29.

The graphical methods employed for steam may also advantageously be used for air, but are much more simple, because the internal energy of air, reckoned from the absolute zero, is always represented by a rectangle constructed on the base V, with a height 2·45 P. (See page 80.) With this modification the construction representing the heat supplied during expansion, or for the adiabatic curve, may be carried out exactly as explained in detail in the case of steam. (See also Appendix, Note C.)

CHAPTER VIII.

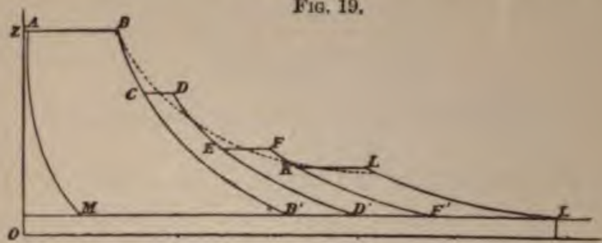
ENGINES RECEIVING HEAT AT VARYING TEMPERATURE.

70. In all that was said in Chapter V., it was supposed that a single source of heat, at a given fixed temperature, existed, from which the engine was wholly supplied: but I now resume the subject, for the purpose of considering cases in which the heat is drawn from various sources at various temperatures. It is true that, in general, the heat is actually derived all from the same source, but we shall find that the results are not dependent on the temperature of the source, but on the temperature at which the engine receives the heat: so that they apply to the large class of engines which receive a portion of their heat at *varying* temperature, and not at the fixed temperature necessary for maximum efficiency.

Take a simple engine, such as that of Fig. 9, Art. 36, and imagine, instead of a single source of heat and a single refrigerator, various sources of temperatures, T_1, T_2, &c., the temperature of the refrigerator being T_0. Suppose the engine working with any fluid, as, for instance, a mixture of steam and water, and, at the beginning of the operation, let the temperature of the fluid be T_1, the temperature of the corresponding source of heat: then, if that source be applied, the fluid receives heat and increases in volume. In Fig. 19 it is supposed that the fluid is initially water, and evaporates partially, till its volume has increased from Z A to Z B, during reception of quantity of heat Q_1 from the first source; the pressure, course, in this case remains constantly at its initial valu Now remove the first source and allow the fluid to expand without gain or loss of heat: the adiabatic curve B C is

described, while the temperature falls from that of the first source to that of the second (say T_2); next, instead of allowing the expansion to proceed, and the temperature to fall,

Fig. 19.



apply the second source of heat, which causes a further evaporation at the constant pressure, corresponding to T_2, during the reception of a quantity of heat from that source which may be called Q_2: the corresponding increase of volume is represented in the figure by C D. Repeat this process for all the sources of heat, the last adiabatic curve being L L^1, while the temperature finally falls to T_0, the temperature of the condenser; then let heat be abstracted by the refrigerator, till the volume M L^1 of steam has been condensed, the point M being so taken, as in the simpler case of Art. 46, that the mixture of steam and water returns, after adiabatic compression, represented by the curve M A to the state of water at temperature T_1. As in all other heat engines, the area of the diagram represents the energy exerted by the fluid during the circular process to which it is subjected; and the only question is to find the relation between that area and the quantities of heat Q_1, Q_2, Q_3, &c., received at each temperature. This can easily be done by reference to Chapter V.: for imagine the adiabatic curves B C, D E, &c., prolonged to meet the horizontal line M L^1 in B^1, D^1, F^1, &c., then each of the curved quadrilaterals A B^1, C D^1, &c., may be considered as the indicator diagram of a simple engine, such as was considered in Chapter V., which receives heat from its own source, and rejects heat into the refrigerator: satisfying the condi-

tions of maximum efficiency for engines working between given limits of temperature. But from Art. 46 it appears that the area of such a diagram must be $Q \,.\, \dfrac{T - T_0}{T}$ where Q is the heat received, and T the temperature of the source; hence, if U be the whole area of the complete diagram, it follows that

$$U = Q_1 \,.\, \frac{T_1 - T_0}{T_1} + Q_2 \,.\, \frac{T_2 - T_0}{T_2} + Q_3 \,.\, \frac{T_3 - T_0}{T_3} + \ldots . \qquad \text{(A)}$$

an equation which shows that the area of the diagram, that is to say, the work done by the engine, depends solely on the quantity of heat supplied from each source, and the temperatures at which it is supplied.

Moreover, let Q be the whole heat supplied from the various sources, and R the heat which passes into the refrigerator, then

$$U + R = Q = Q_1 + Q_2 + Q_3 + \ldots .$$

whence by subtraction of the preceding equation and division by T_0, we obtain the general relation

$$\frac{R}{T_0} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} + \ldots . \qquad \text{(B)}$$

In drawing Fig. 19 and in the description, it was supposed that the engine was a steam engine, but this is not at all necessary, precisely the same reasoning applies to any engine, but the lines A B, C D, E F, &c., of the figure will now no longer be straight, but will be the isothermal curves proper to the particular fluid considered: thus for a perfect gas these lines would be rectangular hyperbolæ. Hence the equations (A) and (B) are true for any engine receiving heat in the way supposed, provided only that none of the expansive energy of the fluid is dissipated by unbalanced expansion. (Art. 39. See also p. 70.)

71. Engines rarely receive heat from various sources in

the way supposed, and the great importance of the result arises from the fact that it is immaterial from whence the heat is derived, provided that the fluid receives heat at the temperatures supposed. For example, in the perfect steam engine of Art. 46, the heat is derived from the hot gases of the furnace at a temperature much higher than that of the boiler: yet, in considering the efficiency of the engine, it is the temperature of the boiler which is considered, not that of the furnace, because it is at the temperature of the boiler that the fluid receives heat.

Thus the results are applicable to any engine whatever, receiving heat at any temperatures, provided only that the whole expansive energy of the fluid is properly utilized. Subject to this proviso, the following general statements may be made, which are equivalent to the foregoing equations.

FIRST.—*The efficiency of every possible heat engine depends solely on the mode in which it is supplied with heat, and not at all on the nature of the fluid or arrangement of the engine.*

This principle is the last step of a gradual generalization, of which the principle laid down in Art. 29, Chapter III., and Carnot's principle explained in Art. 40, Chapter V., are special cases. In Chapter III. it was shown that it is a necessary consequence of the principle of work, that the energy exerted, by a given quantity of a *given kind* of fluid, is independent of the particular kind of machinery, by means of which that energy is utilized. In Chapter V. we found that although the magnitude of the energy exerted varies according to the nature of the fluid, yet, when it receives and rejects heat at *given fixed temperatures*, the proportion which that energy bears to the heat expended, that is to say, the efficiency of the engine, is the same for all fluids, and consequently for all possible engines. Now we go a step farther, and assert that this will be the case, not only when the engine receives

heat at one fixed temperature and rejects heat at another fixed temperature, but also when it receives heat at *any* number of temperatures, provided only that the quantities of heat received at the various temperatures are in the same proportion.

Thus, if the law according to which heat is supplied be supposed given, an engine will be of maximum efficiency, and hence may be said, in a certain sense, to be "perfect," even though it receives heat at varying temperature, the only condition of maximum efficiency being that there shall be no unbalanced expansion. Actually, however, it is only the highest temperature at which heat is supplied which can be considered given, and hence it is only engines which receive the whole of their heat at that highest temperature which can properly be considered as "perfect": all others are of less efficiency, because they receive some of their heat at a lower temperature.

I have supposed for simplicity, as being sufficiently general for my purpose, that the heat is all rejected at a single temperature, but this restriction is not necessary for the truth of the reasoning employed.

SECONDLY.—*If the heat received at any temperature by a heat engine be divided by that temperature (absolute), the sum of the quotients is unaltered by the passage of the heat through the engine.*

This principle is merely the expression in words of equation (B), and amounts to saying, that although heat disappears (being changed into work), during its passage through a heat engine, yet that a certain quantity, found by dividing the heat by the temperature at which it is used, is unchanged, if the opportunity of turning heat into work, presented by the available difference of temperature, has been duly utilized.

Zeuner has called this quantity a "*heat-weight*," thus developing further Carnot's analogy between a heat er

and a water-power engine.* Let W be the weight of water used in a perfect water wheel, $h - h_0$ the fall, then

$$U = W(h - h_0)$$

is an equation which gives the work done by the machine: or, if there be various quantities of water, falling through various heights,

$$U = W_1(h_1 - h_0) + W_2(h - h_0) + \ldots.$$

Now compare this with the general formula (A),

$$U = \frac{Q_1}{T_1}(T_1 - T_0) + \frac{Q_2}{T_2}(T_2 - T_0) + \ldots.$$

and it will be seen that just as the difference of temperature corresponds to the fall, so the quotient $\frac{Q}{T}$ corresponds to the weight.

I shall use this term "heat-weight" occasionally as a name for the quotient in question; but analogies of this kind, though useful at the outset of the subject, must not be pushed too far, and should be considered merely as aids to the imagination: the things compared are essentially different. In particular the analogy fails for imperfect engines; in such engines the "heat-weight" increases during the passage through the engine.

72. Again, let two engines of the same power be imagined having the same refrigerator, but different sources of heat at temperatures T_1, T_2; then if U be the work done by each engine, Q_1, Q_2, the heat expended,

$$U = \frac{Q_1}{T_1}(T_1 - T_0) = \frac{Q_2}{T_2}\cdot(T_2 - T_0).$$

Let one of these engines be applied to drive the other backwards, as in Art. 39: then, since the power of the engines is the same, the combination is self-acting, and we see that a small "heat-weight," descending through a great difference

* 'Grundzüge der Mechanische... ...orie,' p. 68.

of temperature, is able to raise a large "heat-weight" through a small difference, or conversely.

In applying the second law of thermodynamics (Art. 40), this must be taken into account, for let T_2 be greater than T_1, and consequently Q_2 less than Q_1, and let the second engine work the first backwards, then an amount of heat $Q_1 - Q_2$ is drawn from the refrigerator, and transferred to hotter bodies, by a self-acting process, in apparent contradiction to the second law. But it must be remembered that this is accompanied by a descent of the heat Q_2, partly to the level of the refrigerator, partly to the level of the first source, and, when that descent is taken into account, it will be seen that the result in question is not a contradiction of the law, but merely shows that the passage of heat from a cold body to a hot one, spoken of in the second law as impossible, is one which is *uncompensated* by an equivalent passage from a hot body to a cold one.

An interesting process was invented by M. Hirn, and applied by him as an objection to the second law, at a time when he had not fully accepted the mechanical theory of heat. Suppose we have a pair of cylinders, open at the top, placed side by side, and provided with pistons and rods. The piston rods have racks attached to them, which gear into the same pinion, in such a way that when one piston rises, the other necessarily falls by the same amount. The cylinders are connected by a pipe, and contain fluid of any kind capable of expanding by the action of heat; then, since the cylinders freely communicate, the pressure necessarily is the same throughout, and, if the pistons be of the same area, the pressures are then obviously exactly balanced, so that they may be moved up or down without doing any work neglecting friction.

Now imagine a source of heat to be applied to the pipe, by means of which the cylinders communicate, and let one piston be at the top of its stroke, the other, of course, bei

at the same time at the bottom; further, let the temperature of the fluid be less than that of the source. Then, if the first piston descends the fluid passes into the second cylinder, and, in doing so, receives heat from the source, as it passes through the pipe and increases in pressure. This increase of pressure goes on continually as more and more fluid passes through the pipe, and hence the portion of fluid which first passes through is compressed by the pressure of the remainder, and its temperature rises so as to be greater than when it first passed through. But when it first passed through, its temperature was that of the source of heat, and thus its final temperature is greater than that of the source.

Here, then, we have an apparent contradiction to the second law, for it would seem that, without doing any work, the temperature of a certain quantity of heat has been raised above that of the source from which it was derived: the final result being that every particle of gas has a temperature at least equal to that of the source, while the greater part of it has a greater temperature.

The contradiction, however, is only apparent, and the process is well suited to show the real nature of the second law. If the process be examined it will be seen that the rise of temperature is only possible when the gas has a lower temperature initially than the source of heat, and that the heat from the source, in the first instance, descends from the temperature of the source to the temperature of the gas. It is this descent only which renders possible the subsequent elevation, and the elevation can never be more than equivalent to the descent. For example, suppose that we attempted, by a self-acting process of this kind, to cause the heat of a condenser to pass into the boiler of a steam engine, and so to utilize it: for that purpose we must have a source of heat of a higher temperature than that of the condenser, from which a supply of heat may be drawn, and the heat, which could be elevated from condenser to boiler by the process, is no more than been utilized by

the direct employment of the heat from the source to work a heat engine between the temperature of the source and the temperature of the condenser.

It is, however, not inconceivable that some process of this kind might be invented to utilize small differences of temperature, or inconveniently low temperatures: for example, the difference of temperature between the condenser of a steam engine and the atmosphere is at present wasted: and it is within the range of possibility that a process might be invented which should utilize it by transferring a smaller quantity of heat from the condenser to the boiler: such a process would be no violation of the second law, but, if perfectly carried out, would simply be equivalent to widening the limits of temperature within which the steam engine works, as may otherwise be done (theoretically) by the addition of an ether engine. (Art. 43.)

73. In Art. 70 it was supposed that the supply of heat took place by successive steps at successive fixed temperatures: it will, however, seldom happen that this is the case, the engine receiving some or all of its heat, in general, continuously during a gradual change of temperature. By taking the successive temperatures near enough, however, this case becomes equivalent to that originally supposed: let us suppose that the amount of heat ΔQ is supplied at temperature T, then equation (B) becomes

$$\frac{R}{T_0} = \Sigma \frac{\Delta Q}{T} + \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \dots$$

Q_1, Q_2, &c., being supplied as before at fixed temperatures T_1, T_2. . . . Thus in Fig. 19, just now given to show the operation of a steam engine receiving heat at varying temperature: let us suppose the sources of heat indefinitely multiplied, then the broken line B C D E F K L becomes a continuous curve, dotted in the figure, representing the expansion of steam under a continuous supply of heat, the form of the curve depending, as shown in the last chapter,

on the quantity of heat supplied at each step of the expansion.

An important case of this is, when the heat supplied during a continuous change of temperature is, at every step, proportional to the change, as in Case III. of the expansion of steam in Art. 67 of the last chapter. Here we shall have

$$\Delta Q = K \,.\, \Delta T,$$

where K is some constant, whence

$$\frac{R}{T_0} = K \,.\, \Sigma \,.\, \frac{\Delta T}{T} + \&c.$$

Proceeding to the limit and integrating,

$$\frac{R}{T_0} = K \,.\, \log._\epsilon \frac{T}{T^1} + \&c.;$$

and thus it appears that when a quantity of heat is applied continuously, in exact proportion to the change of temperature, its "heat-weight" is equal to $K \log._\epsilon \frac{T}{T^1}$ where T, T^1 are the limits of temperature within which the heat is supplied: or, to put the same thing in other words, it is the same as if the heat were supplied at the fixed temperature (θ) given by

$$\theta = \frac{T - T^1}{\log._\epsilon \frac{T}{T^1}}.$$

When the difference $T - T^1$ is not very great,

$$\theta = \frac{T + T^1}{2} \text{ (nearly).}$$

74. The extension of Carnot's principle, given in the preceding articles, was discovered by Professor Clausius and Sir W. Thomson about the year 1854. For further information the reader is referred to the second edition,* lately published, of Clausius' papers on heat, and to Professor Tait's treatise on Thermodynamics.

* 'Die Mechanische Wärmetheorie.' Erster Band. Von R. Clausius. Braunschweig, 1876.

Nature of the Process in an ordinary Steam Engine.
Expansion of Steam.

75. We are now in a position to discuss the process undergone by the steam in an ordinary steam engine, and at the same time to complete the study of the expansion of steam left partly unfinished in the last chapter.

I shall begin by supposing a non-conducting cylinder, complete expansion down to the pressure of the condenser, clearance and wire-drawing neglected: also the back pressure is imagined that which corresponds to the temperature of the condenser. Then the indicator diagram is shown in Fig. 20*a*, where O N represents *s* ($= \cdot 016$) the volume of 1 lb. of water, A N the rise of pressure as the water is

Fig. 20*a*.

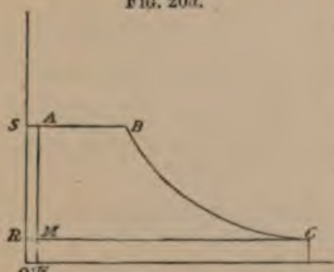
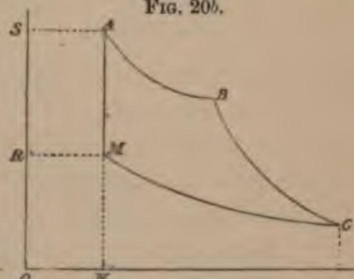


Fig. 20*b*.



forced into the boiler, A B the evaporation of the water till the volume has increased from A S to B S, B C the expansion of the steam from volume B S to volume C R, C M the condensation of the steam which is carried on till *all* the steam is condensed and it becomes once more water.

The efficiency of this engine is now to be found by comparing it with an air engine, which receives and rejects heat in the same way. In Fig 20*b* the diagram of such an engine is shown: M A represents a rise of pressure at constant volume during reception of heat, A B an increase of volume at constant temperature during a further application of heat, B C expansion without gain or loss of heat till

temperature has fallen to that of the refrigerator, C M compression at constant temperature till the air has returned to its original state. Different as the two diagrams may appear when actually drawn, in a thermodynamical point of view they are precisely similar provided the temperatures be the same, and the quantities of heat, supplied at each temperature, be in the same proportion: hence the efficiency of the two engines must be the same according to the generalized form of Carnot's principle given in Art. 71.

Now in the air engine the heat expended or rejected at each step of the process is, employing the notation of Chapter IV., per lb. of air,

During elevation of temperature	$K_r (T_1 - T_0)$
„ expansion	$c T_1 \log._\epsilon r$
„ compression	$c T_0 \log._\epsilon R$

where T_1, T_0 are the temperatures at which the air receives and rejects heat during its expansion A B and compression C M at constant temperatures, r R the ratios of expansion and compression (isothermal).

In the steam engine, on the other hand, the heat expended per lb. of steam is

During elevation of temperature	$T_1 - T_0$
„ evaporation	$x_1 L_1$
„ condensation	$x_0 L_0$

where $1 - x_1$, $1 - x_0$ are the weights of water mixed with the steam at the beginning and end of the expansion respectively, and L_1, L_0 are as usual the latent heats of evaporation at T_1, T_0.

We must now suppose the air engine so arranged that the quantity of heat received at constant temperature is in the same proportion to that received at constant volume as in the case of the steam engine, that is to say, we must suppose

$$\frac{c T_1 \log._\epsilon r}{K_r (T_1 - T_0)} = \frac{x_1 L_1}{T_1 - T_0};$$

but by Art. 31, page 82,

$$\frac{c}{K_v} = \gamma - 1;$$

$$\therefore \ (\gamma - 1)\, T_1 \,.\, \log._\epsilon r \,. = x_1 L_1 \,.$$

This condition being satisfied, the air engine and the steam engine will receive heat, at the same temperatures, in the same proportions, which is the needful condition that the efficiency may be the same: thus it will necessarily follow that the heat rejected in the two cases is in that same proportion, so that by similar reasoning

$$(\gamma - 1)\, T_0 \,.\, \log._\epsilon R = x_0 L_0 \,;$$

but, if r' be the ratio of adiabatic expansion in the air engine, it is clear that

$$R = r r'.$$

And by Art. 36 we know that

$$r' = \left(\frac{T_1}{T_0}\right)^{\frac{1}{\gamma - 1}};$$

$$\therefore \ (\gamma - 1) \log._\epsilon r' = \log._\epsilon \frac{T_1}{T_0},$$

and

$$(\gamma - 1) \log._\epsilon R = (\gamma - 1) \log._\epsilon r + \log._\epsilon \frac{T_1}{T_0};$$

whence replacing $(\gamma - 1) \log._\epsilon r$, $(\gamma - 1) \log._\epsilon R$ by their values just given, it is clear that

$$\frac{x_0 L_0}{T_0} = \frac{x_1 L_1}{T_1} + \log._\epsilon \frac{T_1}{T_0}. \qquad \text{(I)}$$

76. The very important result here obtained may be deduced more briefly without any reference to an air engine by application of equation (B), Art. 70, expressed in words in the second general statement of Art. 71, as modified in Art. 73: for the terms $\frac{x_1 L_1}{T_1}$, $\log._\epsilon \frac{T_1}{T_0}$ on the left-hand side are no other than the "heat-weights" corresponding to the heat received during evaporation at temperature T_1, and elevation

of temperature from T_0 to T_1, while $x_0 L_0$ is the heat rejected during condensation at temperature T_0: hence the equation (I) is only a repetition of the equation of Art. 65.

It should be remarked that the variation in the specific heat of water (Art. 3) is neglected, and it is interesting to remark where the reasoning of Art. 75 would fail if it were taken into account. If the specific heat of water vary, the same amount of heat will not be received at each degree of rise of temperature, and hence the processes in the air engine and the steam engine will not precisely correspond; hence the resulting equation is not quite exact. For some purposes it is desirable to take this into account, which may be done, with sufficient approximation, by employing a mean value, greater than unity, for the value of the specific heat, chosen, by Table II., according to the range of temperature under consideration.

Again, it is possible to make equation (I) more general, so as to include the case in which a plate of metal is supposed in contact with the steam, as in Case III., Art. 67, of the last chapter. If $m c$ be the heat supplied to the steam, as its temperature falls, as explained in detail in the article cited, then, by Art. 73, its "heat-weight" is $m c \log._\epsilon \frac{T_1}{T_0}$, and thus, if q be the mean value of the specific heat of water, we obtain the more general equation

$$\frac{x_0 L_0}{T_0} = \frac{x_1 L_1}{T_1} + (q + m c) \log._\epsilon \frac{T_1}{T_0}, \qquad \text{(II)}$$

which applies to all the three cases treated in the articles cited. It was there shown how to determine the expansion curve of a mixture of steam and water by applying a graphical or arithmetical process in successive stages; the formula now given enables us to obtain a final result with accuracy without going through the intermediate stages; and it may here be noticed that it is possible to deduce the formula by application of suitable mathematical processes to

the equation given in Art. 64 of the last chapter, as in fact was done in the first edition of this work.*

The adiabatic curve, then, is determined by the equation, derived from equation (I) by omitting the suffix 0,

$$x\frac{L}{T} = x_1\frac{L_1}{T_1} + \log_{\epsilon}\frac{T_1}{T} \text{ (approximately)},$$

from which the amount of water in the steam can be determined, after expansion till the pressure (and therefore the temperature) has fallen by a given amount. Let V be the volume of a lb. of steam at the end of the expansion, then

$$V = (v - s)\,x + s = v\,x + s \text{ (approximately)}$$
$$= v\,x \text{ (nearly, unless the steam is nearly all water)};$$
$$\therefore\ V\frac{L}{vT} = x_1 \cdot \frac{L_1}{T_1} + \log_{\epsilon}\frac{T}{T_1} \text{ (approximately)}$$

is an equation which determines the final volume of the steam after expanding till the pressure has fallen by a given amount. In using the equation it is necessary to suppose the final pressure given, and then to find T from Table I*a*, of course adding 461 as the temperatures are absolute. The calculation is simplified by finding x first and then V. The equation is applicable, however much water be mixed with the steam initially, or even when there is no steam but only water, as in Case II., Art. 65; it differs from Rankine's equation given in his work on the Steam Engine (p. 385) only in notation and in being more general, as Rankine has considered only the case in which the steam is originally dry. The formula was discovered by Rankine and Clausius working independently.† Two numerical examples will show how it is applied.

(1) Dry steam at 302° expands without gain or loss heat till its temperature has fallen to 212°, or, what is

* See Appendix, Note D.

† The extension of the formula to the case where heat is su steam in exact proportion to the fall of temperature appears Zeuner. See 'Grundzüge der Mechanischen Wärmetheorie,' p. 1

same time its pressure to 14·7 lbs. on the square inch; it is required to find how much moisture it contains, and what is its volume. Here we have

$$x_1 = 1;\ T_1 = 302° + 461° = 763°;$$
$$T = 212° + 461° = 673°.$$

From Table II. the value of L_1 is 902 thermal units, and of L 966 thermal units;

$$\therefore x \cdot \frac{966}{673} = \frac{902}{763} + \log_e 763 - \log_e 673.$$

Performing the numerical calculations,

$$x = \cdot 911,$$

showing that rather less than 9 per cent. of the steam is condensed.

(2) Let the steam instead of being initially dry contain initially 30 per cent. of water, then $x_1 = \cdot 7$, and the other data are unaltered:

$$\therefore x \cdot \frac{966}{673} = \cdot 7 \cdot \frac{902}{763} + \log_e 763 - \log_e 673,$$

whence

$$x = \cdot 662,$$

showing that about 4 per cent. by weight of steam is condensed, being less than when the steam is initially dry, as already explained (Art. 65).

(3) Let 1 lb. of water at 302° expand, as in Art. 65, till the pressure has fallen to 14·7, then $x_1 = 0$ and the other data are unaltered, whence

$$x = \cdot 0872,$$

a result which signifies that about $8\frac{3}{4}$ per cent. of the water has evaporated.

The volumes of the steam in these three cases are now found by the equation just mentioned,

$$V = ux$$

in which s may be disregarded in the first two cases, but *not* in the third, whence

$$V = 24 : V = 17 \cdot 5 : V = 2 \cdot 35.$$

The first and last results serve to fix the four corners of an indicator diagram of a perfect steam engine working with initially dry steam between the temperatures 302°, 212°; that is to say, the pressures 69·21 and 14·7 lbs. per square inch. It is this diagram which is shown in Fig. 11, Art. 46.

The calculation here described is for some purposes inconvenient, because the form of the curve is not ascertained directly, there being no direct relation between p and V, but only an indirect one by means of the temperature. Hence, if it be required to find the pressure after expanding r times, this can only be done by trial and error. To avoid this inconvenience it was suggested by Rankine that the expansion curve might be represented approximately by the equation

$$p\, V^n = \text{constant} = p_1\, V_1^{\,n},$$

if the index n be found by trial so as to give results agreeing with the foregoing calculation.

To test this suggestion, we have the equation

$$\log. p + n \,.\, \log. V = \log. p_1 + n \,.\, \log. V_1,$$

from which is obtained

$$n = \frac{\log. p_1 - \log. p}{\log. V - \log. V_1}.$$

If the suggestion is correct, we ought to find the same value of n, whatever be the values of p V, provided only the expansion start from the same point represented by the suffix 1.

For example, let us calculate n in the first of the two preceding cases; then we have

$$n = \frac{\log. 69 \cdot 21 - \log. 14 \cdot 7}{\log. 24 - \log. 6 \cdot 153} = 1 \cdot 138.$$

Now if the calculation be repeated with a different value of the terminal pressure, the result ought to be the same.

On trial it is found to be so approximately, and the result is also nearly the same if the initial pressure be, not 69·21, but some other pressure, provided the steam be initially dry. Zeuner has examined the question with great care and accuracy, and finds the best average value of n to be 1·135. But if the steam be initially [illegible]t, a smaller result is obtained; thus, if as in the second [illegible]xample given above, the steam contain 30 per cent. moistur[illegible] then

$$n = 1{\cdot}1 \text{ (ne[illegible]}$$

By making numerous calculat[illegible] of this kind, Zeuner found the best average value of t[illegible]ndex to be given by the empirical formula *

$$n = 1{\cdot}035 + \frac{x_1}{10},$$

where x is the initial dryness-fraction of the steam supposed not less than ·7; and Zeuner's calculations have since been verified by Grashof.†

When Rankine suggested the equation $p\,V^n =$ constant he gave the value $\frac{10}{9}$ or 1·11 for the index, on what grounds cannot now be determined; it is certain that numerical calculations from his own equations give a larger result, except when the steam is wet initially.

77. The equation $p\,V^n =$ constant is of less value than might be supposed, for it will be seen hereafter that when adiabatic expansion has to be considered in practical questions, the data of the question are, nearly always, not the initial pressure and ratio of expansion, but the initial and final pressures, in which case the equation may just as well be employed in its original form as in the form $p\,V^n =$ constant.

* 'Grundzüge der Mechanischen Wärmetheorie.' p. 342.

† 'Theoretische Maschinenlehre.' Band 1. p. 175.

By a slight alteration in the form it becomes more convenient in application. For it has been already shown that

$$L = (v - s)\, T\, \Delta P, \qquad \text{(Art. 46)}$$

where ΔP is the rise of pressure for 1° rise of temperature in lbs. per square foot, and L is expressed in foot lbs.

Also,

$$V - s = x\,(v - s);$$

$$\therefore\ x \frac{L}{T} = 772\,(V - s)\,\Delta P \quad \text{(L in thermal units)};$$

and equation (II) becomes

$$V - s = \frac{(V_1 - s)\,\Delta P_1 + 772\,(q + mc)\log_\epsilon \frac{T_1}{T}}{\Delta P};$$

or, reducing the increments of pressure ΔP from lbs. per square foot to lbs. per square inch,

$$V - s = \frac{(V_1 - s)\,\Delta p_1 + 5{\cdot}36\,(q + mc)\log_\epsilon \frac{T_1}{T}}{\Delta p}, \qquad \text{(III)}$$

an equation which by use of the values of Δp, given in Table I*a*, can very conveniently be used in rough calculations; the quantity s may, as usual, be generally omitted. A much more simple formula is available in some cases. (Appendix, Note C.) When accuracy is desired, the equation should be employed in its original form.

78. Although the adiabatic curve, for a mixture of steam and water, is of a very complicated character, incapable of being expressed exactly by any algebraical equation, yet all adiabatic curves are connected together in such a way that, when any two are given between given limits of pressure, all the others are at once determined by a calculation of the simplest character.

Let $V_1\ \overline{V}$ be the volumes corresponding to a given pressure p of a mixture of steam and water for the two extreme cases in which the mixture is wholly steam, or wholly water. at some other greater pressure; and let us suppose

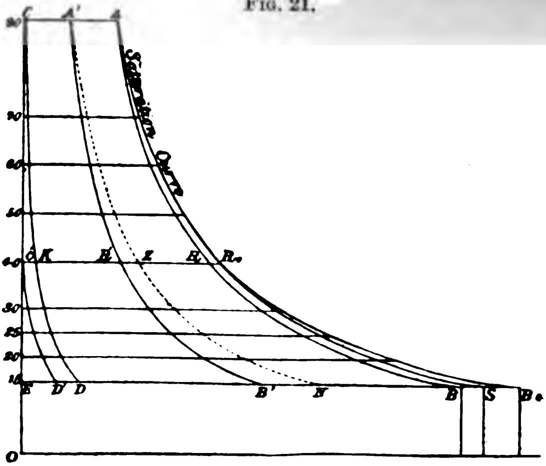
volumes already known. Then, to find the corresponding volume (V′), when the mixture consists at the same initial pressure of x_1 lbs. of steam, and $1 - x_1$ lbs. of water, we have from equation (III) of the last article,

$$V' = x_1 V + \bar{V}(1 - x_1),$$

an equation which determines all the other adiabatic curves: thus, when, by methods already g n, V^1 $\bar{V}$ have been determined, the volume of steam in ny given state initially, after adiabatic expansion down pressure p, is readily calculated; and the formula may sily be extended to cases in which the steam expands in c act with a metallic plate.

The nature of the connection etween the curves may, however, be most easily seen by rence to a figure. Fig. 21 shows various adiabatic cu lying between 90 lbs.

Fig. 21.



and 15 lbs. pressure, of which we will suppose, in the first instance, the curves A B, C D (known either by the calculation just indicated, or by the methods of Chapter VII.), corresponding to dry steam at 90, and water at 90, respectively. Then, to find the adiabatic curve for a mixture containing, at 90 lbs. pressure steam and water, it is only necessary to , A′ B′,

intercepts between the original curves into equal parts. Or, again, to determine the adiabatic curve corresponding to dry steam at 40 lbs. pressure, draw the saturation curve A R_0 B_0, then the required curve starts from R_0, the point in A B_0 which corresponds to 40 lbs. pressure, and divides externally the horizontal intercepts between the two original curves in a fixed ratio; for example, the point S, where that curve cuts the line of 15 lbs. pressure, is found by the proportion

$$DS : DB :: KR_0 : KR;$$

and, on the same principle, C′ D′, the curve corresponding to water at 40 lbs. pressure, or any other required curve, is at once found.

Still further, the expansion curve of steam in contact with a metallic plate may be found: take, for instance, a mixture of steam and water, containing originally one-half water, and let it expand in contact with $8\frac{1}{3}$ times its weight of iron, as in the example of Chapter VII., then the expansion curve is found by setting off at every pressure, such as, for example, 40 lbs. in the figure, R^1 Z equal to C^1 K, then the required curve is A Z N in the figure.

79. The equation obtained in the last article,

$$V' = x_1 V + (1 - x_1) \overline{V},$$

is equivalent to saying that, when a mixture of steam and water expands, the water evaporates, and the steam condenses, just as each would do when taken alone: so that the actual total condensation or evaporation is the difference between the condensation of the steam and the evaporation of the water. When a certain proportion exists between steam and water, the evaporation of the water exactly compensates for the condensation of the steam: a proportion which is given by the equation

$$x = \frac{q \log._\epsilon \frac{T_1}{T_2}}{\frac{L_1}{T_1} - \frac{L_2}{T_2}},$$

[redacted]

where T_1 T_2 are the initial and final temperatures between which the change is supposed to take place. For a very small change at temperature T, the above formula is approximately equivalent to

$$x = \frac{T}{1450}.$$

At 38 lbs. on the sq. inch, this gives ·5 as the value of x, which at high pressures will be at most ·6, and at low pressures at least ·45: if a greater change of temperature be considered, the variation in x will be less, and we may say generally that when a mixture of steam and water in equal proportions by weight expands without gain or loss of heat, the evaporation and condensation approximately balance one another, so that the total change is very small: and further, if water predominates, evaporation takes place, but if steam predominates, condensation.

Similar conclusions may be drawn in the case where a mixture of steam and water expands in contact with a metallic plate: the state of things is the same as if the water received the whole amount of heat given out by the cooling plate, and expanded alone without connection with the steam, while the steam condenses as if the plate were not there. When the water forms a film on the surface of the plate, it is probable that this is really what takes place—a question which we shall resume in a later chapter.

Again, when a mixture of steam and water expands, either adiabatically, or in contact with a plate, we may, if we please, separate mentally any part of the water from the rest, and consider it as a solid plate giving out heat as its temperature falls: so long as any of the remaining water remains unevaporated, the process of expansion will be quite unaltered by this supposition. This is an observation of great importance in the theory of the steam engine, for, generally, it is impossible to determine the *total* amount of water contained in a steam cylinder by direct observation: all that can be done is, to

find the quantity of water discharged from the cylinder per stroke, either as suspended moisture distributed throughout the whole mass of steam discharged, or by re-evaporation during exhaust. We now see, however, that any water remaining in the cylinder after exhaust plays the part of a metallic plate, but with much greater effect, weight for weight, on account of the great specific heat of water.

80. To complete the study of the expansion of steam in the way supposed in the preceding articles, it is now only necessary to find the area of the expansion curves.

Referring to Art. 75, we find that during the operation of the steam engine there supposed, we have

$$\begin{aligned}\text{Heat expended} &= T_1 - T_0 + x_1 L_1 \\ \text{Heat rejected} &= x_0 L_0\end{aligned} \left\{\begin{matrix}\text{approximately,} \\ q = 1\end{matrix}\right\};$$

$$\therefore \text{ Useful work} = T_1 - T_0 + x_1 L_1 - x_0 L_0 ;$$

but this useful work is no other thing than the area of the diagram, which is accordingly given by this formula, in which we observe that T_1, T_0, x_1, L_1, are data of the question, while $x_0 L_0$ is determined by reference to equation (I) of the same article. The formula will be exact instead of approximate if we replace $T_1 - T_0$ by $h_1 - h_0$.

Hence, by substitution, the area of the diagram is given by the formula

$$\begin{aligned}\text{Area} &= T_1 - T_0 + x_1 L_1 - x_1 L_1 \frac{T_0}{T_1} - T_0 \,.\, \log._e \frac{T_1}{T_0} \\ &= T_1 - T_0 - T_0 \,.\, \log._e \frac{T_1}{T_0} + x_1 L_1 \,.\, \frac{T_1 - T_0}{T_1}.\end{aligned}$$

Referring now to Fig. 21, Art. 78, and supposing T_1 T_0 to refer to the limiting temperatures, corresponding to the limiting pressures, represented by that figure, which need not now have the numerical values there supposed, we s that the area of the whole diagram C A¹ B¹ E, correspondi to the case of an initial dryness-fraction x_1, may be separat into the parts C D E, C A¹ B¹ D, whereof the sec the diagram of a perfect engine, working betwe

limits of temperature with an expenditure of heat $x_1 L_1$: hence for the area C D E we have

$$\text{Area CDE} = T_1 - T_2 - T_2 \cdot \log_e \frac{T_1}{T_2}.$$

The same result is obtained by putting $x_1 = 0$ in the preceding general formula.

Losses of Efficiency in Heat Engines in general.

81. When a heat engine works between given limits of temperature, it has already been sufficiently explained that the conditions of maximum efficiency are, that the engine shall utilize the whole difference of temperature between the source and the refrigerator, and that none of the expansive energy of the fluid shall be dissipated by unbalanced expansion. Hence, any diminution of efficiency below that maximum value—common to all engines satisfying those conditions, and hence called "perfect"—may be considered as due to losses of efficiency which may be ranged in two great classes:—

(1) Losses by non-utilization of the whole available difference of temperature.

(2) Losses by non-utilization of the whole available expansive energy of the fluid.

I propose to consider briefly these losses as regards heat engines in general, before passing on to study in detail the losses of efficiency in the steam engine in particular.

Class I.—The first class of losses arises from *misapplication* of heat, the engine not receiving the whole of its heat at the superior limit of temperature, as it should for maximum efficiency, but at some lower temperature; and not rejecting the whole of its heat at the lower limit, but at some higher temperature. Losses of this kind are independent of the particular engine considered, provided that the mode of application of the heat be the same, as appears from Art. 71.

Let us imagine T_1 T_0 the limits of temperature, and let us suppose that a certain quantity of heat, Q, instead of being received by the engine at the temperature T_1, as it should, is received at the lower temperature t, then the work due to that amount of heat in a perfect engine is

$$U = Q \cdot \frac{T_1 - T_0}{T_1},$$

while the actual work done is, from Art. 62,

$$U' = Q \cdot \frac{t - T_0}{t};$$

$$\therefore \text{ Loss} = U - U' = Q \cdot \left\{\frac{T_0}{t} - \frac{T_0}{T_1}\right\} = T_0 \left\{\frac{Q}{t} - \frac{Q}{T_1}\right\},$$

a loss quite independent of the particular nature of the engine, and depending only on the temperatures.

Thus, for example, let the limits of temperature be 320° and 120° Fahrenheit, or 781 and 581 absolute, and let 1000 thermal units be supplied to the engine at temperature 250°, instead of 320°; then

$$U = 1000 \cdot \frac{200}{781} = 256 \text{ thermal units};$$

$$U' = 1000 \cdot \frac{130}{711} = 183 \quad ,, \quad ,,$$

$$\therefore U - U' = 73 \quad ,, \quad ,,$$

that is to say, if the whole available difference of temperature—namely, 200°—had been utilized, we might have transformed 256 thermal units out of the 1000; but actually by wasting the 70° difference of temperature between 320° and 250°, we have obtained only 183 thermal units, or 71·4 per cent., the remaining 28·6 per cent. being wasted by non-utilization of the whole available difference of temperature during the reception of heat.

It may be that the heat is not all rejected at the lowest available temperature, and, in that case, there wi[illegible]

corresponding loss during the rejection of heat, which may be calculated in terms of the amount of heat rejected, and the difference of temperatures on the same principles. For example, suppose that in a steam engine the heat discharged from the condenser is 300 thermal units per I.H.P. per 1′, at the temperature of 100°, while the temperature of the atmosphere is 60°, then the loss by wasting the interval of 40° between the condenser and the atmosphere is $300 \times \frac{40}{561}$, or 21·4 thermal units per I.H.P. per 1′, being about one-half a horse-power.

It will frequently happen that the heat is received during a continuous change of temperature, and in that case

$$U = Q \frac{T_1 - T_0}{T_1} \text{ (as before)};$$

$$U' = \Sigma \, \Delta Q \frac{t - T_0}{t} = Q - T_0 \Sigma \frac{\Delta Q}{t};$$

$$\text{Loss} = U - U' = T_0 \left\{ \Sigma \frac{\Delta Q}{t} - \frac{Q}{T_1} \right\};$$

in which formulæ the whole heat Q is supposed separated into parts, each of which is supplied at the sensibly constant corresponding absolute temperature t.

When, as in Art. 73, the heat is supplied in exact proportion to the change of temperature,

$$\text{Loss} = T_0 \left\{ K \log._{e} \frac{t_1}{t_2} - K \frac{t_1 - t_2}{T_1} \right\},$$

where t_1 t_2 are the extreme values of the temperatures at which the heat is supplied.

Class II.—The second class of losses arises from the fluid not being allowed to expand steadily throughout the whole available difference of pressure overcoming a resistance, and thus doing useful work. Instead of this, the resistance to expansion is wholly or partially removed before the pressure has fallen to the lowest available limit; the expansive

energy of the fluid is then employed, wholly or partially, in generating violent motions, the kinetic energy of which is ultimately transformed into heat by fluid friction. Energy is not destroyed by this process, but it is generally, and probably always, rendered partially unavailable to us; that is to say, it is no longer possible to do as much work at the expense of the internal energy of the fluid, as might have been done had the unbalanced expansion been avoided. Again, even though the whole expansive energy of the fluid be duly exerted on a piston it may be partially employed in overcoming useless resistances.

This second class of losses differs from the first in being different for each kind of fluid, and hence cannot usefully be considered apart from the particular kind of engine which is being examined. Its magnitude is determined graphically by comparing the area of the actual indicator diagram with that of the indicator diagram of an engine, receiving heat according to the same law, in which no such unbalanced expansion takes place and no such useless resistances occur.

Losses of Efficiency in Steam Engines.

82. As in other heat engines, so in the steam engine the principal loss of efficiency proceeds from the narrow limits of temperature within which we are practically compelled to work. In a steam engine at least two-thirds the heat is always wasted in this way, but inasmuch as it is a loss which necessarily occurs whatever be the nature of the engine or its arrangement, an engine in which no other loss exists is conventionally said to be "perfect," and the efficiency of an engine working between given limits of temperature may consequently be properly estimated with reference to that of a perfect engine. When we have to compare engines which work between different limits of temperature, a so
what different course is advisable, but such cases wi

reserved to a later section of this chapter. The present section will be devoted to the comparison between an actual steam engine and the perfect engine working between the same limits of temperature.

No actual steam engine is perfect in the sense just described, and its losses of efficiency may be classified thus:

(1) By radiation to external bo .
(2) By transmission of heat to t xhaust steam.
(3) By clearance and wire-draw .
(4) By misapplication of heat t feed water.
(5) By misapplication of heat d g expansion.
(6) By incomplete expansion.
(7) By excess back pressure.

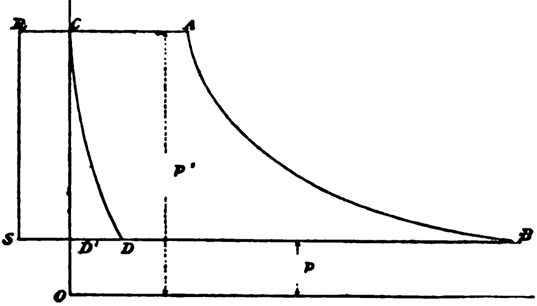
The first of these causes of loss efficiency can only be estimated experimentally, and what is known about it will be considered in a later chapter; the second will only be treated incidentally here, as it properly belongs to Chapter X. on the action of the sides of the cylinder; the third will be discussed in a special chapter (Chapter IX.), and there remains to be considered in detail in the present chapter the last four, which will be treated in order.

83. If the action of a perfect steam engine (Art. 46) be examined, it will be found that an essential part of the process is, that the temperature of the feed water should be raised by compression and not by application of heat; for which purpose it is necessary that the condensing steam should be taken before the condensation is complete, and by means of a large feed pump, compressed until the condensation is complete and the pressure has risen to that of the boiler. That such a process is theoretically possible is manifest from what has already been said, but it has never been carried out in practice, and, probably, would be found very difficult to carry out, without introducing evils greater than that against which it would be intended to provide. In actual steam engines the condensation is completed at the

condenser temperature, and the feed water is raised in temperature by direct application of heat; no doubt, a smaller feed pump is then required, and consequently less power is consumed in driving it, but this advantage is more than counterbalanced by the heat necessary to raise the temperature of the water.

In Fig. 22 the indicator diagram of a perfect engine is represented by C A B D, while C A B D D′ represents the diagram of an engine in which the compression part of the

FIG. 22.



process is not carried out. Then, the cases differ in this that more work is done, and more heat expended in the second case than in the first. It is clear that we shall have

Excess work done .. .. .. = Area C D D′;
Excess heat expended .. .. = $T_1 - T_0$;

in which last result the temperatures of boiler and condenser are supposed T_1, T_0 respectively, and the deviation from unity of the specific heat of water is neglected.

Now, by Art. 80,

$$\text{Area } C D D' = T_1 - T_0 - T_0 \log._\epsilon \frac{T_1}{T_0},$$

and the excess heat, if properly utilized in a perfect engi
would have produced an amount of work expressed by

$$\text{Available heat} = (h_1 - h_0) \frac{T_1 - T_0}{T_1} = (T_1 - T_0) \frac{T_1 - T_0}{T_1} \text{ (D}$$

[redacted]

hence the loss by misapplication of heat must be

$$\text{Loss} = T_0 \log._\epsilon \frac{T_1}{T_0} - \frac{T_0}{T_1}(T_1 - T_0).$$

This kind of loss belongs to Class I. of the general causes of loss considered in Art. 72, and the result might have been written down at once from the general formula there given; its absolute value is the same whether or not the engine is in other respects perfect. Let us call it U_1, then it is conveniently exhibited graphically by drawing a rectangle C S of height C D′, and area equal to U_1. Let p_1 be the initial pressure, and p_0 the pressure corresponding to the temperature of the condenser, then it is clear that for the base of that rectangle

$$S\,D' = \frac{5 \cdot 36\, U_1}{p_1 - p_0}.$$

Then, the whole area R A B S represents the work which would be done by a perfect engine expending the same quantity of heat as the imperfect engine, and the rectangle C S represents the loss due to the cause we are now considering.

For example, suppose the boiler pressure 95 lbs. per square inch, and the condenser temperature 120°, then we have

$$T_1 = 325° + 461° = 786°;$$

$$p_1 = 95; \qquad p_0 = 1 \cdot 68;$$

$$T_0 = 120 + 461 = 581°.$$

Hence we obtain

$$\text{Log.}_\epsilon \frac{T_1}{T_0} - \frac{T_1 - T_0}{T_1} = \cdot 3022 - \cdot 2608 = \cdot 0414;$$

and multiplying by T_0,

$$U_1 = \cdot 0414 \times 581 = 24 \cdot 05 \text{ thermal units,}$$

which is the loss of heat required, being the thermal equivalent of the additional work which might have been done had the

heat been properly used. To graphically represent that loss, we have

$$S D' = \frac{5 \cdot 36 \times 24 \cdot 05}{93 \cdot 3} = 1 \cdot 38 \text{ cubic foot,}$$

which result is to be set off on the scale of volumes, then C S represents the loss on the same scale that the remainder of the diagram represents the work done.

To find the percentage of loss, when the boiler supplies dry steam, it is sufficient to remember that

$$\text{Total heat expended} = H_1 - H_0 = 1093,$$

being simply the total heat of evaporation from 120° at 325°, of which in a perfect engine would be utilized

$$\text{Available heat} = \frac{T_1 - T_0}{T_1} \times 1093 = \cdot 261 \times 1093$$
$$= 285 \text{ thermal units.}$$

The loss then is 24 thermal units out of 285, or about 8·4 per cent. The maximum value of this loss in practical cases is 10 per cent., but, unless the pressure be high, it is much less than the value now found, as is also generally the case in non-condensing engines. If the boiler pressure be 60·4 instead of 95, and the temperature of the condenser 102° instead of 120°, the loss will be found to be 21·85, a result which will be employed in a subsequent article.

The loss here considered must not be confounded with the loss by non-utilization of the waste heat of a furnace in heating the feed water. The heat utilized in a feed-water heater is no doubt better employed than if it were altogether wasted, but it would be still better employed if it could be used in the boiler to generate steam from water at the boiler temperature. A feed-water heater then is to be considered as increasing the efficiency of the boiler, not that of the engine.

The compression shown in this article to be theoretically

advantageous i[illegible] entirely different from the compression taking place i[illegible] actual steam cylinders; this kind of compression is in [illegible]general also advantageous, but from quite different causes, as will be seen hereafter.

84. The next cause of loss of efficiency, namely, misapplication of heat during expansion, likewise belongs to Class I. of Art. 81, being due to supplying the steam with heat after its temperature has fallen. The [illegible] in question is supplied by the cylinder, which itself obta[illegible]at heat partly from the steam jacket, if there is one, bu[illegible]iefly from the steam itself during admission. Its val[illegible] found by the general formula of the article cited, th[illegible] to say, if U_2 be the loss,

$$U_2 = T_2 \left\{ \Sigma \, [illegible] \right\}.$$

in which the whole heat Q supplied during expansion is supposed divided into parts ΔQ, each supplied at a corresponding temperature t; thus, t is the mean absolute temperature of the expanding steam during a small part of the expansion, in which the heat supplied is ΔQ.

For example, in Art. 53, in considering hyperbolic expansion from 60·4 to 8·5, that is to say, from 293° to 185° the whole expansion was divided into four stages, thus

293°		266°		239°		212°		185°
	41		48		56		62·3	

in which the heat supplied during expansion in thermal units per stage of 27° was found to be as shown by the numbers placed below. Then taking for t the mean absolute temperature for each stage, we find

$$\Sigma \frac{\Delta Q}{t} = \frac{41}{741} + \frac{48}{714} + \frac{56}{687} + \frac{62 \cdot 3}{659}$$

$$= \cdot 2986;$$

and further

$$\frac{Q}{T_1} = \frac{41 + 48 + 56 + 62 \cdot 3}{754} = \frac{207 \cdot 3}{754} = \cdot 2750.$$

[redacted]

Hence, if the condenser temperature be 102°,

$$U_2 = 563 \times \cdot 0236 = 13 \cdot 3 \text{ thermal units},$$

which represents an amount of heat which might have been wholly transformed into work, had the heat Q been properly used. Had the heat Q been used in a perfect engine, the portion $Q_1 \frac{T_1 - T_0}{T_1}$ would have been turned into work, that is to say,

$$\text{Available part of Q} = 207 \cdot 3 \times \frac{293 - 102}{754} = 52 \cdot 51\,;$$

thus 25·3 per cent. of the heat supplied during expansion is wasted.

So it is in every case where heat is supplied during expansion, that heat is not wholly wasted, for it increases the work done by the expanding steam, but this increase is by no means as great as if the same heat had been applied in the boiler to generate more steam. The actual amount of the loss will vary considerably, according to the nature of the expansion curve, and will be greater the greater the fraction of the whole which is supplied near the end of the stroke. If the expansion curve be accurately given, then, by the methods of Chapter V., the heat supplied can be found for each step of the expansion, and hence the process just now used in the particular case of hyperbolic expansion can be applied; but the final result can also be obtained by a different method in any case in which the area of the expansion curve and the terminal state of the steam are known. For, by formula (B) Art. 70 applied to the operation represented in the diagram, Fig. 22, by A B D′C,

$$\text{Log.}_e \frac{T_1}{T_2} + \frac{L_1 x_1}{T_1} + \Sigma\,.\,\frac{\Delta Q}{t} = \frac{L_2 x_2}{T_2},$$

the suffix 1 as usual referring to the initial, and the suffix 2 to the final, state of the steam. Also, comparing the whole heat expended with the area of the diagram,

$$T_1 - T_2 + L_1 x_1 + Q = L_2 x_2 + (P_m - P_2)\, V_2,$$

where P_m is as usual the mean forward pressure, and the term containing it is reckoned in thermal units. Divide this last equation by T_1, and subtract it from the first, then

$$x\frac{\Delta Q}{t} - \frac{Q}{T_1} = L_2 x_2 \left\{\frac{1}{T_2} - \frac{1}{T_1}\right\} + \frac{T_1 - T_2}{T_1} - \log._\epsilon \frac{T_1}{T_2}$$
$$- (P_m - P_2) V_2 \,.\, \frac{1}{T_1} ;$$

and thus U_2 is found by the equation

$$\frac{U_2}{T_0} = \frac{T_1 - T_2}{T_1}\left\{1 + \frac{x_2 L_2}{T_2}\right\} - \log._\epsilon \frac{T_1}{T_2} - \frac{(P_m - P_2) V_2}{T_1}.$$

Thus, in the example of hyperbolic expansion just considered,

$$T_1 = 754^\circ : T_2 = 646^\circ : x_2 = 1 : r = 7 \cdot 22 ;$$

hence we obtain

$$\frac{T_1 - T_2}{T_1} = \cdot 1432 : \frac{x_2 L_2}{T_2} = 1 \cdot 525 : \log._\epsilon \frac{T_1}{T_2} = \cdot 1546.$$

Also, since the expansion is hyperbolic,

$$(P_m - P_2) V_2 = P_2 V_2 \log._\epsilon r = 138 \cdot 4 \text{ thermal units,}$$

dividing which by T_1 we obtain $\cdot 1835$; hence, performing the calculation,

$$U_2 = \cdot 0235 \, T_0,$$

a result nearly identical with that found above.

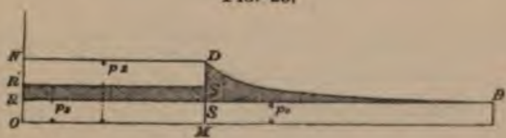
It is to be remarked that when, as is sometimes the case, heat is taken away during expansion, the loss, as calculated by this formula, may prove negative, and then expresses the saving occasioned by taking away heat from the steam during expansion, instead of at the higher temperature of admission. The waste occasioned by the action of the sides of the cylinder is the sum of this loss and the loss by re-evaporation during exhaust, and hence is greater or less than that indicated by the re-evaporation during exhaust according as this loss is positive or negative. This point will

be illustrated hereafter, in considering actual indicator diagrams. (Art. 134.)

85. The next cause of loss is incomplete expansion: in a perfect engine expansion is carried on till the pressure has fallen to that corresponding to the temperature of the condenser, but in actual engines this is never possible; in the first place the back pressure behind the piston is always greater than that corresponding to the temperature of the condenser, for reasons to be explained presently, and it never can be profitable to expand the steam below that limit while practically the greatest expansion is still further limited by other causes. Hence the work obtained from 1 lb. of steam in actual engines is always less than in a perfect engine from this cause: and the magnitude of the loss is thus investigated.

In Fig. 23 let D M represent the pressure and D N the volume of 1 lb. of steam at the end of the stroke, while R B shows the line of condenser pressure corresponding to

FIG. 23.



the temperature of the condenser. Through D trace an adiabatic curve by the preceding rules till it reaches that line in B: then the area D B S represents an amount of work which would have been done in the perfect engine and which is not done in the actual engine, and is consequently the loss by incomplete expansion. This loss belongs to Class II. of Art. 81, and takes effect by generating kinetic energy in the exhausting steam, which is afterwards changed into heat by fluid friction and becomes part of the waste heat given out in the condenser. Its amount is calculated thus:

$$\text{Area N D B R} = T_2 - T_0 + \frac{x_2 L_2}{T_2} \cdot (T_2 - T_0) - T_2 \cdot \log_e \frac{T_2}{T_0} \text{ [Art. 80]},$$

[redacted]

in which the suffix 2 refers to the terminal state of the steam and the suffix 0 to the condenser. But the area of the rectangle D R in thermal units is

$$\text{Rectangle D R} = 3\cdot 38\,(p_2 - p_0)\,V_2 = \left(1 - \frac{p_0}{p_2}\right) P_2\,V_2,$$

where V_2 is the actual volume of 1 lb. of steam; hence for the loss (U_2) we have

$$U_2 = (T_2 - T_0)\left\{1 + \frac{x_2 L_2}{T_2}\right\} - T_0 \log_e \frac{T_2}{T_0} - \left(1 - \frac{p_0}{p_2}\right) P_2\,V_2,$$

in which formula $P_2 V_2$ is supposed expressed in thermal units.

For example, let the expansion terminate when $p_2 = 8\cdot 4$ or $T_2 = 461° + 185° = 646°$, and let $T_0 = 102° + 461° = 563°$ be the temperature of the condenser corresponding to $p_0 = 1$, then assuming the steam dry at the end of the stroke or $x_2 = 1$, we have, as in a previous example,

$$P_2 V_2 = 70; \qquad \frac{x_2 L_2}{T_2} = 1\cdot 525;$$

$$\therefore\ U_2 = 83\,(1 + 1\cdot 525) - 563\,.\,\log_e \frac{646}{563} - 70\left(1 - \frac{1}{8\cdot 4}\right)$$
$$= 209\cdot 57 - 77\cdot 42 - 61\cdot 17$$
$$= 70\cdot 48 \text{ thermal units,}$$

which is the loss by incomplete expansion.

The loss here calculated depends solely on the state of the steam at the end of the stroke and the temperature of the condenser: it is rarely less than 15 per cent., and more often from 20 to 30 per cent., in condensing engines. In non-condensing engines it may be and often is of trifling amount, because in these the pressure p_0 is the pressure of the atmosphere.

An approximate value of this loss is given by a more simple equation which will be used in Chapter XI.,

$$U_2 = (T_2 - T_0)\,\frac{x_2 L_2}{T_2} - (P_2 - P_0)\,V_2.$$

The results of this formula are too small by an amount which is greater the higher the terminal pressure; but it admits of ready calculation. (See Art. 132.)

86. Lastly, the process in an actual engine differs from that of a perfect engine in the back pressure (p_3) being greater than that (p_0) corresponding to the temperature of the condenser. This difference is due partly to frictional resistance in the exhaust ports, coupled with the difference of pressure necessary to exhaust the cylinder in the short space of time in which that operation takes place, and partly to the presence of air in the condenser. It may conveniently be called the *excess* back pressure: its value depends, as formerly stated, on the speed of piston, the dimensions of the exhaust ports, and on the terminal pressure.

In Fig. 22 let the line $R^1 S^1$ be the line of mean back pressure, then the loss by excess back pressure is clearly represented by the rectangle $S^1 R$, or if U_4 be the loss in thermal units,

$$U_4 = 5 \cdot 36\,(p_3 - p_0)\,V_2 = \frac{p_3 - p_0}{p_2} \,.\, P_2 V_2 \,.$$

For example, let the terminal pressure be as before $8 \cdot 4$ and the temperature of the condenser 102°, so that p_0 is unity, while p_3 the real mean back pressure is 3, then we have

$$U_4 = 70 \times \frac{2}{8 \cdot 4} = 16 \cdot 67 \text{ thermal units.}$$

87. Let us now add together the four losses just found for the particular case considered of an engine working with hyperbolic expansion of about $7\frac{1}{2}$ times from initial pressure $60 \cdot 4$, the steam being dry at the end of the stroke and the temperature of the condenser 102°: we have

		Thermal units.
Loss in heating feed	=	21·85
„ during expansion	=	13·30
„ in incomplete expansion	=	70·48
„ in excess back pressure	=	16·67
∴ Total loss	=	122·3

Now the useful work per lb. of steam is given by the formula

$$\text{Useful work} = (P_m - P_2)\,V_2 \text{ foot lbs.}$$
$$= P_2 V_2 \,.\, \frac{p_m - p_2}{p_2} \,.\, \text{thermal units};$$

if $P_2 V_2$ be supposed in thermal units; but

$$p_m = \frac{1 + \log_\epsilon 7\cdot22}{7\cdot22} \,.\, p_1 = 24\cdot9,$$

the expansion being hyperbolic, and thus

$$\text{Useful work} = 182\cdot5 \text{ thermal units.}$$

This result added to the total loss gives about 305 for the number of thermal units per lb. of steam which could have been turned into work had the engine been perfect. We have a means of testing the truth of this result, for by Art. 25 the total heat of formation is given by the equation

$$Q = H_2 - h_0 + (P_m - P_2)\,V_2;$$

but since the temperature of the condenser is 102° and the temperature at the end of the stroke 185°,

$$H_2 - h_0 = 1138 - 70 = 1068,$$

while $(P_m - P_2)\,V_2$ has already been shown to be 138 thermal units, nearly;

$$\therefore\ Q = 1068 + 138 = 1206,$$

in which calculation the jacket heat is included. Now of this there would have been changed into work in a perfect engine the amount

$$U = 1206 \times \frac{T_1 - T_0}{T_1} = 1206 \times \cdot2533$$
$$= 305\cdot5 \text{ thermal units},$$

or practically the same.

The coincidence is not accidental, for if to the algebraical values of the four losses given in the preceding articles we

add the algebraical value of the useful work done, the result will be found to be the general value of Q, the total heat of formation, multiplied by the fraction $\frac{T_1 - T_0}{T_1}$; showing that the whole heat expended in producing the steam, in the state in which we find it at the end of the stroke, is fully accounted for by the useful work done and by the four losses which we have been considering.

We can now express the expenditure of heat in useful work and the losses of heat in terms of the work of a perfect engine by dividing by 305, whence we obtain

		Per cent.
Useful work done	=	59·9
Loss in heating feed	=	7·2
„ during expansion	=	4·3
„ in incomplete expansion	=	23·1
„ in excess back pressure	=	5·5
Available heat	=	100·0

The difference between the consumption of steam in an engine such as that considered in Case I., Art. 25, Chapter III., and that in a perfect engine given in Art. 46, Chapter V., is thus accounted for. The calculations of Chapter III., given in the table p. 58, show that little is gained by increasing the ratio of expansion beyond a moderate limit. The reason of this is that the loss by misapplication of heat during expansion, and excess back pressure, rapidly increases with the ratio of expansion, and thus the direct saving by increased expansion is counterbalanced.

The exhaust waste, together with clearance and wire-drawing, is here neglected, as was also the case in the articles cited: the effect of these causes is so considerable that condensing engines rarely utilize more than 40 or 50 per cent. of the available heat, and often much less. Non-condensing engines, however, have in general a greater *relative* efficiency, almost all the causes of loss of efficiency being less influential in their case: hence they utilize

sometimes as much as 75 per cent. of the whole amount of heat which could be turned into work in a perfect engine working between the same limits of pressure.

88. The foregoing method of expressing the expenditure of heat in heat engines—by comparing the useful work and the several losses of heat with the useful work of a perfect engine, that is to say with the fraction of the whole heat expended which is available for mechanical purposes—is convenient and sufficient, when we are considering engines working between given limits of temperature. When, however, those limits are different, it is necessary to consider the absolute efficiency as well as the efficiency relatively to a perfect engine, for the absolute efficiency is of course the true measure of the practical economy of the engine.

The most usual practical method of expressing the expenditure of heat in a steam engine is by the consumption of steam per I.H.P. per hour: this is not altogether suitable for scientific purposes, because the amount of heat necessary to generate a pound of steam varies with the pressure. Hence I shall employ instead as a measure of the expenditure of heat, the quantity of heat used per I.H.P. per 1′ expressed in thermal units: if the boiler pressure and the temperature of the condenser be known, the corresponding consumption of steam is readily found.

From the results of the preceding articles the corresponding results in this mode of measurement are immediately deducible. For one horse-power is equivalent to $\cdot 42\cdot 75$ thermal units per 1′, and in order to produce it the least possible expenditure of heat is evidently,

$$\text{Least amount} = 42\cdot 75 \, . \, \frac{T_1}{T_1 - T_0},$$

of which $42\cdot 75$ is actually converted into work, and the remainder is what may be called the *necessary* loss of heat, being unavoidable whatever the nature of the engine. All the other losses of heat are, at least theoretically, avoidable,

[redacted]

and they are as given in the preceding articles. In estimating them, it must be remembered that each loss, expressed before as work which might have been done had the engine been perfect, is accompanied by its own necessary loss of heat, and thus, like the useful work, must be multiplied by $\frac{T_1}{T_1 - T_0}$ to obtain its total amount.

Let us for example take the case employed as an illustration in the preceding articles, in which

$$\frac{T_1 - T_0}{T_1} = \frac{293 - 102}{754} = \cdot 2533;$$

$$\therefore \text{ Necessary expenditure of heat} = \frac{42 \cdot 75}{\cdot 2533} = 168 \cdot 77;$$

whence the necessary loss is 126 thermal units per I.H.P. per minute. To estimate the losses we have only to remark that the useful work previously shown to be 182·5 thermal units per lb. of steam, corresponds to a necessary expenditure of heat of 168·77 thermal units per I.H.P. per 1′, whence the conversion multiplier is $\frac{168 \cdot 77}{182 \cdot 5}$ or ·925, by using which we find

Loss in heating feed	=	20·2
„ during expansion	=	12·3
„ in incomplete expansion	=	65·2
„ in excess back pressure	=	15·4
Necessary loss	=	126·0
Total losses	=	239·1

The sum of all these losses is the heat rejected which appears in the condenser, and differs from the heat discharged from the condenser in pound degrees per I.H.P. per 1′ only in the heat contained in the condensed steam.

The total expenditure of heat is obtained by adding 42·75 to the above result: as before remarked, the exhaust waste is not included, and hence the results are considerably less than in actual engines.

If we add 42·75 to the necessary loss we reproduce the least amount of heat from which one horse-power could have been produced under the circumstances: this may properly be called the useful heat expended; the proportion which it bears to the total heat expended is the relative efficiency of the engine, while the absolute efficiency is the proportion which the useful work done bears to the total heat.

89. In analyzing the losses of heat in a steam engine, I have partly followed Zeuner in his work on the 'Mechanical Theory of Heat,' already frequently cited. Zeuner, however, does not contemplate any other than adiabatic expansion, and hence the loss by "misapplication of heat during expansion" is not considered by him.

CHAPTER IX.

CLEARANCE AND WIRE-DRAWING.

90. By "clearance" was originally meant the distance between the piston and the cylinder cover when the piston stands at the end of its stroke, some small interval being necessary to provide against a possible variation in the stroke due to wear of the connecting-rod brasses. The term is, however, now employed in the theory of the steam engine to signify a volume, being the whole volume included between the piston and the slide valve at the instant when the stroke commences. Clearance is expressed as a fraction of the whole piston displacement; thus, if X be the piston displacement, c X is the clearance, where c is a fraction which ranges from ·02 to ·1, according to the size and type of engine. Clearance modifies to a greater or less extent almost every calculation relating to the steam engine, and its neglect may give rise to serious errors, but I have thought it advisable to reserve its consideration to a special chapter, on account of the complexity thereby introduced if the question be considered at all thoroughly.

Together with clearance it is convenient to consider two other intimately connected subjects—first, that of compression; secondly, wire-drawing. In practice the exhaust steam is rarely permitted to escape from the cylinder throughout the return stroke; on the contrary, the exhaust port closes before the end of the stroke, and the steam still remaining in the cylinder is compressed by the advancing piston till, at the end of the stroke, the clearance contains steam of higher pressure than before, though (usually) of

lower pressure than that of the boiler. Again, if the pressure shown by the indicator during the admission be compared with the boiler pressure, a difference, always perceptible, and sometimes very great, will be observed: the steam is then said to be "wire-drawn." The amount of wire-drawing depends on the speed of piston, the state of the steam, and the magnitude of the admission ports: its influence is very complicated and cannot be treated exactly, but nevertheless it is necessary to have some idea of its general character.

As usual I shall commence with the simplest case, which is that where compression and wire-drawing are left out of account: the problem is thereby rendered far easier, and its solution offers no considerable difficulty.

Effects of Clearance considered alone.

91. In cases in which there is no sensible compression, the clearance, at the end of the return stroke, is full of steam of low pressure, not more than 2 or 3 lbs. per square inch, and, in this section, I shall imagine that steam wholly neglected, that is to say the clearance is regarded as absolutely empty. On this supposition, when the admission valve opens, the steam rushes in and fills the empty space with steam of the boiler pressure, which then presses against the piston and exerts energy upon it as if there were no clearance.

Now imagine a cylinder without clearance of the same volume as the original cylinder including clearance, then if the state of the steam were exactly the same in the two cases, the only difference would be that in the cylinder with clearance the steam would not do as much work during admission as in the cylinder without clearance, because the piston does not move through the clearance space.

Actually, if the boiler steam be supposed precisely the same in the two cases, the cylinder steam will often be rather drier with clearance than without clearance; but, reserving

this point for future consideration, I shall suppose the steam in the same state, for which it is only necessary to suppose that the boiler steam is slightly wetter for the cylinder with clearance than for the cylinder without clearance.

In Fig. 24 (page 224), $NS = X$ represents the piston displacement for a cylinder, the clearance of which is represented by $ON = cX$: then $OS = (1 + c) X$ represents the total volume of the actual cylinder, and also the volume of the cylinder without clearance with which the comparison is to be made. N M represents the part of the stroke traversed before the steam is cut off: then if $NM = m . NS$, m is the cut-off and $\frac{1}{m}$ is the apparent ratio of expansion. But the real ratio of expansion (r) is O S : O M,

$$\therefore (1 + c) X = r (c + m) X,$$

or

$$r = \frac{1 + c}{c + m};$$

so that r is no longer the reciprocal of the cut-off, as when clearance is neglected, but has a value which is smaller the greater the clearance.

For example, suppose the steam cut off at $\frac{1}{10}$ and the clearance also $\frac{1}{10}$, then

$$r = \frac{1 \cdot 1}{\frac{1}{10} + \frac{1}{10}} = \frac{11}{2} = 5 \cdot 5.$$

The example has been purposely chosen to show how great the influence of clearance may be: but, though not often so great as this, it is always so great as to render a determination of the clearance indispensable before the real ratio of expansion used in an engine can be determined with any approach to accuracy.

Next, to compare the mean pressures on the piston
two cases considered, it is only necessary
areas D A B S O and E A B S N, the fi
presents the energy exerted on the pist

clearance, and the second when there is none. The question arises in two forms.

(1) Being given the ratio of expansion and the law of expansion, it is required to find the actual mean pressure on the piston when the clearance is known. Here, from the data of the question, the mean pressure (P_m) is known on the piston of an engine working without clearance: that is to say,

$$P_m . OS = \text{Are} \quad \text{ABSO.}$$

But if P'_m be the actual mea essure,

$$P'_m . NS = \text{Ar} \quad \text{ABSN;}$$

$$\therefore P_m (1 + c) X - P'_m . \quad = \text{Area DENO}$$
$$= P_1 c X,$$

where P_1 is the initial pressure: us,

$$P'_m . = (1 + c) P_m - c P_1 .$$

For example, let the steam be cut off at $\frac{1}{10}$, and the clearance be $\frac{1}{10}$, then the real ratio of expansion is as shown above $5 \cdot 5$, and assuming hyperbolic expansion,

$$P_m = P_1 . \frac{1 + \log._e 5 \cdot 5}{5 \cdot 5} = \cdot 49 . P_1 ,$$

which gives the mean pressure when there is no clearance with the same ratio of expansion, and

$$P'_m = 1 \cdot 1 . P'_m - \tfrac{1}{10} . P_1 = \cdot 439 \, P_1 ,$$

which gives the true mean forward pressure to be used in conjunction with the area of the piston and the stroke in calculating the horse-power.

(2) But instead of this the datum of the question may be the actual mean pressure on the piston as determined by an indicator diagram in the usual way, and it may be required to find the equivalent mean pressure in an engine of the *same power* without clearance. In that case let P'_m be the

mean pressure in question, and P_m the equivalent pressure, then

$$P_m \,.\, OS = P'_m \,.\, NS,$$

or

$$P_m = \frac{P'_m}{1+c}.$$

The object of this process is to reduce the mean pressure from the piston displacement to the volume of the steam. In practice the base of the indicator diagram always represents the stroke, and the mean pressure is found by dividing the area by the stroke. Hence, when the abscissæ of the diagram are taken to represent volumes, the mean pressure refers to the piston displacement instead of to the volume of the steam as it ought to do when questions relating to the expenditure of heat are under consideration.

92. From what has been said it is clear that the volumes represented by the abscissæ of the indicator diagram are to be reckoned not from E N, the commencement of the stroke, but from D O, so as to include the clearance, and when the diagram is taken to represent the changes of state of 1 lb. of steam this must be borne in mind. Thus in considering the expenditure of heat, O S is to be taken as the base of the diagram, and not the stroke N S.

In finding graphically or by calculation the expenditure of heat, the only difference is that the area of the diagram representing the external work done during the formation of the steam is diminished by the area of the rectangle D N: the internal work done in producing steam in a given condition being quite independent of clearance. Hence the graphic construction of Art. 28 performed on the base O S will be unaltered, and the result in combination with the actual area of the indicator diagram will represent the total heat of formation. The heat pressure representing it however will be different, according as the base O S or the base N S is chosen: in the first case it will compare with the reduced mean pressure of case (2) of the last article: in the

second case it will compare with the actual mean pressure on the piston.

The formula for the total heat of formation,

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) V_2,$$

will be unaltered if by P_m we understand the reduced mean pressure of case (2) of the preceding article.

The loss of work per lb. of steam by clearance when, as in the present section, there is no compression, is

$$\text{Loss} = \text{Area } DN = \frac{c P_1 V_2}{1 + c} = \frac{c r}{1 + c} \cdot P_2 V_1,$$

understanding by V_1, V_2 as usual the actual volumes of 1 lb. of steam, and by r the real ratio of expansion, and remembering that the volume V_2 corresponds to the total volume of the cylinder.

Compression.

93. The compression of the steam behind the piston during the return stroke has no important influence on the total energy exerted on the steam piston, but only on the back pressure and the consumption of steam. The ratio of expansion is just the same whatever the compression, and the only difference in the forward action on the piston consists in the state of the steam being somewhat different according to the amount of the compression. When the boiler steam rushes into the partly empty clearance it becomes drier, as previously stated, but the amount of drying is less, and that, the fuller the clearance, and hence the state of the steam depends on the compression. This source of complication is for the present to be avoided by supposing the boiler steam to be slightly altered according to the amount of compression, so that the state of the steam in the various cases considered may be the same after reaching the cylinder.

Let the exhaust port be imagined to close at a known point of the return stroke, then a certain quantity of steam will be enclosed behind the piston: this steam is conveniently called

the "cushion" steam, to distinguish it from the steam discharged from the cylinder, and its volume, at release, as compared with the total volume of the cylinder will be denoted by n. The value of n may be found approximately from the indicator diagram, as will be seen presently. If the whole contents of the clearance be reckoned part of the cushion steam the ratio of weights will often be much greater than the ratio of volumes, on account of the water remaining after exhaust (Chapter X.): but it will be more convenient to include in the cushion steam only so much of the whole contents of the clearance as is necessary to make the two ratios equal, and to treat the excess water separately as virtually forming part of the sides of the cylinder (Art. 108).

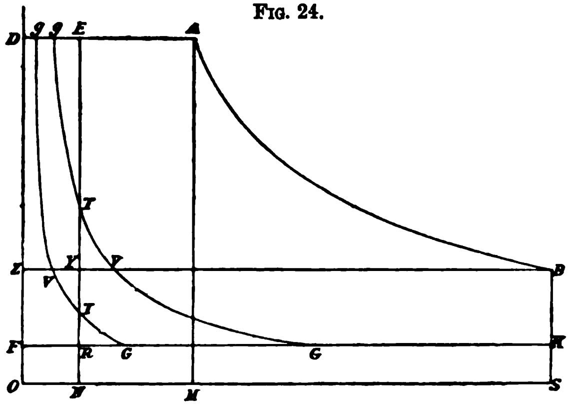
If we trace the changes undergone by the cushion steam from the instant when the exhaust port closes, it appears that, in the first place, it is compressed by the piston till the end of the stroke, or a little before, when it fills the clearance space with steam at a pressure, usually less than that of the boiler, which may be called the cushion pressure; in the second place, it is compressed further by the entrance of fresh steam from the boiler; in the third place, it enters the cylinder, and, after cut off, expands doing work upon the piston; in the fourth place, it suddenly expands doing no work, except in overcoming back pressure; while finally, either it or a precisely similar mass, resumes its original place and recommences its cycle of changes.

Thus the whole mass of steam in the cylinder may be separated into two parts, the cushion steam and the working steam, of which the latter goes through nearly the same changes as if there were no clearance, and the former goes through a series of changes without being condensed at all. The consumption of steam per stroke is, of course, only the working steam, and not the whole amount of steam contained in the cylinder after cut off.

Let us now compare the action of the steam in two

cylinders of the same *total* volume, one without clearance, and the other with clearance and compression. The ratio of expansion is the same in the two cases, and the mean forward pressure, being independent of the compression, is found by the preceding rules. The consumption of steam is, however, diminished in the proportion $1 - n : 1$, while the back pressure is increased; thus, the energy exerted on the steam piston per lb. of steam, is increased in the proportion $1 : 1 - n$, while the useful work done is altered in a more complicated way, according to the amount of compression.

Fig. 24.



To exhibit the process graphically, let O S, Fig. 24, represent, as before, the total volume of the cylinder, and O N the clearance, while K R F is the line of mean back pressure, assumed uniform till the exhaust closes at G; then F G represents the volume of the cushion steam at its lowest pressure. The curve G V T g represents the compression of the cushion steam from the exhaust volume G F to the initial volume D g, of which compression, part, namely G T, is produced by the steam piston, and the remainder g T is produced by the influx of boiler steam into the clearance. The fraction n is given by the equation

$$n = \frac{\mathrm{ZV}}{\mathrm{BZ}},$$

where V is the point of intersection of the compression curve and the line B V Z corresponding to the terminal pressure. The figure represents two compression curves, one for the case in which n is small, and the other where n is large; in the first case the point V lies within the clearance, and the consumption of steam is greater than in a cylinder without clearance with the *same piston displacement;* in the second case V lies without the clearance, and the consumption of steam is less instead of greater. The first case is common in condensing engines, and the second in non-condensing. The area E A B K G T is the area of the diagram, and represents the useful work done per stroke, that is to say, by a weight of steam less than if there were no clearance in the proportion $1 - n : 1$. The space Y V may be called the *effective* clearance, the consumption of steam being the same as if there were no compression with clearance Y V. If we express the effective clearance as a fraction (c^1) of the piston displacement,

$$c^1 = c - n(1 + c),$$

a quantity which may be positive, zero, or negative, according to circumstances.

94. If the base O S of the diagram be now understood to represent the volume of 1 lb. of steam in its terminal state, then

$$\text{Useful work per pound of working steam} = \frac{\text{Area E A B K G T}}{1 - n}.$$

The value of $1 - n$, that is to say, the consumption of steam, diminishes when the compression is increased, while the area expressing the whole useful work done also diminishes; thus the useful work done per lb. is altered, but the alteration may be an increase or a diminution, according to circumstances. In fact, a value can be found for the compression which shall make that useful work a maximum, as may be seen by the following general reasoning.

The n lbs. of cushion steam are alternately compress-

[redacted]

and expanded; now the compression, as far as the cushion pressure is performed by the steam piston, and takes effect as a diminution of the useful work done, while the expansion exerts energy on the piston only till the terminal pressure is reached, when the expansive energy of the cushion is dissipated by sudden expansion. Thus the cushion steam absorbs energy from the steam piston at every stroke, and this loss is greater the greater n. On the other hand, the consumption of steam is less the greater n, and consequently there must be some value of the compression for which the loss is least. A loss there must always be, however, except in one particular case—namely, the purely ideal case, in which the expansion is carried on till the terminal pressure is the same as the back pressure, and the compression is so great that the cushion pressure coincides with the initial pressure, so that, at the end of the stroke, the clearance is full of steam at the boiler pressure. Under these circumstances the useful work done per lb. of steam is unaltered, the diminution in total work being exactly balanced by the diminution in the consumption of steam.

To obtain definite results it is now necessary to make some assumption regarding the form of the compression curve of the cushion steam and the value of n.

In the first place it is to be observed that the expansion (or compression) curve of steam is always approximately an hyperbola, unless the quantity of water mixed with the steam be excessive; and we may therefore fairly suppose the curve V T G (Fig. 24) to be an hyperbola, starting from G, the point where the exhaust closes.

Let now the ratio of compression, that is to say, F G : F R, be denoted by R, then, returning to the supposition that the base of the diagram represents the total volume of the actual cylinder,

$$\mathrm{FG} = \mathrm{R}c\mathrm{X},$$

$$\mathrm{GK} = (1 + c)\,\mathrm{X} - \mathrm{R}c\mathrm{X};$$

therefore the area T G K S N representing the whole back pressure work done

$$= P_3(\overline{1+c}\,.\,X - RcX) + P_3\,.\,RcX\,.\log._e R$$
$$= P_3 X(1 + c - cR + cR\log._e R)$$

where P_3 is the back pressure before the exhaust closes. Thus the mean back pressure on the piston for the whole stroke is

$$P_B = P_3(1 + c - cR + cR\log._e R).$$

The value of R is easily found when the point where compression begins is known. For example, let compression begin at ·8 of the return stroke, then

$$1 + c - cR = \cdot 8;$$

or assuming $c = \frac{1}{10}$ as in a previous example,

$$R = (\cdot 2 + \cdot 1)\,10 = 3;$$

hence

$$P_B = P_3(1 + \cdot 1 - \cdot 3 + \cdot 3\,.\log._e 3)$$
$$= 1\cdot 129\,.\,P_3.$$

Having thus found the mean back pressure, the mean effective pressure is to be obtained by subtracting the result from the value of the mean forward pressure found in Art. 91, then

$$P'_m - P_B = (1 + c)(P_m - P_3) - cP_1 + cRP_3(1 - \log._e R),$$

and the influence of clearance and compression is thus determined. The total useful work done, expressed graphically by the area E A B K G T, is found by multiplication by X.

95. So far no important error can arise from the assumption that the compression curve is an hyperbola through G: in calculating n, however, the error of the assumption may be greater, and it will be desirable to diminish it by finding the point T directly from the diagram, making an allowance for the effect of lead: for assuming T ve have only to suppose T V an hyperbola, a suppo cannot be far from the truth.

Then, since V T is an hyperbola,

$$\frac{Z\,V}{F\,R} = \frac{P_c}{P_2},$$

where P_c is the cushion pressure represented by N T in the figure, and P_2 is, as usual, the terminal pressure;

$$\therefore\ Z\,V = c \,.\, \frac{P_c}{P_2} \,.\, X,$$

and

$$n = \frac{c}{1+c} \,.\, \frac{P_c}{P_2}.$$

The "effective" clearance (see last article) is consequently

$$c^1 = c - c \,.\, \frac{P_c}{P_2} = c \,.\, \frac{P_2 - P_c}{P_2}.$$

In general we may write further,

$$P_c = R\,P_3,$$

and thus

$$n = \frac{c\,R}{1+c} \,.\, \frac{P_3}{P_2},$$

a value of n which I shall have occasion to use presently.

96. The useful work now becomes (Art. 94),

$$\text{Useful work} = X \,.\, \frac{P'_m - P_B}{1-n};$$

or if V_2 be the terminal volume of the steam per lb.,

$$\text{Useful work} = \frac{V_2}{1+c} \,.\, \frac{P'_m - P_B}{1-n} \text{ (per pound)}$$

$$= V_2 \,.\, \frac{P_m - P_3 - \frac{c}{1+c} \,.\, P_1 + \frac{c}{1+c} R\,P_3\,(1 - \log._e R)}{1-n}.$$

If there had been no clearance the useful work would have been

$$\text{Useful work} = V_2\,(P_m - P_3),$$

and therefore the loss is given by the equation

$$U_s = V_2 \,.\, \frac{-n(P_m - P_3) + \frac{c}{1+c}(P_1 \,.\, - R\,P_3 + R\,P_3 \log._e R)}{1-n}.$$

It has been already pointed out that for some value of the compression the loss will be least. This value is found by applying the usual rules for a maximum or minimum to the value of the useful work per lb. of steam just obtained, the variable being R and the value of n expressed in terms of R by the equation given above. The result of this operation is given by the equation

$$(1 + c)\,P_2 \log._e R - c\,R\,P_2 = (1 + c)\,(P_m - P_2) - c\,P_1,$$

which may be put in a simple form if it be supposed that the expansion is hyperbolic, so that

$$P_m = P_1 \,.\, \frac{1 + \log._e r}{r},$$

whence, substituting and simplifying, the equation reduces to

$$\log._e \frac{R}{r} = 1 - \frac{r\,P_2}{P_1} - \frac{c\,r}{1 + c}\left(1 - \frac{R\,P_2}{P_1}\right), \qquad \text{(A)}$$

and the value of the greatest work becomes simply

$$\text{Greatest work} = P_1\,V_1 \log._e R, \qquad \text{(B)}$$

where in (B) R has a value previously found, by trial, from (A). Corresponding values may be found if the expansion be not hyperbolic, but the result in this supposition is sufficiently approximate for the purpose.

For example, let us take the data

$$r = 4 : c = \frac{1}{15} : P_2 = \frac{1}{8}\,P_1,$$

equation (A) becomes

$$\log._e \frac{R}{4} = 1 - \frac{4}{8} - \frac{4}{16}\left\{1 - \frac{R}{8}\right\},$$

or

$$\log._e \frac{R}{4} = \frac{1}{4} + \frac{R}{32},$$

from which, trying a few values of R by aid of the table of hyperbolic logarithms, we easily find

$$R = 6\tfrac{1}{4}\ \text{(nearly)};$$

or, again, if we take a higher rate of expansion, say

$$r = 8 : c = \frac{1}{15} : P_2 = \frac{1}{16} . P_1,$$

we obtain

$$\log._\epsilon \frac{R}{8} = \frac{R}{32}, \text{ or } R = 11 \cdot 5 \text{ (nearly)}.$$

Placing these values in equation (B),

$$\text{Useful work per pound of steam} = 1 \cdot 832 \; P_1 V_1 : (R = 6\tfrac{3}{4});$$
$$\text{,, ,, ,, ,,} = 2 \cdot 242 \, . \, P_1 V_1 : (R = 11 \cdot 5).$$

Suppose now we calculate the work when $c = 0$, that is to say, when there is no clearance, then

$$\text{Useful work} = V_2 (P_m - P_2) = P_1 V_1 \left\{ 1 + \log._\epsilon r - \frac{r P_2}{P_1} \right\},$$

which, in the two preceding cases, becomes

$$\text{Work} = 1 \cdot 886 \, . \, P_1 V_1 \left\{ r = 4 : P_2 = \frac{1}{8} P_1 \right\}$$
$$= 2 \cdot 579 \; P_1 V_1 \left\{ r = 8 : P_2 = \frac{1}{16} P_1 \right\};$$

the difference between these results and the former ones gives the least possible loss by clearance.

$$\therefore \text{ Loss} = \cdot 054 \, . \, P_1 V_1, \quad \text{or} \quad \cdot 137 \; P_1 V_1.$$

Hence the percentage of total work lost by clearance is at least 3 per cent. in the first case and 5·3 per cent. in the second. The loss will of course be greater if the clearance be greater, as is not uncommon in practice.

The point in the stroke at which compression should begin for least loss is obtained from the equation

$$z = 1 + c - c R,$$

where z is the fraction of the stroke required, whence taking the cases considered above,

$$z = \cdot 65, \quad \text{or} \quad z = \cdot 33,$$

results which show that in condensing engines it is usually inconvenient or impossible to commence compression sufficiently early to reduce the loss to a minimum. In non-condensing engines P_3 is at least 15 lbs. per square inch, and a moderate ratio of compression produces a great cushion pressure.

Let us next suppose no compression, and inquire how great the loss will be by clearance in the special cases considered. Putting $R = 1$ in the general value for the useful work:

$$\text{Useful work} = \frac{P_m - P_3 - \frac{c}{1+c}(P_1 - P_2)}{1 - n},$$

where

$$n = \frac{c}{1+c} \cdot \frac{P_3}{P_2},$$

which, still supposing the expansion hyperbolic, and taking the numerical data above given, becomes

$$1{\cdot}721\, P_1 V_1, \quad \text{or} \quad 2{\cdot}178\, P_1 V_1.$$

Comparing these values with those just given, when there is no clearance, we find for the loss

$$\cdot 165\, P_1 V_1, \quad \text{or} \quad \cdot 401\, P_1 V_1,$$

showing a loss of $8\frac{3}{4}$, or $15\frac{1}{2}$ per cent.

The numerical value assumed for the clearance in the preceding examples is not unusual in practice, and, indeed, is often exceeded; and the results obtained therefore show that the loss by clearance is very considerable, when there is no compression, and may be diminished by compression, conclusions which are doubtless in the main correct, though the value of the loss cannot be exactly determined and may be somewhat exaggerated in the preceding calculations. In the next chapter it will be shown that compression has the important additional advantage of diminishing the action of the sides

of the cylinder. The partial compensation in consequence of the drying effect of clearance will be considered presently.

The fact that there always is a loss by clearance, except in the ideal case where the expansion is complete, was first pointed out by Mr. Macfarlane Gray, in a paper read before the Institution of Naval Architects in 1874; and in the same paper it was shown that for a particular value of the compression the loss was reduced to a minimum.

97. In finding, graphically or otherwise, the heat expended on the steam when clearance and compression are taken into account, B V (Fig. 24) must be regarded as the base of the diagram, because it is B V which determines the consumption of steam and represents its terminal volume. If this be done all internal-work-pressures will be unaltered, but external-work-pressures, such as, for instance, the mean forward pressure, must be reduced, when necessary, from the base N S, representing the piston displacement, to the base B V, representing the terminal volume of the steam.

The energy exerted by the working steam is employed, partly in compressing the cushion steam and partly in doing work on the steam piston; while the energy exerted by the cushion steam during its expansion is also employed in doing work on the steam piston; hence the whole work done on the steam piston is equal to the energy exerted by the working steam, subject to a correction corresponding to the expansion or compression T V, occasioned by the compression T g of the cushion steam by the working steam being not exactly equal to its expansion g V. This correction is positive or negative, according as V lies within or without the clearance, that is to say, according as n is small or large: it is, however, of small amount and may usually be disregarded. In applying the formula for the total heat of formation, the mean forward pressure is therefore to be estimated as if there was no compression and then reduced as just described.

Drying Effect of Clearance.

98. It has been already explained that, in the preceding articles, the steam has been supposed in the same state, when comparing together the case with clearance and the case without clearance, and that to realize this it is necessary that the boiler steam should be imagined of various degrees of dryness, according to the circumstances of each special case. The reason of this is that when the steam rushes into an empty or partly empty clearance the expansive energy is employed in generating kinetic energy which, *if time enough elapses*, is absorbed by fluid friction, and appears as heat. Any complete investigation of the effect of clearance and compression must then include the consideration of the drying effect of clearance, at any rate so far as to ascertain the magnitude of the error produced by its neglect.

The effect in question is greatest when there is no compression, as in Art. 91: in that case the admission work is diminished by the quantity $c\, P_1 X$; that is to say,

$$\text{Diminution of admission work} = \frac{c P_1 V_1}{1+c} = \frac{c r}{1+c} \cdot P_1 V_1$$

for each lb. of steam admitted.

Now, the total heat of formation of the steam in its initial state is diminished by an exactly equal amount, and hence this result, expressed in thermal units, is the difference between the heat necessary to produce the boiler steam and the heat necessary to produce the actual steam in the state in which it actually enters the cylinder after being dried by absorption of kinetic energy by fluid friction.

Let, then, x_1 be the actual dryness-fraction after admission, and x'_1 the boiler dryness-fraction: $x_1 - x'_1$ represents the number of lbs. of moisture evaporated per [illegible] of steam, and clearly

$$[illegible] - x'_1 = \frac{c r}{1+c} \cdot \frac{P_1 V_1}{L_1},$$

where L_1 is, as usual, the latent heat of evaporation expressed in the same units as $P_1 V_1$: thus,

$$x_1 - x'_1 = \frac{c\,r}{1+c} \cdot \frac{x_1}{1+k},$$

where k is the number given in Tables III. and V.: a simple formula for the drying effect of clearance where there is no compression. Evidently it increases greatly when the expansion increases. The extreme case possible, and that only ideally, is when $c\,r = 1$, so that no admission work at all is done: in that case,

$$x_1 - x'_1 = \frac{x_1}{1+k} \text{ (nearly).}$$

Referring to the Tables for the value of k, it will be seen that the drying effect of clearance may be as much as 8 per cent. in this extreme case. In all ordinary cases it is far less, but nevertheless is sufficient perceptibly to diminish the loss by clearance.

For example, let us take

$$c = \frac{1}{15} : r = 4 : p_1 = 60 \cdot 4 ;$$

$$\therefore\ x_1 - x'_1 = \frac{1}{4} \cdot \frac{x_1}{11\frac{1}{4}} = \cdot 022 \,.\, x_1,$$

or

$$x'_1 = \cdot 978\, x_1,$$

and the heat expended per lb. of steam will be approximately diminished by clearance in the proportion ·978 : 1. Now the useful work done per lb. of steam was shown to be 1·886 $P_1 V_1$ without clearance, and 1·721 $P_1 V_1$ with clearance, which last result must now be divided by ·978 to enable us to make a true estimate of the effect of clearance. Thus the effective work is 1·76 $P_1 V_1$, instead of 1·72 $P_1 V_1$; and the true loss is ·126 $P_1 V_1$, or 6·7 per cent., instead of $8\frac{3}{4}$ per cent., as was found to be the case when the drying effect of clearance was omitted.

The partial compensation due to the cause here considered

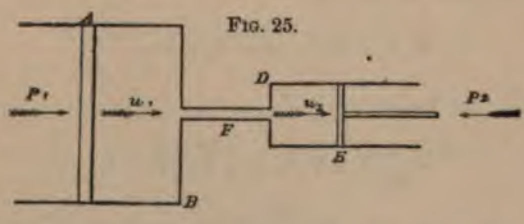
is proportionally less when there is compression, than when there is none: it can be calculated in each special case by a proper modification of the foregoing process, but enough has been said to render the method of procedure intelligible. No great reliance can be placed on the results of the calculations, for reasons to be stated presently (Art. 103).

Steady Wire-drawing.

99. It has been already said that steam is said to be "wire-drawn" when a sensible difference of pressure exists between the boiler and the cylinder during the admission of the steam. Such a difference proceeds partly from frictional resistances, and partly from the fact that the motion of the piston is not uniform, but gradually increases from the commencement to the middle of the stroke, and of these two causes I consider the frictional resistances first.

In Fig. 25, A B is a large cylinder, and D E a small one,

FIG. 25.



the two being connected by a pipe, F, provided with a stop-cock which can be opened or closed at pleasure. The large cylinder contains steam of pressure P_1, confined by a suitably loaded piston, which flows through the pipe into the small cylinder D E, the piston of which is loaded with pressure P_2, the difference between P_1 P_2 depending on the frictional resistance overcome in the pipe F, which may be made to vary at pleasure by opening and closing the stop-cock. The pistons are imagined to move forward with velocities u_1 u_2, which in the present case are uniform.

Under these circumstances, let a time be considered during which 1 lb. of steam flows from the large cylinder to the small one; then, clearly, the only change produced in the whole mass of steam included between the pistons, is due to the transfer of 1 lb. of steam from the large cylinder to the small one, the remainder of the steam remaining in the same condition as before.

Applying the principle of work, we have

$$\text{Energy exerted} = \text{Work done} + \frac{u_2^2 - u_1^2}{2g},$$

the last term being the change of kinetic energy consequent on the increased velocity with which the lb. of steam moves. But the energy exerted is $P_1 V_1$, where V_1 is the specific volume of the steam in the large cylinder; and the work done consists partly of external work, $P_2 V_2$, done in overcoming the resistance to motion of the small piston, and partly of internal work, which is given by rules previously stated. Hence we obtain

$$P_1 V_1 = P_2 V_2 + I_2 - I_1 + \frac{u_2^2 - u_1^2}{2g},$$

which may be written

$$\frac{u_2^2 - u_1^2}{2g} = P_2 V_2 + I_2 - P_1 V_1 - I_1;$$

but with the notation employed in previous chapters (Art. 12).

$$I = h + \rho x = h + (L - Pv)\,x = h + Lx - PV;$$

thus,

$$\frac{u_2^2 - u_1^2}{2g} = h_2 - h_1 + L_2 x_2 - L_1 x_1,$$

which is a general formula applicable to any case in which the motion is *steady*—that is to say, when the pistons move uniformly. The only suppositions made are, that sufficient time elapses to permit the kinetic energy, corresponding to the rushing motion through the pipe F, to be converted into

heat by fluid friction when the eddying motions subside into the simple motion of translation of the small piston, and that any heat generated by friction against the sides of the pipe passes into the steam, not into the metal of the pipe.

The very important general formula here given is applicable to many special cases, of which that to be considered in the present article is when u_1 u_2 are small, while the difference of pressures is not very small. Let the velocity of the small piston be not more than 16 feet per second, or 960 feet per minute, then the height due to that velocity is under 4 feet, and the kinetic energy is consequently less than 4 foot lbs. and may be neglected; then, as u_2 is in any case less than u_1 we have simply

$$0 = h_2 - h_1 + L_2 x_2 - L_1 x_1,$$

a formula which enables us to find x_2 the final dryness-fraction of the steam when its initial dryness is known, and also the pressures in the two cylinders.

Hence

$$x_2 = \frac{h_1 + L_1 x_1 - h_2}{L_2} = \frac{t_1 - t_2 + L_1 x_1}{L_2},$$

where L_1 L_2 are expressed in thermal units, a formula which is applicable to every case of wire-drawing which is steady and not too rapid.

By subtracting x_1 from each side,

$$x_2 - x_1 = \frac{t_1 - t_2 - x_1 (L_2 - L_1)}{L_2};$$

or using the formula for L (Art. 4),

$$x_2 - x_1 = \frac{t_1 - t_2}{L_2} (1 - \cdot 71\, x_1),$$

a formula which shows that steam is made drier by wire-drawing, and enables us to calculate the amount of drying.

For example, let steam s

wire-drawn to 14·7 lbs. per square inch, and let it contain initially 5 per cent. of moisture, so that $x_1 = \cdot 95$, then

$$t_1 = 293^\circ : t_2 = 212^\circ : L_2 = 966;$$

$$\therefore\ x_2 - x_1 = \frac{81}{966}(1 - \cdot 95 \times \cdot 71)$$

$$= \cdot 0274,$$

showing that more than half the moisture is evaporated. It will be seen, however, that the amount of drying is nearly proportional to the difference of temperature, and hence is generally much less than in the extreme case here supposed.

100. When clearance is combined with wire-drawing, the effect in drying the steam is of course much greater. It will be sufficient to consider one case, namely, that in which there is no compression and the clearance is regarded as sensibly empty when admission commences.

The difference between this case and the last is, that now the piston does not move through the clearance space, and that, consequently, the work done in driving the piston is diminished in the proportion

$$\frac{1}{r} : \frac{1}{r} - \frac{c}{1+c}, \quad \text{or} \quad 1 : 1 - \frac{cr}{1+c},$$

the volume of the cylinder being regarded as unity for the purposes of the calculation. Hence the equation of the last article becomes, omitting the velocities for the reasons previously stated,

$$P_1 V_1 = P_2 V_2 \left\{1 - \frac{cr}{1+c}\right\} + I_2 - I_1;$$

or, making the same changes as before,

$$L_1 x_1 = L_2 x_2 + t_2 - t_1 - \frac{cr P_2 V_2}{1+c};$$

that is,

$$x_2 = \frac{t_1 - t_2 + L_1 x_1}{L_2} + \frac{cr}{1+c} \cdot \frac{P_2 V_2}{L_2}$$

$$= \frac{t_1 - t_2 + L_1 x_1}{L_2} + \frac{c r x_2}{1+c} \cdot \frac{1}{k+1},$$

from which x_2 may be calculated.

For example, let

$$c = \frac{1}{15} : r = 4 : x_1 = \cdot 95;$$

and let the wire-drawing be from 66 to 60 lbs. per square inch, then

$$t_1 - t_2 = 6^\circ \cdot 2 : L_2 = 909 : L_1 x_1 = 859 \cdot 4;$$

$$\therefore \ x_2 = \frac{866}{909} + \frac{1}{46} \cdot x_2;$$

$$x_2 = \cdot 973.$$

From this example it appears that the effect of clearance is much greater than that of moderate wire-drawing in drying the steam; but, as before, it is much diminished by compression.

101. The loss by wire-drawing may be differently stated, according to the particular cases which are compared together. The most natural case to take is, perhaps, to compare the performance of a perfect engine with one which is in all respects perfect except that the steam is wire-drawn down to a given pressure.

Let T_1, T_2 be the absolute temperatures, and let the other notation be as before, then the expenditure of heat in the first perfect engine is $L_1 x_1$, and the equivalent expenditure in the second engine is $L_2 x_2$; hence if T_0 be the temperature of the condenser,

$$\text{Loss} = L_1 x_1 \,.\, \frac{T_1 - T_0}{T_1} - L_2 x_2 \,.\, \frac{T_2 - T_0}{T_2},$$

whence substituting for $L_2 x_2$ from the preceding equation,

$$\text{Loss} = L_1 x_1 \,.\, \frac{T_1 - T_0}{T_1} - (L_1 x_1 + t_1 - t_2) \,.\, \frac{T_2 - T_0}{T_2}$$

$$= T_0 \left\{\frac{1}{T_2} - \frac{1}{T_1}\right\} L_1 x_1 - (T_1 - T_2) \,.\, \frac{T_2 - T_0}{T_2}.$$

For example, let

$$T_1 = 764^\circ : T_2 = 754^\circ :$$

corresponding to wire-drawing from 70 to 60 lbs. per square inch, then

$$L_1 x_1 = 856 : \frac{T_2 - T_0}{T_2} = \cdot 253;$$

$$\therefore \text{ Loss} = 8 \cdot 39 - 2 \cdot 53 = 5 \cdot 86 \text{ thermal units.}$$

The loss here, as in Chapter VIII., means the loss of available heat; that is to say, the heat-equivalent of work which might have been done in a perfect engine working at 70 lbs., but is not, in consequence of wire-drawing. It must consequently be compared, as in the case of the other losses considered in the chapter cited, not with the whole heat expended, but with the fraction available for mechanical purposes.

In practical cases the loss would probably be less, and in some cases may even be a gain.

Unsteady Wire-drawing. Influence of the shortness of the period of admission.

102. Wire-drawing in practice is much more complicated than in the comparatively simple cases which have just been considered, on account of the varying motion of the piston. This has a twofold effect on the difference of pressure between boiler and cylinder, the result being that, at the beginning of the stroke, the difference is not large, but that it goes on increasing as the stroke progresses, the admission line drawn by the indicator exhibiting a gradually falling pressure, with a rounded cut-off corner instead of a constant pressure with sharp cut-off. Moreover, the period of admission is so short that it frequently happens that there is not sufficient time for the absorption of kinetic energy by fluid friction, as supposed in the preceding section.

In the first place the acceleration of the piston has a direct influence, apart from frictional resistances, an idea of which may be obtained by considering the simpler case of

an incompressible fluid flowing through a pipe into a cylinder provided with a piston which moves with velocity u_2. Let the ratio of areas be m, then obviously

$$u_1 = m\, u_2$$

is the velocity of the fluid flowing through the pipe. Further, let the piston be connected with a uniformly revolving crank, then if the obliquity of the connecting rod be neglected, the acceleration of the piston is well known to be $\omega^2 y$, where ω is the angular velocity and y the distance of the piston from the middle of its stroke. Then if n be the revolutions per 1″,

$$\omega = 2\,\pi\, n;$$

and since the acceleration (f) of the water in the pipe is obviously m times the acceleration of the piston, we must have

$$f = m\,\omega^2\, y = m\, 4\,\pi^2\, n^2\, y.$$

Now take two points in the pipe, A and B, distant l from each other, then, if the water moved uniformly, there could be no difference of pressure between A and B, except that due to frictional resistances; but if there be an acceleration in the direction A B, P_A must be greater than P_B in order to produce that acceleration. Let A be the area of the pipe, w the weight of a cubic foot of the fluid, then

$$(P_A - P_B)\,A = \frac{w}{g}\,.\,A\,l\,.\,f;$$

or putting $l = 1$,

$$P_A - P_B = m\,\frac{w}{g}\;4\,\pi^2\,n^2\,y,$$

a formula which gives the difference of pressu
due to acceleration) for each foot length of
pounds per square foot.

In the theory of reciprocating pun
paid to this effect of unsteady moti

siderably the speed at which it is possible to drive them, and renders an air vessel necessary to avoid shocks. Some idea of its magnitude, in the case of a steam cylinder, may be obtained by considering that a maximum value of f would be about 100, when the units are feet and seconds, while w, the weight of a cubic foot of steam, will not often exceed ·33, hence a maximum value of the fall of pressure per foot length of pipe would be m lbs. per square foot. If now there were no initial condensation, m would be a little, but not much, greater than the ratio of areas of piston and port, and hence would not exceed 20, so that the fall of pressure per foot length of pipe should not in any case exceed ·14 lb. per square inch at the commencement of the stroke, gradually diminishing as the piston approaches half stroke; but if there be much initial condensation, m will depend much more on the rate of condensation than the mere area of port, and hence may be very greatly increased. The practical conclusion appears to be that this effect cannot be large, except where there is much initial condensation. (See Art. 106.)

Frictional resistances, and the head necessary to generate motion, are doubtless usually far more influential in the production of a difference of pressure between the boiler and the cylinder; they vary as the square of the velocity with which the steam rushes into the cylinder, and hence, if there were no initial condensation, would be zero at the beginning of the stroke, and gradually increase up to half stroke if cut off has not previously occurred. The line drawn by the indicator pencil during admission, often called the "steam line" by writers on the indicator, is accordingly not horizontal, but falls as the stroke proceeds. When the ports and passages are too small for the speed of piston, the fall is very marked. When there is any great amount of initial condensation, the velocity of the steam through the ports, at a particular instant, will depend in great measure on the rate of condensation at that instant, and will be

increased; thus the fall of pressure is increased, and is not perhaps *necessarily* greater at the end of the admission than at the beginning, though no doubt that generally occurs in practice.

103. The other reason, mentioned above, why wire-drawing, in an actual steam cylinder, is more complicated than in the simple case considered in the preceding section, is the shortness of the time allotted for admission, in consequence of which, the eddying motions, due to frictional resistances and especially to the sudden expansion from the area of the port to the area of the cylinder, have not time to subside, but continue for some instants after cut off, or even perhaps, in some cases, till the end of the stroke.

At any point in the stroke, either before or after cut off, imagine the piston suddenly held fast, and, if open, the port suddenly closed, the whole mass of steam in the passages and cylinder will be in a state of violent motion, and its pressure will not be uniform throughout, but will generally be greater in the passages than in the cylinder. In a very short space of time, however, the pressure will be equalized, and the kinetic energy of the motion will be absorbed in the form of heat: thus the pressure in the cylinder will rise slightly, and there remain stationary. Suppose this operation carried out at every point of the stroke, and the corresponding pressures laid off on a diagram, which may be called the "equilibrium" diagram, that diagram will in general lie above the actual diagram, and will not coincide with it until after the steam is cut off, and where the piston speed is high may even, in extreme cases, be conceived to deviate from it at the end of the stroke.

In the investigations relating to the drying action of clearance and wire-drawing in the present chapter, and to the heat supplied during expansion in Chapter VII., it has been presupposed that there was no sensible difference between the two diagrams, and each particular case must be carefully

examined and allowance made for possible error before relying on the results of such calculations. It may be in some cases that instead of clearance making the steam drier, it makes it actually wetter during admission than it was in the boiler. And the heat supplied during expansion may be in some cases considerably less than would appear from an actual diagram, however carefully taken.

The question here considered may also be dealt with without any reference to the "equilibrium" diagram by treating the kinetic energy, not yet absorbed by fluid friction, as part of the internal energy of the steam, in addition to the energy stored up in steam of the same quality at rest. The heat supplied during expansion, found by the processes of Chapter VII., is therefore to be corrected by the subtraction of the difference between the kinetic energy of the steam in its initial and final states respectively; or if, as will usually be the case, the kinetic energy in the terminal state may be neglected, the correction will simply be the initial kinetic energy of the steam. Hence it is possible to find a maximum value of the correction so far as due to wire-drawing: for, if the difference of pressure between boiler and cylinder be $P-P^1$ and the mean specific volume of the steam admitted be V_0, the kinetic energy in question must be less, and will, probably, be much less, than is given by the formula

$$\text{Kinetic energy} = (P-P^1)\,V_0 = \frac{p-p^1}{p_0}\,.\,P_0V_0\,,$$

which readily may be calculated in thermal units by use of Table IV*a*. Now in the American experiments (Chapter XI.), from which the data were taken employed in Chapter VII., the difference of pressure between boiler and cylinder does not appear to have exceeded one-tenth the boiler pressure (absolute), and therefore seven or eight thermal units is an excessive estimate of the correction in question. Where the pressure is reduced by throttling to one-half the boiler

pressure or less, the case of course is very different. It is more difficult to find a maximum limit in the case of clearance, but, as the correction cannot exceed a small fraction of the work done, I think that if the data are given correctly there is no reason to doubt the substantial accuracy of the results given in the examples in question.

The influence of the shortness of the time allotted for admission on the effects of wire-drawing was first pointed out by Zeuner in a paper published in the 'Civil Ingénieur' for 1875, to which I shall have occasion to refer in the next chapter.

The formula and graphical process of Chapter III. for determining the total heat of formation of the steam at the end of the stroke are quite unaffected by the circumstances considered in the present article, but the exhaust waste in some extreme cases may include kinetic energy stored up in the exhausting steam, instead of being exclusively due to re-evaporation during exhaust and external radiation.

104. In Art. 30, Chapter III., it has already been explained that the figure drawn by the pencil of an indicator represents the average changes of state of the whole mass of steam shut up in the cylinder, but that, when clearance and wire-drawing are taken into account, these changes are not exactly the same as the actual changes of state of any particular portion of the steam. The diagram of energy of a particle of steam can never have any compression curve differing greatly from the expansion curve, nor can there be a rounded cut-off corner such as appears in actual indicator diagrams.

For the forms of actual indicator diagrams, I must refer to any good treatise on the indicator.

CHAPTER X.

ACTION OF THE SIDES OF THE CYLINDER AND OF WATER REMAINING AFTER EXHAUST.

105. When the volume of steam actually delivered from a steam cylinder at release, is compared with the volume of dry steam at the terminal pressure, corresponding to the amount of feed water used, after deduction of the jacket supply, it is always, or nearly always, found that the first is far less than the second, showing that at the end of the stroke the steam discharged from the cylinder must contain more or less water, which is either re-evaporated during exhaust, or is carried out with the exhaust steam in the shape of suspended moisture. Some of this effect is no doubt due to the fact that the steam supplied by the boiler is rarely dry: but in general the difference in question is far too great to be thus accounted for, and it is therefore necessary to suppose that liquefaction takes place after the steam enters the cylinder. Moreover, when the expansion curve drawn by an indicator is examined, it is almost always found, even when the greatest care has been taken to eliminate disturbing causes, to show that evaporation takes place during expansion.

Now, these unquestionable facts can only be explained by supposing that liquefaction takes place during the admission of the steam to the cylinder, and evaporation during expansion and exhaust. This alternate liquefaction and evaporation is chiefly due to the action of the sides of the cylinder, in many cases combined with the effect of water remaining in the cylinder after the exhaust is completed. Let us first consider the liquefaction during the admission.

Initial Condensation.

106. It is in the first place clear that the amount of steam liquefied on admission must depend on the area of the surface exposed to the steam: it is true that other circumstances may, or rather must, have influence, and especially the time during which the contact of the steam with the surface lasts, and the temperature of the surface; but the first consideration is the actual area of the surface.

Let d be the diameter of the cylinder in feet, λ the stroke, then

$$\text{Cubic content} = \frac{\pi}{4}\lambda d^2;$$

$$\text{Exposed surface} = \frac{\pi}{2}d^2 + \pi\lambda d;$$

including in the surface the cylinder cover and piston, although of course it will not always follow that these have the same influence as the surface of the cylinder itself.

Let V_1 be the volume per lb. of the steam filling the cylinder, then

$$\text{Weight of steam} = \frac{\pi}{4}\cdot\frac{\lambda d^2}{V_1}.$$

Hence the exposed surface per lb. of steam is given by the formula

$$S_0 = V_1 \cdot \frac{4\left\{\frac{\pi}{2}d^2 + \pi\lambda d\right\}}{\pi\lambda d^2},$$

or

$$S_0 = V_1\left\{\frac{2}{\lambda} + \frac{4}{d}\right\} \text{ square feet,}$$

where V_1 is as usual in cubic feet and λ, d, in feet.

Now, let r be the ratio of expansion, then, treating the clearance space as if it all formed part of the actual cylinder, the admission surface will be

$$S = 2V_1\left\{\frac{r}{\lambda} + \frac{2}{d}\right\},$$

a formula from which numerical results are readily obtained. The formula shows that the admission surface per lb. of

steam is increased, (1) by diminishing the size of the cylinder, (2) by lowering the pressure of the steam, (3) by increasing the expansion.

In Chapter VII. methods have been explained by means of which the quantity of heat abstracted from each lb. of steam during admission may be determined, and hence we are in a position to find the heat abstracted by each square foot of admission surface. It is true the results of these processes are not free from possible errors of considerable magnitude, occasioned by difficulties of observation, ignorance of the quality of the steam supplied by the boiler, and certain effects of wire-drawing (Art. 103). With proper care, however, they may be applied to obtain roughly approximate results and a rough approximation is all that is required for the present purpose.

INITIAL CONDENSATION IN STEAM CYLINDERS.

Dimensions of Cylinder.	Ratio of Expansion.	Initial Pressure.	Surface per Lb.	Initial Condensation.	Heat abstracted		
					Per Lb.	Per Sq. Ft.	Per Sq. Ft. per 1'.
Bache. Stroke = 2′ Diam. = 2′ 1″	12·6	89	26·1	62·8	558	21·4	5136
	8·6	90	26·6	47·4	421	15·8	3792
	5·1	91	23·4	30·3	269	11·5	2760
	2·2	$41\frac{1}{2}$	31·8	22·4	207	6·5	1560
Dallas. Stroke = 2′ 6″ Diam. = 3′	5·1	47	36·2	34·5	317	8·8	2112
	3·9	47	29·0	26·4	243	8·4	2016
	2·9	46	26·0	21·5	160	6·1	1464

The annexed table gives some examples of the results of such calculations in the case of two of the engines experimented on by the American Board of Naval Engineers, an analysis of which will be given in the next chapter. The

engine of the *Bache*, though compound, was operated as a simple engine in the particular experiments here considered: the cylinder was jacketed, but the jacket does not appear to have supplied much heat, as the liquefaction in it was small. The engine of the *Dallas* was simple, and the cylinder was not jacketed.

These results are probably too large, and perhaps considerably too large, for the reasons indicated above: yet I believe we may safely draw the following conclusions from them—conclusions which are fully confirmed by the other experiments made at the same time, as well as by those recently made in France by M. Hirn and others, and by numerous other previous experiments, of which I may especially mention those made on locomotives by Mr. D. K. Clark, which will be again referred to presently:—

(1) The initial condensation, and the heat abstracted per square foot of admission surface, increase with the ratio of expansion.

(2) The initial condensation at high rates of expansion may exceed 50 per cent.

(3) The heat abstracted per square foot of admission surface is enormously great when compared with the time occupied by admission. The table shows the rate per 1′ on the supposition that the period of admission was one-fourth of a second: the actual period varied, but was nearly always considerably less.

Much difficulty has been felt in admitting that any considerable fraction of the steam can be condensed in the very short time in which the contact lasts, and, no doubt, the ordinary processes of radiation and convection are utterly inadequate for the purpose, with differences of temperature such as occur in steam cylinders, even if acco[illegible] taken of the great radiating power of aqueous vapou[illegible]red with the permanent gases. Yet all experien[illegible] the rate of condensation of pure [illegible] practi[illegible]

only by the rate at which the heat given out can be absorbed at the other side of the tube, plate, or other surface on which the condensation takes place: * and when the process of condensation is examined, it appears that radiation and convection cannot much influence the result. For the instant the steam comes in contact with a cold surface, a film of water is deposited, which cannot have a temperature materially different from that of the steam itself, and which would intercept the radiation from the central mass of steam, even if the surface could be imagined to have a much lower temperature. It is more probable, however, that the surface rises at once nearly to the temperature of the steam, and that the rate of condensation depends mainly on the rate at which heat can be conducted from the surface to the interior of the body in contact. The principal subject of inquiry is, therefore, as to the rate of conduction of heat through metallic bodies. I first, however, remark that, large as are the quantities of heat with which we have to do compared with the time of transmission, yet that time is so short that the mean temperature of the cylinder cannot vary much. A square foot of iron 1 inch thick weighs about 40 lbs. The specific heat of iron may be taken about ·12, and consequently the heat requisite to change the temperature of such a plate by 1° will be about 4·8 thermal units, and the change of temperature corresponding to the greatest condensation indicated in the table above will be about 4°·5, while in general the change will be considerably less.

Flow of Heat by Conduction.

107. When heat is transmitted from a medium A to a medium B of lower temperature through a metallic plate which separates them, the rate of transmission of heat depends, first, on the area of surface in contact, secondly, on the

* The experiments of Professor Osborne Reynolds show that the presence of air considerably checks condensation.

difference of temperature. Let us suppose the temperatures to be T_A T_B, then all the heat passing from it to B must pass, first, *into* the plate, second, *through* the plate, and third, *out* of the plate: and each of these three passages requires a certain difference of temperature. Thus the temperature (t_A) of the outer surface of the plate is less than T_A, and that of the inner surface (t_B) is less than t_A, but greater than T_B. For our purposes it is only necessary to consider the transmission of heat *through* the plate, which takes place by conduction—for which purpose we suppose the temperatures t_A t_B to be known.

The steady flow of heat by conduction is governed by very simple laws: let F be the flow per square foot per 1′, y the thickness, and $t_A - t_B$ the difference of temperature; then

$$F = f \cdot \frac{t_A - t_B}{y},$$

where f is a coefficient of conductivity which is known roughly for iron by Forbes' experiments. Forbes found that a difference of temperature of 1° would cause the transmission per 1′ through a surface of 1 foot square and 1 foot thick of sufficient heat to raise about ·01 cubic foot of water through 1°. Hence if y be reckoned in inches,

$$f = 7{\cdot}5.$$

Forbes' results show considerable differences in different bars experimented on, owing perhaps to differences in the quality of the iron, and also show that the conductivity of iron diminishes as the temperature rises. Also the rate of transmission through a curved surface is not exactly the same as through a plane surface. I believe, however, that the formula

$$F = 7{\cdot}5 \cdot \frac{t_A - t_B}{y} \text{ (thermal units per 1′)}$$

gives approximately th[illegible] of transmission, so far as du[e] to conduction alone. [illegible] experiments give a much

smaller result, but it is not certain that the thermal resistances at entrance and exit were completely eliminated by the methods he adopted.

In any case, the results given by the formula must be considered *maximum* values, and it follows at once that, in no practical case, could even 2000 thermal units per 1′ be transmitted, much less 5000, as in one of the examples given above, if the circumstances were such as the formula supposes. In fact, however, the flow of heat is by no means steady, but the temperature t_A of the surface of the cylinder, in contact with the steam, undergoes great and very rapid changes, the effect of which is greatly to augment the rate of conduction.

FIG. 26.

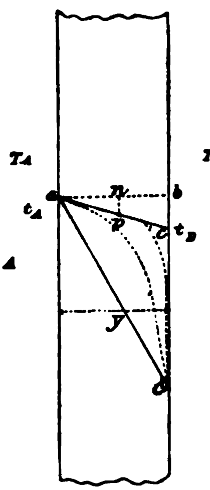


Fig. 26 shows a section of a plate through which heat is flowing: a is a point on the hotter surface, b a point on the cooler surface, while P lies within the plate, and, for convenience, is taken on the line $a\,b$. When the temperatures t_A t_B are fixed, the difference of temperature at P is shown by the formula to be simply proportional to the distance from the surface, a fact represented graphically by the straight line a P b, the ordinate P n of which, at any point P, shows the difference of temperature between P and a. The flow of heat is then the same, at every point of the plate, at all times, and hence is said to be steady: the tangent of the angle $c\,a\,b$, that is to say $\frac{\mathrm{P}\,n}{a\,n}$, measures what we may conveniently call the "thermal gradient" in degrees to the inch.

Imagine, however, the temperature t_B to be suddenly lowered, as indicated by the thermal gradient $a\,c^1$, $c\,c^1$ being

the sudden fall: then, if sufficient time elapse, no doubt the flow becomes sensibly steady as before, only at a greater rate. But, before the flow becomes steady, the plate must cool down through half the sudden fall in question, and consequently, at first, more heat must flow into B than flows out of A, while the temperature of each point of the plate gradually falls: that is to say, at each instant, and at each point of the plate, the temperature varies. We may express this by drawing a temperature curve for each instant considered: curves, which at first will nearly coincide with $a\,b$ except in the immediate neighbourhood of B, but which will gradually change their form till at last they sensibly coincide with $a\,c^1$. The figure shows two of these curves, the first immediately after the change, and the second a little later.

The slope of each curve increases on passing from A to B, and, at each point, graphically represents the thermal gradient; whence it is obvious that the rate of conduction into B may be ten or twenty times greater, in the first instant of the change, than the rate of steady flow corresponding to the difference of temperature and total thickness. Thus, when steam comes in contact with a cold surface, the rate of condensation is, in the first instance, excessive, and afterwards rapidly diminishes: for example, let the difference of temperature between the steam and a thick plate with which it comes in contact be 100°, then the surface next the steam instantly rises nearly to the temperature of the steam, and the heat given out by the condensing steam is transmitted into the interior of the plate at a rate which, near the surface, is very great, but which rapidly diminishes as the interior of the plate is penetrated, and as time elapses. If, then, the time of contact be very short the condensation will be the same as if the plate had a certain very small thickness, and rose up to the temperature of the steam.

There is, therefore, reason to believe th

steam is admitted to a cylinder, the condensation is the same as if the cylinder were very thin and rose instantly to the temperature of the steam; and it is hence easily seen that a great amount of initial condensation is perfectly possible. For example, if the cylinder be imagined $\frac{1}{100}$th of an inch thick, and to rise through 200° from the temperature of the exhaust to the temperature of admission, the heat absorbed would be nearly 10 thermal units per square foot of admission surface, or from the table above, from 200 to 300 thermal units per lb. of steam.

An important conclusion, however, follows from the consideration of the shortness of the period of admission, and from the fact that the temperature of the surface is approximately the same as the temperature of the steam, and that is, that the condensing action of the sides must be mainly *superficial;* the central mass of steam, not in immediate contact with the sides, remaining practically unaffected: a conclusion which is probably correct even in the case when the steam is superheated.

108. Although the foregoing explanation of the condensing action of the sides is, in many cases, sufficient, and probably complete, yet another cause, by means of which that action may be, and no doubt constantly is, indefinitely augmented, must not be overlooked, and that is the effect of water remaining in the cylinder after exhaust.

It is not necessary to imagine a quantity of water collected at one point; a most powerful influence will be exercised by a film of water spread over the whole internal surface. Such a film, only $\frac{1}{100}$th of an inch thick, covering 20 square feet, would weigh 1 lb., and hence, by the table given above, would be at least equal in weight to the steam enclosed in the cylinder; at admission it would rise from the temperature of the exhaust to the temperature of the steam admitted, [illegible] on account of the great specific heat of water, would be [illegible] influential as an iron cylinder of the same thickness.

It has been shown already, in Chapter VIII., that water remaining after exhaust has the same effect as a metallic plate which follows the temperature of the steam, and hence may conveniently be regarded as an integral part of such a plate; hence there is no occasion for the present to distinguish between the action of the metallic cylinder walls themselves, as explained above, and the action of the water here considered. When the initial condensation is 40 or 50 per cent. or upwards, it is almost certain that an accumulation of water must be the principal cause.

Re-Evaporation.

109. The heat abstracted from the steam by the cylinder during admission must all be given out again during expansion and exhaust, together with the difference (if any) between the heat communicated by the steam jacket and the heat lost by radiation. At least this must be the case when a permanent regime has been attained, though of course, for a limited period, the cylinder may be imagined to have a rising or falling mean temperature.

If then the condensing action of the sides is great, so also must be the power of giving out heat. Now all that we know of the processes of radiation and convection leads us to believe them to be quite inadequate to the communication of any such amount of heat as is necessary for the purpose, and if the surface of the cylinder was dry, it would be impossible to explain how so large an amount of heat was abstracted from the cylinder as is represented by the action in question. In fact, however, the surface of the cylinder is covered with a film of water deposited there during admission: this film falls in temperature with the expanding steam, and, as it does so, receives heat from the cylinder and evaporates. No doubt, in ordinary circumstances, where a mass of water is in contact with a hot

surface, evaporation is a comparatively slow and difficult process, but this proceeds from the difficulty of obtaining a sufficiently rapid convection through the mass of water. In the present case we have a thin film of water spread over the surface, and it may well be supposed that a very small difference of temperature is sufficient to enable the film to abstract heat from the surface; so that the surface closely follows the temperature of the steam. The flow of heat, by conduction from the interior to the surface of the mass of metal, is augmented in the manner already explained, and here also it appears that the action of the sides is equivalent to that of a thin cylinder which follows the temperature of the steam.

Water remaining after exhaust virtually augments the weight of such a cylinder as just explained in the case of the initial condensation, and probably follows the temperature of the steam much more closely than the surface of the cylinder itself.

And, as before, the true conclusion to be drawn from the fact that the action of the sides must be supposed very energetic to have any material influence on the working of the engine, is not that the influence in question is really unimportant, but that the *action of the sides is mainly superficial*, the central mass of steam remaining practically unaffected. Hence, when steam expands in a cylinder, the central mass of steam, whether wet or dry, or whether or no the cylinder be jacketed, expands adiabatically and partially liquefies, just as if the cylinder were non-conducting; the liquefied steam, being suspended throughout the mass, is carried out into the condenser during exhaust without abstracting any considerable amount of heat. But the film of water, deposited during admission, evaporates, partly during expansion and partly during exhaust, and absorbs nearly the whole amount of heat given out by the cylinder.

From this view of the action of the sides it follows that,

when a permanent regime has been reached, not only is there an equality between the heat abstracted by the cylinder and the heat given out by the cylinder, but also between the condensation during admission and the superficial evaporation during expansion and exhaust; and matters so arrange themselves in the cylinder that these two equalities may coexist, a condition which governs the actual intensity of the action of the sides.

Action of a Simple Plate.

110. Each square foot of the cylinder surface must operate to a great extent independently of every other, for the amount of heat which can be communicated by lateral conduction from one part to another is evidently very limited: hence it is desirable to consider more closely the action of a thin plate of known weight, attached to the piston, the rest of the surface being for the time left out of consideration. The influence of such a plate on the expansion curve has already been thoroughly considered in Chapters VII. and VIII., and it has just been pointed out that the actual action of the sides probably resembles the action of a plate the temperature of which follows the temperature of the steam.

Let $m\,c$ be the heat supplied by such a plate as in Art. 67, and let it be placed in a cylinder which at each stroke discharges 1 lb. of steam. Let the initial temperature be t_1, and, at the instant when the steam is admitted to the cylinder, let the temperature of the plate be θ, then, clearly, the temperature of the plate at once rises to t_1, the requisite heat being obtained by condensation of part of the steam admitted, which forms a film on the plate of weight $1-x_1$ suppose. Then, assuming the boiler to supply dry steam,

$$L_1\,(1-x_1) = m\,c\,(t_1-\theta).$$

This film partially evaporates during expansion, as it receives heat from the plate, while the mass of steam, not in

immediate contact with the plate, expands adiabatically and partially liquefies, as if the plate were not there, the liquefied steam being distributed as suspended moisture through the whole mass (Art. 109). Hence if y be the fraction of the film evaporated during expansion,

$$y = \frac{\left\{q + \frac{m\,c}{1 - x_1}\right\} \log_{\epsilon} \frac{T_1}{T_2}}{\frac{L_2}{T_2}} \quad \text{(Art. 76, Eq. II.)};$$

or by substitution for $1 - x_1$,

$$y = \frac{q + \frac{L_1}{t_1 - \theta}}{L_2} \,.\, T_2 \log_{\epsilon} \frac{T_1}{T_2},$$

where the capital letters, as usual, refer to absolute temperatures, and the suffix 2 to the end of the expansion.

The values of y obtained from this equation are of course greatest when θ is greatest, that is when $\theta = t_2$, and even then are always fractional, showing that the whole of the film, produced by initial condensation, will not be evaporated during expansion, but that a certain portion, $1 - y$, will remain, which can be calculated from the foregoing formula.

When the exhaust opens, the portion in question wholly or partially evaporates, and the temperature of the plate falls until heat enough has been supplied to produce that evaporation. It will fall no further, because, when dry,—as just explained,—it communicates no sensible amount of heat to the steam. If a permanent regime has been attained, the fall of temperature must be exactly $t_2 - \theta$, the plate returning to its original temperature, and the whole process will then be repeated indefinitely. When this is the case the exhaust waste will be $m\,c\,(t_2 - \theta)$.

Now, if the fraction $1 - y$ be evaporated by a fall of temperature $t_2 - \theta$, we must have, since the absolute weight of that fraction will be $(1 - x_1)\,(1 - y)$, and since the evapo-

ration takes place under the approximately constant pressure corresponding to t_0,

$$mc\,(t_2 - \theta) = (1 - x_1)\,(1 - y)\,(L_0 - t_2 + t_0),$$

where the suffix 0 refers to the temperature of the exhaust: hence

$$1 - y = \frac{mc}{1 - x_1} \cdot \frac{t_2 - \theta}{L_0 - t_2 + t_0} = \frac{t_2 - \theta}{t_1 - \theta} \cdot \frac{L_1}{L_0 - t_2 + t_0};$$

and it is obvious that, by comparing this result with the value of y previously obtained, an equation will be found from which θ may be determined. That value will always be less than t_2, and should it be greater than t_0, represents the actual temperature of the plate during exhaust, which will be lower and lower the greater the expansion. But, if the expansion proceed far enough, θ will be first equal and then less than t_0; in the first case, the lowest temperature of the plate will be exactly that of the exhaust, and in the second, as the plate cannot sink lower, the conclusion is that the initial condensation is always greater than the re-evaporation during expansion and exhaust. Water then accumulates in the cylinder, forming a permanent film on the plate, and every successive stroke in this way virtually increases the weight of the plate, and the process of accumulation goes on at a constantly increasing rate.

As a numerical example, suppose that the initial pressure in the cylinder is 90 lbs. on the square inch, and the back pressure $3\frac{3}{4}$ lbs. per square inch, corresponding to an initial temperature (t_1) of 320°, and an exhaust temperature (t_0) of 150°: then, so long as the terminal pressure is not less than about 20 lbs. per square inch, the lowest temperature (θ) of the plate will be above the exhaust temperature, and the effect of the plate will be to produce an initial condensation and an exhaust waste which gradually increase as the expansion increases. Any water which may happen to be in the

cylinder at first, will gradually evaporate till the permanent regime has been reached, and the maximum exhaust waste will be 78·6 thermal units per lb. of steam. But, if it be attempted to carry expansion beyond the critical pressure of 20 lbs. per square inch, accumulation of water will begin, and the action of the plate will be indefinitely increased.

111. By imagining the plate sufficiently thin, it may always be made to follow the temperature of the steam as closely as we please, and therefore, theoretically, the speed of piston will have no influence on the action of the plate, so long as the expansion does not extend below the critical pressure. But, if the plate have a sensible thickness, it will not have time to follow the temperature of the steam, and an increase of piston speed will be followed by a diminution in the action. And, if the critical pressure be overpassed, the accumulation of water and consequent augmentation of the action of the sides will not be indefinite, but will be limited by the time necessary for initial condensation, so that at slow speeds the accumulation of water will be much greater than at high speeds.

112. If heat be supplied to the plate from without by conduction its action will be diminished, because the initial condensation will be diminished and the re-evaporation increased by amounts exactly corresponding to the heat supplied during the periods of condensation and evaporation.

To estimate this effect quantitatively, it would be necessary to know the quantity of heat supplied during the three periods of admission, expansion, and exhaust; and, in the period of expansion, the law of supply must be known as well as the total amount. It would perhaps be possible to make a supposition nearly representing the actual facts in a given instance. I shall not, however, enter on this question here; but confine myself to remarking that quantities of heat supplied in this way produce effects out of all proportion to their magnitudes, in raising the lowest temperature of the

plate and lowering the critical pressure at which accumulation of water commences. The balance between condensation and re-evaporation is a very delicate one, and a small disturbing cause produces a great change in the effect produced. It is important to observe that this will be especially the case when the terminal pressure approaches the critical value.

Conversely, if heat be withdrawn from the plate by conduction to external bodies, the lowest temperature of the plate will be lessened and the critical pressure increased.

113. In a non-condensing engine the action of the plate is less than in a condensing engine, because the exhaust temperature is over 212° and the range of temperature of the plate is greatly diminished. The critical pressure will of course be raised, and if expansion be carried beyond that pressure, water will accumulate to an extent only limited by the piston speed: but if expansion be not carried too far, the exhaust waste and other prejudicial effects of the action of the plate will be considerably diminished. Numerical results can be obtained by use of the preceding formula with a suitable value of t_0.

114. If, as is generally the case in practice, the steam be compressed in the clearance space, by closing the exhaust before the end of the stroke, the action of the plate will be diminished. For, when the steam is compressed, its temperature will rise, and the temperature of the plate will follow it—the heat, needful for the purpose, being absorbed from the compressed steam—and thus when fresh steam is admitted from the boiler, the initial condensation is not so great as when there is no compression. It is true that some of the compressed steam will frequently be liquefied in heating the plate, and that the whole mass of steam and water in the clearance will have its temperature raised at the e
of the heat of the boiler steam: but, nevertheless
result is, probably always, to diminish the initi
tion, and so to diminish the action of th

pression, therefore, is advantageous on this account as well as for the reasons given in the last chapter.

115. If the expansion curve be known, and the state of the steam at any point of the stroke, the weight of the plate can be determined. For, from the initial and terminal pressures, we can find by Table I. the corresponding temperatures: then, applying the fundamental formula,

$$\frac{L_2 x_2}{T_2} = \frac{L_1 x_1}{T_1} + (q + mc) \,.\, \log._\epsilon \frac{T_1}{T_2},$$

we find

$$q + mc = \frac{\dfrac{L_2 x_2}{T_2} - \dfrac{L_1 x_1}{T_1}}{\log._\epsilon \dfrac{T_1}{T_2}}.$$

This problem can also be solved graphically by a proper modification of the construction given in Art. 64.

The value of $m\,c$, thus obtained, includes any water remaining after exhaust; because the value of x_2 is determined in such a way as not to include any of this water, but only the water discharged with the exhaust steam at each stroke, either as suspended moisture or by re-evaporation during exhaust. See Chapter XI.

General Conclusions respecting the Action of the Sides. Jacketing. Superheating.

116. The action of each separate portion of the cylinder surface is probably similar to that of a simple plate, but, in considering the total action, we must remember that different parts of the surface go through different changes of temperature, and must have different mean temperatures. As the piston advances fresh surface is continually uncovered, and initial condensation therefore continues to some extent through the whole expansion. In the early part of the expansion condensation will predominate, in the middle and end re-evaporation. The weight of the equivalent plate

for each portion of the surface will depend partly on the area of exposed surface per lb. of steam, and partly on the depth to which the changes of temperature extend: and therefore the intensity of the action will depend partly on the pressure, partly on the range of temperature, and partly on the speed of piston. On the whole, we may draw the following general conclusions as probably representing the action of the sides approximately :—

(1) Some initial condensation will take place at all ratios of expansion.

(2) At low ratios of expansion the temperature of the internal surface of the cylinder during exhaust is greater than the exhaust temperature, and the range of temperature is less the less the expansion.

(3) At a certain limiting ratio of expansion the temperature of the internal surface during exhaust sinks to that of the exhaust.

(4) In (2) and (3) an exact balance exists between evaporation and condensation, and the initial condensation increases with the expansion only because the surface and the range of temperature are greater. Any water originally remaining in the cylinder after exhaust gradually evaporates till a permanent regime is reached.

(5) Beyond the ratio of expansion mentioned in (3), condensation predominates over evaporation, and water accumulates in the cylinder to an amount depending on the speed of piston. The action of the sides greatly increases from this cause.

(6) The action of the sides is much less in high-pressure cylinders than in low-pressure cylinders, because bo
exposed surface and the range of temperature are
if the ratio of expansion be too great, accumula
may cause very great waste, even with
temperature.

(7) Quantities of heat—small in

total amounts represented by the initial condensation and subsequent re-evaporation—added, by means of a steam jacket, or by the use of superheated steam, will diminish materially the action of the sides. Conversely, radiation from unclothed cylinders will greatly augment that action.

(8) Compression diminishes the action of the sides.

117. These conclusions, though following naturally from the close analogy between the actual cylinder sides and a simple plate, are, if I mistake not, fully confirmed by actual experience, and, indeed, most of them have been suggested by known facts respecting the working of steam engines in practice.

The great influence of initial condensation and subsequent re-evaporation in certain cases was pointed out by Mr. D. K. Clark, so long ago as 1855, in his well-known work on Railway Machinery. Mr. Clark's experiments were made on locomotives, and showed that the prejudicial action of the sides might be avoided in non-condensing engines by properly protecting and heating the cylinders, while, on the contrary, if the cylinders were exposed and the piston speed low the action might be greatly augmented.

In 1860 some experiments were made by Mr. Isherwood on one of the engines of the U.S. steamer *Michigan*, which show a great amount of liquefaction increasing at low speeds and high expansions. These experiments were discussed by the late Professor Rankine in a paper read before the Institution of Engineers for Scotland, and published in their Transactions for 1861–2. In this paper—though his language in this and subsequent papers is often inconsistent with this view—Rankine states distinctly that the temperature of parts of the cylinder rises and falls with the steam, while the great mass of it remains at a nearly uniform mean temperature, a suggestion already made, some time previously, by Mr. Isherwood: and he shows how water accumulates in the cylinder and aggravates the action of the

sides themselves. Further, Rankine explains that the compound engine has an advantage, in splitting up the whole range of temperature, and so diminishing the action of the sides, and points out that the intensity of the action is mainly governed by the necessity of an exact balance between condensation and evaporation.

On the other hand, Rankine does not appear to have ever distinguished between the liquefaction consequent on expansion and the liquefaction during admission; and hence he regarded it as both possible and desirable to prevent liquefaction altogether by the use of a steam jacket. According to the view just explained, a great difference exists between the two; the steam liquefied during expansion is distributed throughout the whole mass of steam, and rushes into the condenser without abstracting from the cylinder any sensible amount of heat; while the steam liquefied during admission is deposited as a film on the cylinder walls, and to it alone Rankine's reasoning respecting the necessary balance between condensation and re-evaporation is conceived to apply. Moreover, experience shows that, in non-condensing engines and in the high-pressure cylinders of compound engines, the prejudicial action of the sides may be, to a great extent, avoided by the use of a steam jacket and by limiting the ratio of expansion; and hence Rankine seems to have concluded that the action of the sides was important only in cases where proper precautions had not been taken, and that consequently it might be excluded from consideration in a theory of the steam engine.

Continental writers, generally, until quite recently, have attached very little importance to the action of the sides. Zeuner, especially, has frequently expressed an opinion that the action of the sides has been overrated: in a paper*

* "Ueber die Wirkung des Drosselus und den Einfluss des schädlichen Raumes auf die bei Dampf-maschinen verbranchte Dampfmenge," 'Civil Ingénieur,' vol. xxi. p. 1.

published only two years ago (1875), he states his beli that a more powerful cause must be found for some of th effects observed, and finds it in certain consequences of wire drawing described at the close of the last chapter.

M. Hirn was among the first to point out the genera nature of the action of the sides, but until recently he wa far from suspecting the magnitude of the effects produced.[*] Experiments, however, made by himself, and, under hi general direction, by Messrs. Hallauer and others, have con vinced him that errors of 30, 40, 50 per cent. and upward may be made by neglecting this action: in the third edition (1876) of his work on Thermodynamics he has entered at great length into the question, and strongly pressed the view that the action of the sides is mainly superficial. The effect of accumulated water in augmenting the action, and the necessity of a balance between condensation and evapora- tion, pointed out by Rankine, are however, if I mistake not, left out of account. These experiments were very carefully made, and checked in various ways, so as to leave no doubt that the effects observed are really in great measure due to the action of the sides, and cannot be attributed to the effects of wire-drawing pointed out by Zeuner.

In 1874–5 experiments were made on jacketed, non- jacketed, and compound engines, by Mr. Emery, representing a Board of American Naval Engineers: these experiments are discussed in the next chapter (Chapter XI.): their results show clearly the great influence of the sides, and especially that although in non-condensing engines the evils of lique- faction can be almost removed by jacketing, yet that in con- densing engines it is not so.

118. Experience shows that a steam jacket is advan- tageous, and the reason why it is so has been pointed out in the present chapter; but the amount of advantage to be thus gained will vary according to circumstances. In many

[*] Société Industrielle de Mulhouse, 'Bulletin Spécial,' 1876, p. 187.

cases it may be that the advantage is small. For example, in the high-pressure cylinder of a compound engine, if the terminal pressure be high as compared with the back pressure, so as to be decidedly above the critical value, the range of temperature will be small and no considerable liquefaction will probably occur even though there be no jacket. And if the terminal pressure approach the back pressure so that the tendency to liquefaction is strong, the jacket may probably be unable materially to influence that tendency on account of the high mean temperature in a high-pressure cylinder, which prevents much heat being drawn from the jacket. In these two cases, then, it may be expected that little advantage will be gained by the use of a steam jacket: but if on the other hand a terminal pressure of intermediate magnitude at or a little below the critical value be imagined, the amount of heat necessary to prevent the accumulation of water will be very small, and a jacket may be expected to prove of material advantage.

Again, if the circumstances be such that water tends to accumulate at one place in the cylinder instead of being spread over the whole internal surface, a jacket will be of little advantage, for its influence extends only to superficial evaporation and condensation. Great caution therefore is necessary in drawing conclusions from any special set of experiments on the influence of jacketing.

119. When the amount of priming water is small and uniformly distributed throughout the whole mass of steam, its influence will not be important on the working of the engine; the exhaust steam will be wetter than it otherwise would be, but it will rush out into the condenser without abstracting much additional heat from the cylinder. The case is, however, very different if the priming water comes ·r at intervals in considerable amounts; the cylinder will ·ntain mu[illegible] water remaining after exhaust, he [illegible] 'enished before it has time to

evaporate, even supposing that the circumstances are such as to favour evaporation; the exhaust waste will in consequence be greatly increased. This effect of priming water has been pointed out by M. Illeck in a criticism of Hirn's views, published in the 'Civil Ingénieur,' vol. xxii. p. 371.

120. The action of the sides is diminished not only by jacketing, but also by the use of superheated steam. When superheated steam enters a cold cylinder, condensation immediately takes place on the surface, but to a less extent than if the steam were saturated: hence there is less water to re-evaporate, and the heat drawn from the cylinder is diminished. In short, the evaporation has an advantage over the condensation. The experiments of M. Hirn show that the exhaust waste may be greatly diminished in this way, as will be seen in the next chapter. It has been suggested by Mr. Dixwell, in a paper read before the Society of Arts at Boston in 1875, that the amount of superheating should be varied according to the cut off.

121. The influence of the speed of piston on the action of the sides cannot at present be estimated even approximately. The speed of the piston affects the depth to which the changes of temperature penetrate the mass of the cylinder, and besides, the rate of condensation and evaporation. As regards the depth, no doubt that must be diminished by increasing the speed, and, if this were the only circumstance to be considered, it would be possible to estimate the influence of speed with tolerable certainty. But as regards the rate of condensation and evaporation, it must be remembered that the effect of speed will mainly depend upon whether the condensation or the evaporation is *most* influenced. It is therefore not inconceivable that under certain circumstances an increase of speed may be accompanied by an increase instead of a diminution in the action of the sides. When the ratio of expansion is so large that water tends to accumulate in the cylinder, it is highly probable that the speed of the

piston affects the condensation much more than the evaporation; and generally we may expect that the nature of the influence will depend very much on the ratio of expansion.

Experience throws little light on the subject. In some instances a marked diminution of liquefaction has accompanied an increase of speed, but in others no important difference has been found when the speed was doubled.

The influence of the state of the piston and cylinder-cover surface is another point for investigation. If these surfaces could be made non-conducting, it can hardly be doubted that considerable advantage might, in some cases, result.

CHAPTER XI.

EXPERIMENTS ON STEAM ENGINES.

122. THE more important parts of the theory of the steam engine having been discussed in the preceding chapters I shall now go on to apply the theory in further detail to the working of steam engines in practice, making use of data furnished by some of the numerous experiments which have been made.

The data required for a complete comparison of theory and experiment are very elaborate and extensive, and are to be met with in very few cases, I might almost say in none excepting those made by M. Hirn, or under M. Hirn's direction, and described in papers read before the Société Industrielle de Mulhouse in various years, ending in 1876. Very instructive results may, however, be obtained from less extensive series of experimental data, and my principal object in the present chapter will be to point out how special data, furnished by experiments of various kinds, may be made use of to determine the amount of heat utilized and lost in different ways by steam engines in practice.

Experimental Determination of the Efficiency of an Engine. External Radiation. Amount of Priming Water.

123. Let the data be the indicated power, and *either* the amount of feed water used in a given time, *or* the condensation water, with its rise of temperature, *or*, preferably, both these last data combined.

The indicated power must be determined from a number

of diagrams, accurately taken every few minutes, for a considerable time; a single diagram is of little value where accuracy is desired. The feed water is to be determined by direct measurement during the same period, and care must be taken that the water level in the boiler is precisely the same at the beginning and end of the experiment; the loss of water, frequently occasioned by imperceptible leakage, is so far as possible to be estimated and allowed for; and the boiler pressure, height of barometer, and temperature of the feed, are to be frequently noted, so that mean values can be obtained. The case where the condensation water is the datum will be mentioned presently.

From these data it is possible to find approximately the efficiency of the engine, both absolutely and relatively to a perfect engine working between the same limits of temperature. I should remark that in considering the efficiency of steam engines, the feed water ought, theoretically, to be considered as being at the temperature of the condenser in condensing engines, or at 212° in non-condensing engines, any loss or gain caused by the feed being actually at a different temperature being considered in estimating the efficiency of the boiler. The result will be exact, instead of approximate, if the amount of water carried over with the boiler steam be known, a matter to be considered presently.

As an example, I select an experiment made with the compound engine of the U.S. steamer *Rush*, particulars of which are given in Mr. Emery's Report. (See Art. 131.) The data are:

Water [redacted] hour	=	18·38 lbs.
Average [redacted] above atmosphere	=	69·06 lbs. per sq. in.
Average [redacted]	=	14·81 „ „
Average [redacted] feed	=	114°

to supply dry steam, then

$$= H_1 - h_0.$$

where the suffix 1 refers to the boiler temperature (315°) corresponding to the absolute boiler pressure of 83·87 lbs. per square inch. Referring to Tables I. and II.,

$$H_1 = 1178 : h_0 = 114 - 32 = 82;$$

$$\therefore \text{ Heat expended} = 1096 \text{ thermal units per pound of steam;}$$

$$\text{Ditto per I.H.P. per } 1' = \frac{1096 \times 18 \cdot 38}{60} = 336 \text{ (nearly).}$$

The absolute efficiency is now obtained by dividing 42·75, the thermal equivalent of 1 H.P., by 336, whence

$$\text{Absolute efficiency} = \cdot 127.$$

The efficiency of a perfect engine working between the same limits of temperature is $\frac{315 - 114}{776} = \cdot 259$, and hence

$$\text{Relative efficiency} = \cdot 49.$$

These results, which are much superior to the results of any other experiment made with the engines referred to, were obtained with both cylinders jacketed, and the moderate total expansion of 6·2.

Secondly. If the boiler be supposed to supply moist steam, the total heat of evaporation must be estimated with regard to such moisture as in Article 6.

Hence it appears that to obtain exact results when the feed water is the datum, the amount of priming water in the boiler steam must be known. This is an observation which it is far from easy to carry out accurately; any experiment professing to determine it being virtually an experiment on the total heat of evaporation of water, and requiring, for accuracy, the elaborate precautions described by Regnault in his memoirs on the subject. A rough determination, however, can be made without difficulty.

The only exact experiments hitherto made on the amount

of priming in an ordinary steam boiler are those of M. Hallauer, described in the 'Bulletin de la Société Industrielle de Mulhouse' for 1874. They show that the proportion is often very small, and nearly always less than 5 per cent., unless there be some special cause of priming, when, as is well known, it may reach any amount. It probably is different for each different case, and varies from time to time according to the circumstances of the evaporation.

124. Let us next suppose that, instead of the feed water being measured, the quantity of water discharged from the condenser per minute is noted, and the difference of temperature between the water entering and the water leaving. The observation has hitherto been made for injection condensers only, and in this case the water leaving the condenser consists partly of condensed steam, and partly of the water entering the condenser, which, while rising in temperature, absorbs the heat given out by the condensing steam.

Let W be the weight of water leaving the condenser in pounds per I.H.P. per minute, and W′ the weight of the condensed steam, while t_0 is the temperature on exit, and θ_0 on entrance, then

$$(W - W')(t_0 - \theta_0) = W'R,$$

where R is the heat given out by each pound of condensing steam.

But if Q be the heat expended reckoned from θ_0,

$$Q = U + R + t_0 - \theta_0 \qquad \text{(Art. 14)},$$

where U is the useful work done per pound of steam, so that

$$W'U = 42 \cdot 75.$$

Substituting these values,

$$W'Q = W(t_0 - \theta_0) + 42 \cdot 75.$$

Thus the heat discharged from the condenser in thermal units per I.H.P. per minute, when added to the constant

42·75, gives the heat expended *reckoned* from the temperat of the water *entering* the condenser.

This result is quite independent of the quality of t steam supplied by the boiler, but does not include any l by external radiation, and, on the other hand, the work do per pound of steam is not exactly the indicated work, b is a smaller quantity, because the friction of the pist generates heat, which forms part of that which appears the condenser.

For example, in an engine of 46·21 I.H.P., the quanti of water discharged from the condenser was observed to l 408·3 lbs. per minute. The water, on entrance, had a tem perature of 53°, and on exit of 89°·54, so that the rise temperature was 36°·54. Dividing 408·3 by 46·21, an multiplying by 36·54, we obtain 322·86 as the therm units per I.H.P. per minute, discharged from the condenser and adding 42·75 we obtain 365·6 as the expenditure heat when the feed water is drawn from the water *enterin* the condenser. When the feed water has a different tempe rature, the expenditure of heat cannot be calculated exactl without knowing the quantity of steam used. If that quan tity be known roughly, a correction may be found; thus, i the present case, let the feed be drawn from the wate leaving the condenser, and let the steam used be estimate as 20 lbs. per I.H.P. per hour, or one-third of a pound pe minute, then the correction is $\frac{36\cdot 54}{3}$ or 12·2 nearly, an the expenditure of heat will be 353·4 thermal units pe I.H.P. per minute. Theoretically, the weight of steam use might be found by measuring the water entering the con denser, a method which was actually adopted in the Frencl experiments (Art. 126); but this measurement is not prac tically easy to carry out, and any error will be multiplie many times, because the weight of condensation water is full twenty times the weight of the feed water.

The measurement of the heat discharged from the condenser is used as a practical method of testing the performance of steam engines by Messrs. B. Donkin and Co., and the test is, no doubt, a valuable one, being to a great extent independent of the performance of the boiler. For details of the mode of practically carrying out the method, I must refer to an article in 'Engineering' for February 5th, 1875.

125. The two observations of feed water and condensation water may be made at once, and in that case we have a means of verifying the first law of thermodynamics, by showing experimentally that the heat appearing in the condenser is actually less than the heat supplied in the boiler; and at the same time by use of the known value of the mechanical equivalent of heat we can obtain some idea of the possible magnitude of the external radiation and of the amount of priming water in the steam supplied by the boiler.

As an example, I select an experiment made by Messrs. B. Donkin and Co. on a 60-H.P. compound engine at the Hele Works, described in 'Engineering' for November 3rd, 1871. For full details, I must refer to the article cited, and I only quote the data necessary for my purpose, as follows:

Boiler pressure (absolute)	=	67·7
Water evaporated per I.H.P. per 1′ ..	=	$\frac{20 \cdot 55}{60} = \cdot 3425$
Water discharged from condenser ..	=	606·5 lbs. per 1′
Rise of temperature in condenser ..	=	31·66
Indicated horse-power	=	56·88
Heat discharged from condenser ..	=	337·6 {thermal units per I.H.P. per 1′

Reckoning the temperature of the feed as 51°·66 being the mean initial temperature of the injection water, for reasons explained in the last article, we find for the total heat of evaporation 1153 thermal units, and hence

Heat expended per I.H.P. per 1′ = 394·9 thermal units.

But on the second mode of reckoning,

$$\text{Heat expended} = 337 \cdot 6 + 42 \cdot 75 = 380 \cdot 35,$$

a result which shows that 14·55 units of heat per I.H.P. minute, together with the heat generated by piston frict were wasted in radiation, inclusive, probably, of the l discharged in the water from the steam jacket. When water is not returned into the condenser, and so recko as part of the discharged water, the difference of temperat between it and the water entering the condenser has to allowed for. This difference in the present case appear have been about 122°, and the quantity discharged was 102 l per hour, or ·0267 per I.H.P. per minute: thus, 122 × ·02 or 3·25 of the above difference must be subtracted, leavi 12·2 units per I.H.P. per minute, which is the differe between the external radiation and the heat supplied friction of pistons and valves. If we estimate the friction one-twentieth of the engine power, it would amount to 2 thermal units per I.H.P. per minute, leaving 14·3 units radiation, or about one-twenty-seventh of the whole h supplied. It is to be observed, however, that this is maximum result proceeding from the supposition that t steam supplied by the boiler was perfectly dry. It would reduced one-half by supposing that steam to contain 2 p cent. priming water. The experiment here made lasted t hours, and careful observations were frequently made throug out that time, so as to obtain average results.

In France many such experiments have been made M. Hirn, with quite analogous results, and it may therefore considered as experimentally demonstrated that the amou of heat appearing in the condenser of an engine is really l than that supplied in the boiler, and if Joule's value of t mechanical equivalent of heat be assumed, we may furth conclude that, when an engine is thoroughly clothed, the l by external radiation is not important, and that the stea

from an ordinary boiler need not contain any important amount of priming water, conclusions which are confirmed by direct experiments on the radiation from surfaces, and on the amount of water contained in boiler steam.

The expenditure of heat reckoned from the temperature of the condenser in the above example is given by

$$\begin{aligned}\text{Heat expended} &= 395 - \cdot 3425 \times 31\cdot 66 \\ &= 384 \text{ thermal units per I.H.P. per } 1';\end{aligned}$$

and therefore

$$\text{Absolute efficiency} = \frac{42\cdot 75}{384} = \cdot 111.$$

Taking the temperature of the condenser as 84°,

$$\text{Efficiency of a perfect engine} = \frac{300 - 84}{761} = \cdot 284;$$

and therefore

$$\text{Relative efficiency} = \cdot 391.$$

This example, compared with the case of the *Rush* in the last article, shows that, although the system of estimating the efficiency of an engine by comparison with a perfect engine is useful when the limits of temperature are the same, yet that it may be practically misleading when this is not the case: the low temperature in an injection condenser, as compared with the surface condenser of the *Rush*, being practically useless, though in a theoretically perfect engine it must be taken into account. Perhaps the adoption of a fictitious temperature of the condenser might meet the difficulty. The minimum back pressure in practice may be about 2 lbs. per square inch, and the temperature corresponding to this, namely 125°, might conveniently be adopted as the inferior temperature in condensing engines, as 212° is in non-condensing engines.

126. Experience appears to show that 18 lbs. of steam per I.H.P. per hour is about the minimum consumption of steam commonly reached in actual engines, and the preceding calculations indicate that, in condensing engines, this is equiva-

lent to saying that about 13 per cent. of the whole h expended may be turned into mechanical work, or ab 50 per cent. of the heat which would be turned into work a theoretically perfect engine working between the sa limits of temperature. The average result, even in econo cally working engines, is much less. Of course losses c nected with the boiler are not included in this statement.

Let us now attempt to study experimentally the vari ways in which heat is wasted in actual engines, leaving of account what may be called the necessary waste, that is say, the heat rejected in every engine, however perfect, account of the narrow limits of temperature within which is restricted to work.

Experimental Determination of the Exhaust Waste.

127. It has already been explained in Chapter III. th when steam is formed in a particular way, a certain defini amount of heat is required to form it, depending on t amount of external work done by it during formation: a hence that whence steam is found to be in a particular st at the end of the stroke in an engine of known power, it possible to find the total amount of heat which has be supplied to it in the boiler and during the passage from t boiler to the end of the stroke. This heat, which we call the total heat of formation, is given by the formula (Art. 2

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) V_2 ,$$

or by the graphical method of art, and the result of the c culation is not in any way hypothetical, provided the d are correctly given.

Now the heat (Q^1) supplied in the boiler, supposing in t first instance that the steam supplied is dry, is known wh the circumstances of the evaporation are known, and eviden it follows that $Q^1 - Q$ must be subtracted or $Q - Q^1$ added

the steam passes from the boiler to the end of the stroke. This difference is due partly to external radiation, but chiefly to waste of heat by re-evaporation during exhaust, as has already been repeatedly explained, and hence $Q^1 - Q$ was called the exhaust waste: * its effects have been considered in hypothetical cases in Chapter III. I propose in the present section to show how the necessary data may be ascertained by experiment and to give some results of the calculation.

The first step is to find the weight of steam discharged from the cylinder per stroke, and this is done by measuring the feed water, as in the preceding section, and also the water discharged from the steam jacket, if any, then by subtraction and division by the number of strokes observed in the given time, the result must be the required weight. These observations, if conducted with care, are not liable to any important error.

Next, let the volume at the terminal pressure of the cushion steam remaining in the cylinder after exhaust be found, as in Art. 84, Chapter IX., and let that volume be subtracted from the total volume of the cylinder, including clearance, then the result must be the volume occupied at release by the steam discharged from the cylinder per stroke, and division by the weight found as above gives the volume of 1 lb. of it. Now, if the terminal pressure be known it will be possible to find the corresponding volume of dry steam, and thus by division the proportion (x_2) of dry steam in the steam contained in the cylinder at the end of the stroke is found.

The determination of the volume of cushion steam cannot be made with accuracy, but the error thus occasioned is not of great importance, the principal source of error being in the determination of the terminal pressure. The want of an

* This quantity may also be described as the heat "not accounted for by the indicator;" but it is not the exact equivalent of the steam "unaccounted for by the indicator."

easy and accurate method of determining the terminal pressure is a most important obstacle to the progress of the theory of the steam engine; if an indicator be used it must be carefully tested before and after the experiment; every precaution must be taken to avoid oscillations as far as possible, and a mean value should be obtained from a large number of diagrams, rejecting those which show exceptional variation from the mean. It appears advisable to select a period from the whole duration of the experiment during which the diagrams vary little, showing an approach to uniformity in the conditions, but of course the feed water used during the period must be separately measured. At low terminal pressures an error of one-tenth of a lb. per square inch will cause a considerable error in determining x_2.

M. Hirn has in some cases replaced the indicator by a flexion dynamometer, formed by the beam itself of a beam engine, and states that he has obtained good results; such a method appears to be of limited application, for the inertia of the piston and other reciprocatory parts would be difficult to provide against, and would cause great errors except in slow-moving engines. For an account of M. Hirn's apparatus, see 'Pandynamomètres,' par G. A. Hirn (Gauthier-Villars, 1876). Possibly, better results might be obtained by means of a special instrument designed for indicating the pressure at or near the end of the stroke only. M. Hirn, however, states that he is satisfied that some indicator diagrams obtained by M. Leloutre are within ½ per cent. of the truth. Any determination of the terminal pressure must be less certain than that of the mean effective pressure, and allowance must be made for possible error.

When, as is generally the case, release occurs before the forward stroke is completed, an estimate is to be made of what the terminal pressure would have been had the release been postponed till the completion of the stroke.

I now proceed to give some examples, chiefly selected

from the American experiments, an account of which is given at the close of this chapter.

128. First I consider an experiment on the *Dexter*, a non-jacketed engine, the data of which are:

Boiler pressure (absolute)	= 81·92
Temperature of feed	= 114°
Terminal pressure	= 16·87
Cushion pressure	= 8·27
Diameter of cylinder	= 26 ins.
Stroke	= 36 „
Clearance	= ·0537
Total weight of feed water	= 178,867
Revolutions, using same	= 125,197

Here the weight of feed water per stroke = ·714 lbs., and this would be the weight of steam discharged at each stroke, if there was no loss of any kind: the loss in the case of the *Dexter* was accurately ascertained to be 4·96 per cent., but although the piston appears to have been perfectly tight, it does not follow that all this should be deducted; I shall, however, assume this, and hence the actual weight of steam per stroke is reduced to ·679 lbs.

From the data it follows that the volume of the cylinder, including clearance, was 11·652 cubic feet, of which the fraction

$$n = \frac{c}{1+c} \cdot \frac{P_e}{P_2} = \cdot 0249 \qquad \text{(Art. 85)}$$

was occupied by cushion steam, so that the volume of the ·679 lb. of working steam was 11·38 cubic feet nearly, and hence for the volume of 1 lb.

$$V_2 = 16\cdot 76 \text{ cubic feet.}$$

If now we seek the volume of dry steam corresponding to the terminal pressure of 16·78 lbs., we find from Table III. that the value is 23·25 cubic feet, whence, by division,

$$x_2 = \cdot 72 \text{ (nearly)},$$

a value which has been already used in Chapter III. in finding the total h[illegible]mation, neglecting clearance, by

the graphical method. I shall now employ the formu
to obtain the same result, that is to say,

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) V_2 \qquad \text{(Art. 26).}$$

The mean forward pressure P_m is given by the experimer
as 41·19 lbs. on the square inch, and this, referring as it doe
to the piston displacement (11·06), and not to the termin
volume of the working steam, must be diminished in th
proportion 11·06:11·38; thus the corrected value is

$$p_m = 40 \cdot 03.$$

The correction in question does not apply to P_2, an
therefore in thermal units

$$(P_m - P_2) V_2 = \frac{(40 \cdot 03 - 16 \cdot 87) \times 16 \cdot 76}{5 \cdot 36}$$
$$= 72 \cdot 4 \text{ thermal units.}$$

Now the temperature corresponding to the terminal pre
sure is found from Table I*a* to be 219° nearly, and th
corresponding value of L is 961: hence

$$h_2 - h_0 = 219 - 114 = 105 \text{ thermal units,}$$

and

$$x_2 L_2 = \cdot 72 \times 961 = 691 \cdot 9 \text{ thermal units;}$$

therefore

$$Q = 105 + 691 \cdot 9 + 72 \cdot 4$$
$$= 869 \cdot 3$$

is the total heat of formation.

But the heat expended in the boiler is the total heat c
evaporation of water from 114° at the boiler temperature c
313°, that is to say, 1088 thermal units;

$$\therefore \text{ Exhaust waste} = 218 \cdot 4 \text{ thermal units,}$$

or rather more than 20 per cent. This calculation does no
include the loss of water mentioned above, and signifies tha
the cylinder abstracts from each pound of the steam passing
through it 218·4 thermal units, which is afterwards give
out by external radiation and re-evaporation during exhaust

Terminal pressures near to the atmospheric pressure, as in the present instance, are perhaps difficult to measure with accuracy, but as a difference of half a pound does not make a difference of more than 20 thermal units, the calculation is probably very near the truth; supposing only that the boiler supplied dry steam.

The quality of the steam was not tested, but there is no reason to think that there was any important amount of priming water. To see the effect of priming water on the result, let us imagine the amount to be 5 per cent.; then the whole calculation of the total heat of formation is unaltered, but the total heat of evaporation becomes

$$\begin{aligned} Q^1 &= h_1 - h_0 + x_1 L_1 \\ &= 313 - 114 + \cdot 95 \times 893 \\ &= 199 + 848 = 1047 \text{ thermal units;} \end{aligned}$$

$$\therefore \text{ Exhaust waste} = 178,$$

or about 17 per cent.

The conclusion then seems inevitable that there must have been a very considerable amount of heat wasted in this way.

The cylinder in this instance was clothed, but not steam jacketed. Let us next consider a case where a steam jacket was used.

129. As a second example, I take the case of a trial made with the machinery of the U.S. steamer *Gallatin*, with steam jacket in use, the data of which are as follows:

Boiler pressure (absolute)	= 86·4
Temperature of feed	= 115°
Terminal pressure	= 12·14
Cushion pressure	= 9·93
Diameter of cylinder	= 34·1″
Stroke	= 30″
Clearance	= ·066
Total weight of feed water	= 8961
Revolutions, using same	= 6809
Water received from steam jacket, steam chest, &c.	= 640

Here the total weight of steam passing through tl cylinder is 8501 lbs., and the weight per stroke consequent ·624 lb.; the volume of the cylinder, without clearance, 15·86 cubic feet, and, including clearance, 16·91 cubic fe The fraction of this volume occupied by cushion steam was

$$n = \frac{c}{1+c} \cdot \frac{P_c}{P_2} = \cdot 0506,$$

and therefore the volume of the ·624 lb. of working stea was 16·05 cubic feet, so that dividing by ·624,

$$V_2 = 25 \cdot 72 \text{ cubic feet.}$$

From Table III. we find the volume of dry steam at th terminal pressure of 12·14 lbs. to be 31·45 cubic feet;

$$\therefore x_2 = \cdot 818.$$

The tabulated mean forward pressure is 31·684, whicl being diminished as before to reduce it to the volume of th steam, gives

$$p_m = 31 \cdot 3;$$

from which is obtained as before,

$$(P_m - P_2)\, V_2 = 92 \cdot 1 \text{ thermal units.}$$

But the temperature corresponding to 12·14 is 202° hence

$$h_2 - h_0 = 202 - 115 = 87; \qquad x_2 L_2 = 973 \times \cdot 818 = 796;$$

therefore

$$Q = 87 + 796 + 92 \cdot 1 = 975.$$

We must now estimate the heat supplied in the boiler and in doing so we must include the steam supplied to the jacket, that is to say, for each lb. of working steam we must reckon 1·054 lb. of water evaporated in the boiler. The total heat of evaporation, calculated as usual, is 1096;

$$\therefore Q' = 1 \cdot 054 \times 1096 = 1155,$$

and hence

$$\text{Exhaust waste} = 180 \text{ thermal units,}$$

being about $15\frac{1}{2}$ per cent. of the whole heat supplied. No allowance is made for loss of water of the kind mentioned in the previous case. The result shows the difference made by a steam jacket; for though the ratio of expansion was more than double that in the previous example, being more than 7 instead of $3\frac{1}{2}$, yet the exhaust waste is diminished from 20 to $15\frac{1}{2}$ per cent.

On the other hand, it is clear that the addition of a steam jacket, though it diminishes the evil effects of liquefaction in the cylinder, by no means removes them: the waste here shown being too great to be accounted for in any other way. In the *Gallatin* experiments the quality of the steam was tested by a method which, though not admitting of much accuracy, was sufficient to show that the amount of priming water was small.

A table of results will be given presently for various other cases.

130. If the quantity and rise of temperature of the condensation water be given, in addition to the other data, it will be possible to distinguish that part of the whole exhaust waste due to re-evaporation during exhaust. Thus in the *Hele* engine trial the total exhaust waste, calculated on the supposition that the boiler supplied dry steam, was probably about 14 per cent. (see page 303); and it was shown in Art. 123 that about 4 per cent. was due to radiation, and the effect of priming water: the remaining 10 per cent., then, was the loss by re-evaporation.

Analysis of the Total Heat of Formation.

131. In Chapter VIII. it was shown that the losses of heat, not included in the exhaust waste, were partly due to waste of the expansive energy of the steam after its formation and partly due to [illegible] application of heat by supplying it at a [illegible] than that of the boiler.

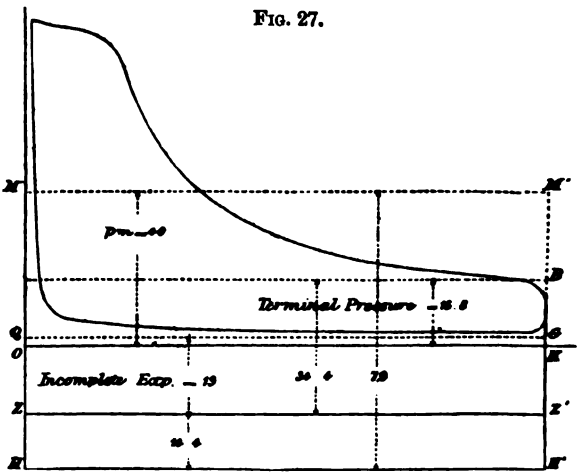
These losses were found to be of four kinds, and proper formulæ were given for determining the magnitude of each loss. The complication of these formulæ renders them inconvenient to apply, and I propose here to give a short roughly approximate method of analyzing the total heat of formation and showing how much of it is wasted from various causes.

Referring to page 66, in Chapter III., let us suppose that the heat pressure has been found from the formula

$$p_h = p_m + \left(k + \frac{k^1}{x_2}\right) \cdot p_2,$$

then the result expresses the total heat of formation, as a pressure on the piston, just as the useful work is represented by the mean effective pressure. Now let T_1 be the absolute temperature of the boiler, T_0 that of the condenser or the temperature corresponding to the atmospheric pressure if

FIG. 27.



there be no condenser: then multiplying p_h by the fraction $\frac{T_1 - T_0}{T_1}$, which is the efficiency of a perfect engine, a pressure is obtained which would have been the mean effective pressure on the piston had the engine been perfect.

In Fig. 27, M M is the line of mean forward pressure, reduced to the base O K, which includes the effective clearance (Art. 93): H H^1 is the heat-pressure line, reduced as just described, so that the area of the whole figure down to H H^1 now represents the work which might have been done had the heat been used in the best manner, while the diagram itself represents the work which actually is done: the difference then represents the total loss as compared with a perfect engine.

Now the loss by incomplete expansion is approximately (Art. 85),

$$U_2 = \frac{T_2 - T_0}{T_2} \cdot x_2 L_2 - (P_2 - P_0) V_2,$$

and the equivalent pressure is consequently

$$p = \frac{T_2 - T_0}{T_2} \cdot \frac{L_2}{144\, v_2} - p_2 + p_0,$$

whence

$$p + p_2 - p_0 = \frac{T_2 - T_0}{T_2} \cdot (\bar{p}_2 + p_2).$$

We have then a very simple rule for finding the loss by incomplete expansion approximately: find the value of $\bar{p}_2$ from Table V., and after adding p_2 multiply by the fraction $\dfrac{T_2 - T_0}{T_2}$: set the result downwards from the terminal pressure and draw Z Z^1, then if Q Q^1 be the back-pressure line, corresponding to the temperature of the condenser, the rectangle Q Z^1 is the loss required. Thus the diagram shows the total loss, and the losses by incomplete expansion and back pressure: whence it follows that the difference, namely, the rectangle Z H^1, must be the loss by the other causes not therein included, namely, by heating the feed water (Art. 83), by applying heat during expansion (Art. 84), by clearance and wire-drawing.

132. In the case of the *Dexter*, the first of the two

examples considered in the last section, it was shown Art. 28, page 66, that the value of $\left(k + \frac{k^1}{x_2}\right) . p_2$ was 239 ll per square inch nearly, and the mean forward pressure r duced for clearance was found to be (Art. 127) 40 lbs. p square inch; therefore

$$p_1 = 239 + 40 = 279.$$

Now the fraction $\frac{T_1 - T_0}{T_1}$ expressing the efficiency of a pe fect engine appears from the data given in the articles cite to be ·258, whence by multiplication is obtained the mea effective pressure in a *perfect* engine working between th same limits, that is to say, 72 lbs. per square inch. In Fig. 2 therefore, the line H H^1 lies 72 – 40, or 32 lbs. below the bas O K. Next, from Table V. the value of $\bar{p}_2$ is found to be 20 and consequently $\bar{p}_2 + p_2$ is 222 nearly; also

$$\frac{T_2 - T_0}{T_2} = \frac{219 - 114}{680} = \cdot 155.$$

Multiplying, then, 222 by ·155, we obtain 34·4, whic must be set downwards from B to find the line Z Z^1. The taking 1·4 as corresponding to 114°, the temperature of th condenser, the distance Q Z is 19 lbs. per square inch, whic represents the loss by incomplete expansion approximately the remainder, H Z, representing the other losses, bein 14·4 lbs. per square inch. About half of the "other losses is due to heating the feed water instead of raising its tempe rature by compression, and the other half, to imprope application of heat during expansion, to clearance, and wire drawing.

The mean back pressure from the diagram is 3·65 lb per square inch, of which, as stated just now, 1·4 lb. is tha corresponding to the temperature of the condenser, and con sequently the excess back pressure is 2·25 lbs. Hence th

whole available heat expended in forming the steam, not reckoning the exhaust waste calculated in the preceding section, is distributed thus:

Useful work done	$= \dfrac{40 - 3 \cdot 6}{72} =$	50·6
Excess back pressure	=	3·1
Incomplete expansion	=	26·5
Other losses	=	19·8
Total available heat	=	100·0

133. The results of the foregoing calculation are almost independent of the amount of water in the steam at the end of the stroke, and hence the process may be applied approximately even when the weight of feed water per stroke is unknown.

Graphical Representation of the Action of the Sides of the Cylinder.

134. As has already been repeatedly explained, the prejudicial action of the sides of the cylinder takes effect partly by re-evaporation during exhaust, and partly by re-evaporation during expansion, and, according to the method of analysis followed in Chapter VIII. and subsequently, this loss is divided between the exhaust waste of Articles 128, 129, and the "other losses" of the last article. It is, however, possible to exhibit this loss (at least approximately) apart from the other losses to which actual steam engines are subject.

The necessary data are (1) the total weight of feed water used in a given time, including the liquefaction in the jacket, which need not be separately measured; (2) an average indicator diagram showing the average pressure at each point of the stroke; (3) the average boiler pressure and height of barometer; (4) the amount of priming water in the steam supplied by the boiler.

From (3) and (4) may be found the specific volume of the

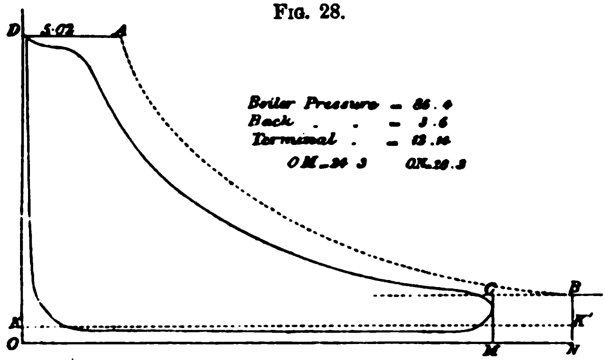
steam supplied by the boiler which is to be set off on a gram, on which are laid down as ordinates the boiler pre and the terminal pressure shown by the indicator. In Fig D A, B C, are lines of boiler pressure and terminal pres and D A represents the specific volume of the boiler st Now, by the construction of Chapter VII., or otherwise, the adiabatic curve through A, terminating in B on the of terminal pressure, then if K K[1] be the line of mean pressure shown by the diagram, the figure D A B K[1] K be the indicator diagram of an engine with non-conduc cylinder, in which the back pressure and the terminal pres are the same as in the actual engine. The losses of wor such an engine are approximately the same as in the ac engine, except so far as due to clearance, wire-drawing, the action of the sides of the cylinder, and hence this diag may for many purposes be adapted as a standard of c parison.

The data furnished by (1) and (2) are sufficient to en us to find, by a process exactly similar to that of Art. the proportion between the volume of steam discharged stroke, and the volume of the whole weight of steam cluding jacket steam) used per stroke, supposed dry at terminal pressure. This proportion is often called the fract of the whole consumption "accounted for by the indicat and in many experiments on steam engines forms part of tabulated results. Now multiply the specific volume of steam at the terminal pressure by the fraction thus fou and set off on the diagram O M to represent the resu then on O M as base construct the actual indicator diagr which will now compare with the indicator diagram of ideal engine with non-conducting cylinder, the *total* consum tion of steam being the same in the two engines.

Fig. 28 shows the construction for the trial of the G *latin*, cited in Art. 111, the jacket being in operation, the ratio of expansion about 7¼. The data already gi

show that the *total* weight of feed water per stroke was ·66 lb., and the effective volume of the cylinder as before 16·05 cubic feet: hence the volume of the cylinder per pound

FIG. 28.



of steam is 24·3 cubic feet. Take then O M 24·3 cubic feet, and, allowing for the *effective* clearance (Art. 93), construct the actual diagram to scale. Next set off O D = 86·4, which was the boiler pressure, and assuming the boiler to supply dry steam, lay off A D = 5·02 cubic feet; trace the adiabatic curve A B, which terminates in B, such that O N, the terminal volume, is 28·3 cubic feet, and draw K K^1 corresponding to the actual back pressure of 3·6 lbs. per square inch. The difference between the real and the ideal diagram represents approximately the loss through the action of the sides combined with clearance and wire-drawing. By continuing the adiabatic curve A B to the line of back pressure, and drawing a new adiabatic curve through C, the comparison may be made exact, but this is hardly necessary.

When the fraction (f) of steam consumed "accounted for by the indicator" forms part of the tabulated results of an experiment, all that is necessary in applying this method is to construct the adiabatic curve, starting from the boiler pressure on a scale enlarged in the proportion 1 : f, due allowance being made for the effects of clearance and compression in drawing the curve.

Diagrams of this kind are highly instructive, and perhaps furnish a clearer idea of the nature of the loss occasioned by the sides of the cylinder than can be otherwise obtained. In applying the method to the case of a compound engine, the diagrams from the high-pressure and low-pressure cylinders must, of course, be combined by the usual process before carrying out the construction.

If the actual expansion curve be similar to the adiabatic curve, the actual diagram (leaving clearance and wire-drawing out of account) will be smaller than the ideal in the proportion f: 1, and the action of the sides will then be represented by the exhaust waste alone. But if (as will generally be the case) the actual expansion curve does not fall so fast as the adiabatic curve, the loss by action of the sides will be greater, or, if it falls faster, less, than the exhaust waste. (Comp. Art. 84.)

Results of Experiments on the Distribution of the Heat expended in Steam Engines.

135. Frequent reference has been made in the course of the present work to certain experiments on steam engines, which have been cited as the "American experiments." These experiments were made in the years 1874–5 on the machinery of the U.S. revenue steamers, *Bache, Rush, Dexter, Dallas,* and *Gallatin,* by Messrs. Emery and Loring, engineers in the service of the United States, representing a board of marine engineers. They are among the most important experiments yet made on steam engines, on account of the great care and judgment with which they were carried out and the fulness with which some of the most important data needed in theoretical investigations are set forth. Their results and the conclusions to be drawn from them are the subject of various reports by Mr. Emery to the U.S. Government, which have been reprinted in 'Engineering,' vols. xix. and xxi.

In the present section I shall give some account of these and other experiments, together with the results of calculation respecting the distribution of the heat expended. For details of the mode of carrying out the experiments I must refer to the original reports.

136. The annexed table shows the results in question for the greater number of the American experiments, and for one other experiment, to be mentioned presently.

The three columns headed "Principal Particulars," and the consumption of steam given in the twelfth column, are of course the direct results of experiment: the remaining columns show the results of calculation by processes already fully explained, but which it may be useful to recapitulate here.

The expenditure of heat given in the last column in thermal units per I.H.P. per minute is reckoned from the temperature of the feed water, whatever that may happen to have been in the particular experiment considered. This is theoretically correct, and the number obtained by dividing 42·75 by the expenditure of heat thus reckoned is the true efficiency of the engine: although it has been already pointed out that it might be advantageous to adopt a fictitious temperature of the condenser, corresponding to the minimum back pressure practically possible. Experiments 42–45 are exceptions, being made without vacuum. Also in calculating the expenditure of heat the boiler has been supposed to supply dry steam: the results obtained are therefore too large, though probably not much too large, as there is no reason to believe the amount of priming water considerable.

The three columns headed "Useful Heat Expended" show the useful work done, together with the corresponding *necessary* loss (Art. 88), expressed as a percentage of the total heat expended. The first of these columns is therefore the absolute efficiency of the engine, and the third the efficiency relatively to a *perfect* engine working between the same limits of temperature a engine.

DISTRIBUTION OF HEAT IN STEAM ENGINES

Description of Engine.	Principal Particulars.			Useful Heat expended per Cent of Total			Losses of Heat per Cent of Total Heat					Total Expenditure of Heat and Steam	
	Boiler Pressure.	Speed of Piston	Ratio of Expansion.	Useful Work	Loss in a Perfect Engine	Useful Heat	Exhaust Waste	Incomplete Expansion	Heating Feed	Radiation, Back Pressure	Other Losses	Lbs. Steam per I.H.P. per Hour	Thermal Units per I.H.P. per 1'
'BACHE.'													
1. Compound engine, low-pressure cylinder jacketed.	95½	150	11¾	6·7	20·6	27·3						26·1	[illegible]
2.	94½	180	7⅝	8·0	24·1	32·1	37·6	11·2	7·9	2·6	8·8	28·0	511
3. Operated as a simple engine, without jacket.	92¾	188	5¼	8·9	27·1	36·0	27·2	18·0	7·9	2·4	8·5	26½	480
4. Operated as a simple engine, with jacket in use.	95½	160	12½	8·6	24·4	33·0	32·0	12·8	8·0	1·8	12·4	27·1	498
5.	95¾	185	8½	9·7	27·5	37·2	27·9	13·4	8·1	1·5	11·9	24·1	441
6.	94	215	5	10·2	29·2	39·4	21·9	21·2	8·2	2·1	7·2	22·1	410
7.	45½	181	2¼	7·1	30·6	37·7	24·4	22·9	6·4	3·3	5·3	31·0	568
8. Operated as a compound engine, with jacket in use.	96¼	155	17	9·2	24·3	33·5	15·4	15·6	7·6	4·0	23·9	22·1	405
9.	95¼	193	9¼	11·0	28·2	39·2	13·5	18·5	8·0	2·8	18·0	20·7	389
10.	95	218	7	11·3	28·8	40·1	16·4	19·1	8·2	2·5	13·7	20·8	378
11.	95	225	5¾	11·2	28·4	39·6	15·6	19·6	8·2	2·5	14·5	20·4	382
12.	94	242	4¼	11·1	30·8	41·9	15·0	19·9	8·0	1·9	13·3	21·2	385

	'DALLAS.'													
13.	Simple engine, not jacketed.	50	243	5	9·4	33·1	42·5	20·4	18·1	6·8	3·2	9·0	26·7	455
14.		50	285	3⅞	9·4	33·1	42·5	16·3	20·8	6·8	3·6	10·0	27·0	455
15.		46½	308	3⅛	9·4	35·8	45·2	17·8	22·1	6·2	4·3	4·4	26·9	455
16.		48½	322	2⅞	8·9	33·5	42·4	19·6	21·9	6·7	3·8	5·7	28·9	480
17.		42	317	2⅜	8·3	33·0	41·3	19·4	23·9	6·4	4·0	5·0	31·0	515
	'DEXTER.'													
18.	Simple engine, not jacketed; leakage not included.	83½	339	4½	10·4	29·7	40·1	21·0	19·1	8·2	3·9	7·7	22·6	411
19.		82	366	3½	10·4	30·0	40·4	20·0	21·1	8·2	3·9	6·4	22·7	411
20.		81¼	437	2¾	10·3	30·6	40·9	13·7	25·2	8·0	5·2	7·0	23·0	415
21.		55½	305	3¼	8·6	27·1	35·7	24·9	20·2	7·4	4·5	7·3	27·4	497
22.		56¾	364	2	7·8	26·4	34·2	26·4	21·7	7·3	4·4	6·0	30·2	548
	'RUSH.'													
23.	Compound engine, both cylinders jacketed.	84	319	6¼	12·7	36·3	49·0	..	..	..	..	..	18·4	336
24.		51½	250	4	10·6	36·3	46·9	..	..	..	..	..	22·1	404
	'GALLATIN.'													
25.	Simple engine, jacketed; jacket in use.	86½	255	7·3	11·4	32·3	43·7	15·2	20·1	7·8	3·1	10·1	20·5	374
26.		82	343	4·9	10·9	32·9	43·8	12·9	25·4	7·8	3·1	7·0	21·5	390
27.		60¼	221	6·1	10·2	30·1	40·3	13·9	22·7	7·2	5·0	10·9	22·9	420
28.		58	230	5·1	9·8	30·1	40·9	20·3	17·8	6·7	3·7	10·6	24·0	437
29.		56	246	3·7	10·2	33·8	44·0	18·3	25·7	7·1	3·9	1·0	23·2	420
30.		52¼	291	2·2	9·0	32·8	41·8	13·8	27·1	6·7	3·5	7·1	26·5	476
31.		30	206	2·0	7·2	31·9	39·1	16·9	26·6	5·8	4·3	7·3	33·3	591
32.		28	212	1·5	6·4	30·4	36·8	17·7	25·6	5·4	4·7	9·8	37·4	664

DISTRIBUTION OF HEAT IN STEAM ENGINES—*continued.*

	Description of Engine.	Principal Particulars.			Useful Heat expended per Cent. of Total.			Losses of Heat per Cent. of Total Heat.					Total Expenditure of Heat and Steam.	
		Boiler Pressure.	Speed of Piston.	Ratio of Expansion.	Useful Work.	Loss in a Perfect Engine.	Useful Heat.	Exhaust Waste.	Incomplete Expansion.	Heating Feed.	Excess Back Pressure.	Other Losses.	Lbs. Steam per I.H.P. per Hour.	Thermal Units per I.H.P. per 1′.
33.	'GALLATIN' (*continued*). Jacket not in use.	86	261	7·8	9·4	26·6	36·0	22·8	19·5	8·5	3·4	9·8	25·0	454
34.		83¼	300	5	10·6	29·7	40·3	17·8	24·4	8·3	2·0	7·2	21·9	401
35.		59½	215	5·9	9·0	26·9	35·9	24·5	14·7	7·7	3·0	14·2	26·0	476
36.		58	254	3·7	9·8	30·3	40·1	17·7	24·9	7·5	2·0	7·8	24·0	437
37.		52½	278	2·2	8·6	35·4	44·0	16·0	22·0	6·3	2·0	9·7	28·1	496
38.		29½	200	2·0	5·9	25·8	31·7	31·5	21·9	5·9	4·1	4·9	40·4	720
39.		27½	204	1·5	5·4	25·8	31·2	28·7	24·8	5·4	4·0	5·9	44·2	787
40.	Jacket supplied with steam of 85 lbs. Difference of temperature 68°.	28½	205	1·8	6·9	27·5	34·4	15·0	31·9	5·8	4·1	8·8	34·1	616
41.		29	211	1·5	7·1	29·5	36·6	13·6	33·7	5·7	4·4	6·0	34·9	605
42.	Without vacuum; jacket in use.	84½	247	4·1	9·9	64·1	74·0	4·0	1·8	4·7	..	15·5	25·9	431
43.		82	266	3·5	9·4	61·7	71·1	6·7	2·7	4·5	..	15·0	27·3	456
44.	Without vacuum; jacket not in use.	85½	233	4·4	8·6	55·4	64·0	18·4	1·5	4·9	..	16·2	30·0	499
45.		81	258	3·5	8·8	58·1	66·9	10·1	8·8	4·5	..	14·7	29·4	488
	'HELE' ENGINE.													
46.	Compound engine, low-pressure cylinder jacketed; jacket in use.	67½	279	2·2	11·1	28·0	39·1	14·0	14·4	7·7	5·7	19·1	20·5	384

The five remaining columns show the losses of heat, being the heat unnecessarily lost.

(1) By the exhaust waste, that is to say, by transmission of heat to the exhaust steam and by external radiation less the heat given out by piston friction and the effects of compression. The results are maximum values, because the steam supplied by the boiler has been supposed dry: the method of calculation has been fully detailed in the present chapter.

(2) By incomplete expansion, that is to say, by the amount of work which the steam discharged from the cylinder might theoretically be made to do by allowing it to expand adiabatically down to the pressure corresponding to the temperature of the condenser. The method of calculation is explained in Chapter VIII. In comparing the values given with each other and with the work done it must be remembered that the quantity of work in question depends not merely on the terminal pressure, but also on the temperature of the condenser and on the terminal dryness of the steam: moreover, it includes the loss of heat *necessarily* accompanying the performance of that amount of work, as explained fully in the chapter cited, so that it compares directly with the useful heat expended, not the useful work done.

(3) By misapplication of heat in heating the feed, that is to say, by raising the temperature of the feed water by direct application of heat instead of by compression of the exhaust steam. The nature of this loss and the method of calculation are explained in detail in Chapter VIII.

(4) By excess back pressure, that is to say, by the difference between the actual back pressure and the pressure corresponding to the temperature of the condenser. The mode of calculating this loss is also explained in Chapter VIII.

(5) By other losses, that is to say, by clearance and wire-drawing and by misapplication of heat during expansion. The results given in this column represent the heat re-

maining unaccounted for after the useful heat and the four preceding losses have been considered, and are therefore not so reliable as those given in the other columns, because small errors in the experiments and calculations become by accumulation large ones in estimating the residuary heat.

The results shown in the table are, with one or two exceptions, fairly consistent, when the great complexity of the question is considered, and lead us to believe that the difficulty of obtaining exact experimental data has been to a great extent overcome in the experiments in question. It will be observed that no attempt has been made to carry the analysis farther by calculating separately the several losses by misapplication of heat during expansion, by clearance and wire-drawing: to do this it would be necessary to know the initial pressure exactly, and this is complicated by questions relating to the effect of wire-drawing, noticed in Chapter IX., Art. 103, which cannot at present be satisfactorily dealt with. It is to be particularly noticed that none of the calculations of the present chapter are affected in this way unless it be conceived possible that some of the kinetic energy generated during admission may remain unabsorbed by fluid friction at release, in which case a part of the exhaust waste will be due to this cause.

All the results are given as percentages of the total expenditure of heat, but by multiplying by that total expenditure as given in the last column, and dividing by 100, they may be expressed in thermal units per I.H.P. per minute, or by multiplying by the consumption of steam and dividing by 100, they may be expressed in lbs. of steam per I.H.P. per hour.

137. The engine of the *Bache* is compound, of the Woolf type,* the small cylinder 16 and the large cylinder 25 inches diameter, the stroke of both 2 feet; the large cylinder alone

* In the example of Art. 60 the engine has frequently been spoken of as if it possessed an intermediate reservoir, the treatment of the steam between the two cylinders being quite immaterial for the purposes of the article in question.

is jacketed. Though compound, the engine could also be operated as a simple engine by use of the large cylinder alone: hence there were four sets of experiments according as the engine was simple or compound, with or without jacket. The results of twelve of these experiments are shown in the table, all but one made with a boiler pressure of about 95 lbs. per square inch absolute. The first three without jacket show an exhaust waste of over 27 per cent., increasing with the ratio of expansion to such an extent, that working expansively becomes very wasteful. A discrepancy in the tabulated data has prevented the calculation of the losses in the first of these, but no doubt at the highest ratio of expansion that loss exceeded 40 per cent.

The next four made with jacket in use, show a diminution in the exhaust waste from 27 to 22 at ratio of expansion 5, but, as in the former case, this cause of loss increases so fast with the expansion, that higher rates of expansion are wasteful. The last of these experiments was made at a lower pressure, and shows an increase in the exhaust waste from 22 to 24, notwithstanding a diminution in the expansion from 5 to $2\frac{1}{8}$; the cause of this is the increased surface per pound of steam, as pointed out already in Chapter X.

The last five experiments show in a very striking way the effect of compounding. The exhaust waste is diminished to 15 per cent., but this improvement is gained at a considerable sacrifice, the "other losses" increasing to a great extent. The reason of this is that the "other losses" include the wasteful liquefaction and re-evaporation in the high-pressure cylinder, the nature of which was explained in detail in Chapter VII., together with the wire-drawing between the cylinders. Nevertheless, in the present case, compounding is advantageous, but it is clear that even here expansion must not be carried too far; a ratio 7 appears the best in the present instance, whic diminished to 5 without material loss, whi ble increase is decidedly disadvantageous.

138. The *Dexter* and *Dallas* have simple engines of somewhat larger size than that of the *Bache*, the cylinder being 26 inches diameter and 3 feet stroke, and 36 inches diameter and $2\frac{1}{2}$ feet stroke respectively. The cylinders are without jackets in both cases.

In the *Dexter* a loss of water by imperceptible leakage of 4·96 per cent. was accurately measured, and is taken into account in the results given. In the other experiments, a smaller loss of this kind doubtless occurred, but was not measured.

The piston speed was much greater in these experiments than in those on the *Bache*, for which reason, probably, the results are considerably better. In one of these experiments (No. 20) the exhaust waste is diminished to 13·7 per cent. by increasing the piston speed to 437 feet per minute. It is much to be wished that a series of experiments should be made on the effect of piston speed, all other conditions remaining the same. It has been already pointed out in Chapter X. that an increase of speed may perhaps not always be advantageous.

139. The *Rush* has a compound engine with intermediate receiver and with both cylinders jacketed, the diameters are 24 inches and 38 inches respectively, with a stroke of 27 inches. The results are considerably better than those obtained with any of the other engines, as the table shows. No particulars being given of the water discharged from the jacket, the losses of heat have not been calculated: there can, however, be no doubt that the exhaust waste is greatly diminished by compounding, as in the case of the *Bache*, while the corresponding increase in "other losses," due to liquefaction and re-evaporation in the high-pressure cylinder, is to a great extent avoided by the use of a steam jacket or by other means (Art. 118). The fraction of steam "accounted for by the indicator," at the end of the te of the high-pressure cylinder, was so large as to

show that the steam passing through the cylinder must have been nearly dry at the end of the stroke, whereas in the *Bache* it was then very wet.

140. The engine of the *Gallatin* is simple, with jacketed cylinders, 34 inches diameter and 30 inches stroke. A great variety of experiments were made on this engine with various steam pressures, with jacket in and out of use. Though not always perfectly consistent, as may be expected from the nature of the case, these experiments are highly instructive, and the results of twenty-one of them are given in the table.

Experiments 25–39 show the marked influence of the steam jacket in diminishing the action of the sides of the cylinder, and increasing the best ratio of expansion when other circumstances remain the same: while lowering the initial pressure in the cylinder, other things being equal, is always followed by a diminution of efficiency.

In experiments 40–41 the initial cylinder pressure was lowered to about 29 lbs., while the jacket was supplied with steam of pressure 85 lbs., and therefore of temperature 68° above that of the steam in the cylinder: the result, when compared with other experiments at a similar pressure, shows a considerable diminution in the exhaust waste, though it is still as great as it would have been if the steam had been used in the cylinder at 85 lbs., no doubt in consequence of the increase of surface due to the lower pressure.

Experiments 42–45 were made without vacuum, two with and two without jacket in use. In this case the engine is virtually non-condensing, and, as is the case generally in non-condensing engines, the high exhaust temperature greatly diminishes the action of the sides. Thus, when the jacket was used, the exhaust waste was reduced to from 4 to $6\frac{3}{4}$ per cent., with a ratio of expansion of $3\frac{1}{2}$ to 4; an amount which is hardly more than would probably be due to external radiation and the effect of priming water, showing that the

transmission of heat to the exhaust steam must have been trifling.* The influence of the steam jacket is very marked, and it is probable that the much greater compression characteristic of non-condensing engines has something to do with the diminution of action in question. The "other losses" are large, partly because the excess back pressure has been included, but chiefly because the expenditure of heat has been reckoned in these experiments from 212°, as in non-condensing engines; which has also been taken as the inferior temperature in calculating the relative efficiency. Hence the "necessary" losses are high, and the relative efficiency great.

141. Mr. Emery has deduced from his experiments an empirical rule for the best ratio of expansion, which may be written

$$r = 1 + \frac{p}{22},$$

where p is the *absolute* pressure in pounds per square inch; and he considers that the results of this formula are "nearly correct for single engines of large size, with details of good design, too large for single engines of ordinary construction, and too small for the better class of compound engines."

142. The *Hele* engine trial has already been discussed in the earlier part of the present chapter (Art. 125). The average terminal pressure, taken from forty-two indicator diagrams for the low-pressure cylinder, was 3·7 lbs. per square inch,† from which, estimating the *effective* clearance as ·05, the results are deduced which are given in the table.

* In experiment 42 the tabulated value of the fraction of the consumption of steam "accounted for by the indicator" leads to a negative value of the exhaust waste, a result in all cases impossible. On examining the other data, however, the tabulated value was found to be erroneous. In one or two other instances discrepancies in the results were found to be due to errors in the tabulated values of the fraction in question, but in most instances they were found to be correct.

† I am indebted for this information to the courtesy of Messrs. B. Donkin Co.

The exhaust waste is found to be 14 per cent., being somewhat less than in the case of the *Bache* compound engine. The "other losses" are high, for the same reason as before, namely, the liquefaction and re-evaporation in the high-pressure cylinder, due to the absence of a steam jacket, coupled with a great ratio of expansion (10). The liquefaction in the high-pressure cylinder was probably aggravated by the circumstance that the steam supplying it passed through the low-pressure steam jacket, and hence, probably, carried with it some of the liquefied steam there condensed. The great loss by "incomplete expansion," notwithstanding the low terminal pressure, which at first sight appears anomalous, is due to the low condenser temperature corresponding to a pressure of only ·6 lb. per square inch. The effects of variation of temperature in modifying the *apparent* results of calculation have already been commented on in Art. 125, and elsewhere.

143. By far the most complete experiments on steam engines, from a theoretical point of view, are those commenced in 1873, and still in progress, by MM. Dwelshauvers-Déry, W. Grosseteste, and O. Hallauer, under the general supervision of M. Hirn. A summary of these experiments, with an introductory note by M. Hirn, is given in the 'Bulletin Spécial' of the Société Industrielle de Mulhouse, published in 1876, page 187. I have already referred to these experiments in Chapter X. as furnishing decisive evidence on the subject of the action of the sides of the cylinder. Their general results show a close agreement with those of the American experiments, but they extended to superheated steam, a part of the subject not therein included.

The comparative results of certain experiments with saturated and superheated steam respectively, are shown in the table annexed.

EFFECT OF SUPERHEATING.

Description of Experiment.	Admission Pressure. Lbs. per Square Inch.	Ratio of Expansion.	Heat transmitted to Exhaust Steam per cent.
Steam superheated 157° F.	60·6	4	7·8
Steam saturated	54	4	15·6
Steam superheated 95° F...	56	7	12·43
Steam saturated	54·7	7	21·8

The numbers given in this table for the percentage of heat transmitted to the exhaust steam, do not include the external radiation as the exhaust waste of preceding articles does; a suitable allowance having been made in the calculation for it. (Comp. Art. 130.)

The result shows the great diminution in the action of the sides which can be produced by superheating, a diminution, the cause of which has already been pointed out in Chapter X., and which is the true reason why superheated steam has been found in practice to be economical.

[redacted]

TABLES OF THE PROPERTIES OF SATURATED STEAM.

TABLE Ia.—RELATION BETWEEN PRESSURE AND TEMPERATURE.

Temperature Fahrenheit, t.	Pressure in Lbs. per Sq. In. at the Level of the Sea in Lat. 44°, p.	Difference, Δp.	Temperature Fahrenheit, t.	Pressure in Lbs. per Sq. In. at the Level of the Sea in Lat. 44°, p.	Difference, Δp.
°			°		
432	350·73	3·64	414	289·48	3·15
431	347·09	3·61	413	286·33	3·12
430	343·48	3·58	412	283·21	3·09
429	339·90	3·55	411	280·12	3·07
428	336·35	3·53	410	277·05	3·04
427	332·82	3·50	409	274·01	3·02
426	329·32	3·47	408	270·99	2·99
425	325·85	3·44	407	268·00	2·97
424	322·41	3·42	406	265·03	2·94
423	318·99	3·39	405	262·09	2·92
422	315·60	3·36	404	259·17	2·89
421	312·25	3·33	403	256·28	2·87
420	308·92	3·31	402	253·41	2·84
419	305·61	3·28	401	250·57	2·82
418	302·33	3·25	400	247·75	2·79
417	299·08	3·23	399	244·96	2·77
416	295·85	3·20	398	242·19	2·75
415	292·65	3·17	397	239·44	2·72

Table Ia.—*continued.*

t	p	Δp	t	p	Δp
°			°		
396	236·72	2·70	374	182·63	2·21
395	234·02	2·67	373	180·42	2·19
394	231·35	2·65	372	178·23	2·17
393	228·70	2·63	371	176·07	2·15
392	226·07	2·61	370	173·92	2·13
391	223·46	2·58	369	171·79	2·11
390	220·88	2·56	368	169·69	2·09
389	218·32	2·53	367	167·60	2·07
388	215·79	2·51	366	165·53	2·05
387	213·28	2·49	365	163·49	2·03
386	210·79	2·47	364	161·47	2·01
385	208·33	2·45	363	159·46	1·99
384	205·88	2·42	362	157·48	1·97
383	203·46	2·40	361	155·51	1·95
382	201·06	2·38	360	153·56	1·93
381	198·68	2·36	359	151·63	1·91
380	196·32	2·34	358	149·72	1·89
379	193·98	2·32	357	147·82	1·87
378	191·67	2·29	356	145·95	1·85
377	189·38	2·27	355	144·10	1·83
376	187·11	2·25	354	142·27	1·82
375	184·86	2·23	353	140·45	1·80

Table Ia.—*continued.*

t	p	Δp	t	p	Δp
°			°		
352	138·65	1·78	330	103·43	1·41
351	136·87	1·76	329	102·02	1·40
350	135·11	1·74	328	100·62	1·39
349	133·37	1·73	327	99·23	1·37
348	131·64	1·71	326	97·86	1·35
347	129·93	1·70	325	96·51	1·34
346	128·23	1·68	324	95·17	1·32
345	126·55	1·66	323	93·85	1·31
344	124·89	1·64	322	92·54	1·29
343	123·26	1·63	321	91·25	1·28
342	121·63	1·61	320	89·97	1·27
341	120·02	1·59	319	88·70	1·25
340	118·43	1·57	318	87·45	1·24
339	116·86	1·56	317	86·21	1·22
338	115·30	1·54	316	84·99	1·21
337	113·76	1·52	315	83·78	1·19
336	112·24	1·50	314	82·59	1·18
335	110·74	1·49	313	81·40	1·17
334	109·25	1·48	312	80·23	1·15
333	107·77	1·46	311	79·08	1·14
332	106·31	1·45	310	77·94	1·13
331	104·86	1·43	309	76·81	1·12

Table Ia.—*continued.*

t	p	Δp	t	p	Δp
308	75·69	1·10	286	54·24	·843
307	74·59	1·09	285	53·39	·832
306	73·50	1·08	284	52·56	·821
305	72·42	1·06	283	51·74	·810
304	71·36	1·05	282	50·93	·799
303	70·31	1·04	281	50·13	·790
302	69·27	1·03	280	49·33	·780
301	68·24	1·018	279	48·55	·771
300	67·22	1·006	278	47·78	·761
299	66·22	·994	277	47·02	·752
298	65·23	·982	276	46·27	·743
297	64·25	·970	275	45·53	·733
296	63·29	·957	274	44·79	·724
295	62·33	·945	273	44·07	·714
294	61·38	·933	272	43·35	·705
293	60·45	·921	271	42·65	·696
292	59·53	·909	270	41·96	·687
291	58·62	·898	269	41·27	·678
290	57·72	·887	268	40·60	·669
289	56·83	·876	267	39·93	·660
288	55·96	·865	266	39·27	·651
287	55·09	·854	265	38·62	·642

Table Ia.—*continued.*

t	p	Δp	t	p	Δp
°			°		
264	37·98	·633	242	25·92	·465
263	37·35	·624	241	25·46	·458
262	36·72	·615	240	25·00	·452
261	36·11	·607	239	24·55	·446
260	35·50	·599	238	24·11	·439
259	34·90	·591	237	23·67	·433
258	34·31	·583	236	23·25	·426
257	33·73	·575	235	22·82	·419
256	33·15	·567	234	22·40	·413
255	32·59	·559	233	21·99	·407
254	32·03	·551	232	21·59	·400
253	31·48	·543	231	21·19	·394
252	30·94	·535	230	20·80	·388
251	30·41	·527	229	20·41	·382
250	29·88	·520	228	20·03	·376
249	29·36	·513	227	19·66	·370
248	28·85	·506	226	19·29	·364
247	28·34	·499	225	18·93	·358
246	27·84	·492	224	18·57	·352
245	27·35	·485	223	18·22	·346
244	26·87	·478	222	17·87	·340
243	26·39	·471	221	17·53	·335

Table Ia.—continued.

t	p	Δp	t	p	Δp
[illegible]	[illegible]	·281	198	11·05	·229
[illegible]	[illegible]	[illegible]	197	10·82	·225
[illegible]	[illegible]	[illegible]	196	10·60	·221
[illegible]	[illegible]	[illegible]	195	10·38	·217
[illegible]	[illegible]	[illegible]	194	10·16	·213
[illegible]	[illegible]	[illegible]	193	9·95	·209
[illegible]	[illegible]	[illegible]	192	9·74	·205
[illegible]	[illegible]	[illegible]	191	9·53	·202
[illegible]	[illegible]	[illegible]	190	9·33	·199
[illegible]	[illegible]	[illegible]	189	9·13	·195
[illegible]	[illegible]	[illegible]	188	8·94	·191
[illegible]	[illegible]	[illegible]	187	8·75	·188
[illegible]	[illegible]	·271	186	8·56	·185
[illegible]	[illegible]	[illegible]	185	8·37	·181
[illegible]	[illegible]	·262	184	8·19	·178
205	12·77	·258	183	8·01	·175
204	12·51	·254	182	7·84	·171
203	12·26	·249	181	7·67	·168
202	12·01	·245	180	7·50	·165
201	11·76	·241	179	7·34	·162
200	11·52	·236	178	7·17	·159
199	11·29	·232	177	7·01	·156

Table Ia.—*continued.*

t	p	Δp	t	p	Δp
°			°		
176	6·85		154	4·091	
		·153			·0991
175	6·70		153	3·992	
		·150			·0969
174	6·55		152	3·895	
		·147			·0948
173	6·40		151	3·800	
		·144			·0928
172	6·26		150	3·707	
		·141			·0909
171	6·12		149	3·616	
		·139			·0890
170	5·98		148	3·527	
		·136			·0872
169	5·85		147	3·440	
		·134			·0854
168	5·71		146	3·354	
		·131			·0836
167	5·58		145	3·270	
		·129			·0818
166	5·45		144	3·188	
		·126			·0801
165	5·32		143	3·108	
		·123			·0784
164	5·20		142	3·030	
		·121			·0767
163	5·08		141	2·953	
		·119			·0751
162	4·961		140	2·878	
		·1166			·0735
161	4·844		139	2·805	
		·1144			·0720
160	4·730		138	2·733	
		·1123			·0705
159	4·618		137	2·663	
		·1101			·0690
158	4·508		136	2·594	
		·1079			·0675
157	4·401		135	2·526	
		·1057			·0660
156	4·295		134	2·461	
		·1035			·0645
155	4·192		133	2·397	
		·1013			·0630

Table Ia.—*continued.*

t	p	Δp	t	p	Δp
°			°		
132	2·334		112	1·342	
		·0615			·0382
131	2·273		111	1·304	
		·0601			·0372
130	2·212		110	1·267	
		·0588			·0363
129	2·154		109	1·230	
		·0574			·0355
128	2·096		108	1·195	
		·0561			·0346
127	2·040		107	1·160	
		·0548			·0337
126	1·985		106	1·127	
		·0535			·0328
125	1·932		105	1·094	
		·0522			·0319
124	1·880		104	1·062	
		·0509			·0311
123	1·829		103	1·031	
		·0497			·0302
122	1·779		102	1·001	
		·0485			·0294
121	1·731		101	·971	
		·0475			·0287
120	1·683		100	·942	
		·0464			·0280
119	1·637		99	·914	
		·0454			·0274
118	1·591		98	·887	
		·0444			·0267
117	1·547		97	·860	
		·0433			·0260
116	1·504		96	·834	
		·0423			·0252
115	1·462		95	·809	
		·0412			·0246
114	1·421		94	·785	
		·0402			·0239
113	1·381		93	·761	
		·0392			

TABLE Ib.—RELATION BETWEEN PRESSURE AND TEMPERATURE.

Temperature.	Force in Inches of Mercury at 32° at Sea Level. (Lat. 53° 21′.)	Temperature.	Force in Inches of Mercury at 32° at Sea Level. (Lat. 53° 21′.)	Temperature.	Force in Inches of Mercury at 32° at Sea Level. (Lat. 53° 21′.)
°	ins.	°	ins.	°	ins.
150	7·540	123	3·720	96	1·6971
149	7·354	122	3·619	95	1·6457
148	7·173	121	3·520	94	1·5958
147	6·996	120	3·423	93	1·5471
146	6·822	119	3·329	92	1·4998
145	6·651	118	3·237	91	1·4537
144	6·485	117	3·147	90	1·4088
143	6·322	116	3·059	89	1·3652
142	6·162	115	2·974	88	1·3228
141	6·006	114	2·890	87	1·2815
140	5·854	113	2·809	86	1·2413
139	5·704	112	2·729	85	1·2023
138	5·558	111	2·652	84	1·1643
137	5·415	110	2·576	83	1·1274
136	5·275	109	2·502	82	1·0915
135	5·139	108	2·430	81	1·0566
134	5·005	107	2·360	80	1·0227
133	4·874	106	2·292	79	0·9898
132	4·747	105	2·225	78	0·9577
131	4·622	104	2·160	77	0·9266
130	4·500	103	2·097	76	0·8964
129	4·381	102	2·035	75	0·8671
128	4·264	101	1·975	74	0·8386
127	4·150	100	1·917	73	0·8109
126	4·039	99	1·8595	72	0·7841
125	3·930	98	1·8039	71	0·7580
124	3·824	97	1·7498	70	0·7327

TABLE IIa.—TOTAL AND LATENT HEAT OF EVAPORATION IN THERMAL UNITS.

t	$t-32$	h	Δh	H	ΔH	L	ΔL
°							
401	369	375·16		1204·2		829·1	
			1·043		·305		·738
374	342	347·0		1196·0		849·0	
			1·037		·305		·732
347	315	319·0		1187·8		868·8	
			1·032		·305		·727
320	288	291·14		1179·5		888·4	
			1·027		·305		·722
293	261	263·41		1171·3		907·9	
			1·023		·305		·718
266	234	235·8		1163·1		927·3	
			1·0185		·305		·713
239	207	208·3		1154·8		946·5	
			1·0144		·305		·709
212	180	180·9		1146·6		965·7	
			1·011		·305		·706
185	153	153·6		1138·4		984·8	
			1·008		·305		·703
158	126	126·4		1130·1		1003·8	
			1·006		·305		·701
131	99	99·2		1121·9		1022·7	
			1·004		·305		·699
104	72	72·09		1113·7		1041·6	
			1·002		·305		·697
77	45	45·03		1105·4		1060·4	
			1·000		·305		·695
32	0	0		1091·7		1091·7	

Explanation of Symbols.

H.—Total Heat of Evaporation (Art. 4) expressed in thermal units.

h.—Heat necessary to raise a lb. of water from 32° to t° (Art. 3) expressed in thermal units.

L = H − h.—Latent Heat of Evaporation (Art. 4) expressed in thermal units.

TABLE IIb.—TOTAL AND LATENT HEAT OF EVAPORATION IN FOOT POUNDS.

t	$\frac{h}{100}$	Δh	$\frac{H}{100}$	ΔH	$\frac{L}{100}$	ΔL
401	2896		9297		6401	
		805		235		570
374	2679		9233		6554	
		801		235		565
347	2463		9170		6707	
		797		235		561
320	2247		9106		6858	
		793		235		557
293	2033		9042		7009	
		790		235		554
266	1820		8979		7159	
		786		235		550
239	1608		8915		7307	
		783		235		547
212	1396		8852		7455	
		780		235		545
185	1186		8788		7603	
		778		235		543
158	975·8		8725		7749	
		777		235		541
131	765·8		8661		7895	
		775		235		540
104	556·5		8598		8041	
		773		235		538
77	347·5		8534		8186	
		772		235		536
32	0		8428		8428	

Explanation of Symbols.

The symbols in this table have the same meanings as in Table IIa, but quantities of heat are [illegible] in foot pounds instead of thermal units. The diffe[illegible] tables are the mean differences per degree betw[illegible] indicated.

TABLE III.—DENSITY AND SPECIFIC VOLUME OF DRY STEAM.

Pressure in Lbs. per Sq. In., p.	Volume of 1 lb. in Cub. Feet, v.	$v - s = u$	Weight of a Cubic Foot in Lbs. by direct Experiment.	Weight of a Cubic Foot in Lbs. by Calculation, w.	Difference per Lb. Pressure, Δw.
250	1·841	1·825	..	·5432	·002056
200	2·270	2·254	..	·4404	·002080
170	2·645	2·629	..	·3780	·002107
140	3·177	3·161	..	·3148	·002130
110	3·986	3·970	..	·2509	·002150
90	4·810	4·794	..	·2079	·002185
70	6·090	6·074	·1682	·1642	·002210
60	7·037	7·021	·1457	·1421	·002230
50	8·347	8·331	·1228	·1198	·002270
40	10·30	10·28	·09937	·0971	·002297
30	13·49	13·47	·07550	·07413	·002332
25	16·01	15·99	·06339	·06247	·002364
20	19·74	19·72	·05117	·05065	·002398
15	25·87	25·85	·03878	·03866	·002437
12	31·9	..	·03131	·03135	·002470
10	37·8	..	·02630	·02641	·002490
8	46·6	..	·02126	·02143	·00253
6	61·1	..	·01620	·01637	·00258
5	72·5	..	·01258	·01379	·00265
	89·8	..	·01112	·01114	

TABLE IVa.—EXTERNAL WORK.

t	PV Steam Gas.	P v Foot Pounds.	P u Foot-Pounds.	Δ P u	P u Thermal Units.	Δ P u
°						
401	73,670	66,250	65,600		85	
				38		·0492
374	71,360	65,030	64,570		83·6	
				43		·0557
347	69,050	63,720	63,400		82·1	
				48		·0622
320	66,750	62,320	62,100		80·4	
				53		·0687
293	64,440	60,820	60,670		78·6	
				57		·0739
266	62,130	59,230	59,130		76·6	
				61		·0791
239	59,830	57,550	57,480		74·5	
				65		·0842
212	57,520	55,780	55,730		72·2	
				69		·0894
185	55,210	53,910	53,880		69·8	
				73		·0946
158	52,910	51,950	51,930		67·3	
				77		·0998
131	50,600	49,890	49,880		64·6	
				80		·104
104	48,290	47,740	47,740		61·8	

Explanation of Symbols.

The third column of Table IV*a* gives the product in foot pounds of the pressure (P), and the volume (v) of dry saturated steam at the temperature indicated in the first column. The fourth, fifth, sixth, and seventh columns give P u, that is to say, P $(v - s)$ in foot pounds and thermal units together with the differences needed for interpolation: thus the external work done during evaporation under constant pressure and its heat-equivalent are known for any temperature, and hence by Table I. for any pressure.

TABLE IVb.—INTERNAL WORK DURING EVAPORATION.

t	ρ Foot Pounds.	$\Delta\rho$	ρ Thermal Units.	$\Delta\rho$	$k = \frac{\rho}{Pu}$
°					
401	574,500		744·1		8·75
		608		·787	
374	590,900		765·4		9·16
		608		·788	
347	607,300		786·7		9·59
		609		·789	
320	623,800		808·0		10·05
		610		·791	
293	640,300		829·3		10·55
		611		·792	
266	656,800		850·7		11·10
		611		·792	
239	673,300		872·1		11·71
		612		·793	
212	689,800		893·5		12·39
		614		·795	
185	706,300		915·0		13·11
		616		·797	
158	722,900		936·4		13·92
		618		·800	
131	739,600		958·1		14·82
		620		·803	
104	756,400		979·8		15·78

Explanation of Symbols.

Table IV*b* gives ρ the internal work done in producing dry saturated steam of a given temperature, and hence, by Table I., of a given pressure from water of the same temperature, or what is the same thing the intrinsic energy of dry saturated steam reckoned from water at the same temperature. The results are given in foot pounds and thermal units with the differences necessary for interpolation. The last column shows the proportion (k) which the internal work bears to the external work, a number also given in Table V. for a series of pressures.

TABLE IVc.—TOTAL INTERNAL WORK.

t	I Thermal Units.	Δ I	I ÷ 100 Foot Pounds.	Δ I
°				
401	1119·2		8641	
		·256		197
374	1112·4		8587	
		·249		192
347	1105·7		8536	
		·242		187
320	1099·2		8485	
		·236		182
293	1092·7		8435	
		·231		178
266	1086·5		8378	
		·226		174
239	1080·3		8340	
		·221		170
212	1074·4		8295	
		·216		166
185	1068·6		8249	
		·210		162
158	1062·8		8205	
		·205		158
131	1057·3		8162	
		·201		155
104	1051·9		8121	

EXPLANATION OF SYMBOLS.

Table IVc gives I the total internal work done in producing dry steam at any given temperature from water at 32°, or what is the same thing the intrinsic energy of dry saturated steam reckoned from water at 32°. As in the preceding tables, the differences are given for a difference of temperature of 1°.

TABLE V.—TABLE showing the INTERNAL-WORK-PRESSURE and the HEAT-PRESSURE during Evaporation under a constant External Pressure.

External Pressure, p	Internal-Work-Pressure. Lbs. per Sq. Foot, $\overline{P}$.	Difference, $\Delta\overline{P}$.	Lbs. per Sq. Inch, $\overline{p}$.	Difference, $\Delta\overline{p}$.	$k = \frac{\overline{p}}{p}$	Heat-Pressure p_h
250	315,100		2188		8·75	2438
		1100		7·64		
200	260,100		1806		9·03	2006
		1130		7·83		
170	226,200		1571		9·24	1740
		1162		8·07		
140	191,300		1329		9·49	1469
		1208		8·39		
110	155,100		1077		9·79	1187
		1247		8·66		
90	130,200		903·6		10·04	993·6
		1282		8·90		
70	104,500		725·5		10·36	795·5
		1327		9·21		
60	91,200		633·4		10·56	693·4
		1347		9·36		
50	77,750		539·8		10·80	589·8
		1391		9·65		
40	63,850		443·3		11·08	483·3
		1435		9·97		
30	49,500		343·6		11·45	373·6
		1480		10·28		
25	42,100		292·2		11·69	317·2
		1518		10·54		
20	34,500		239·5		11·98	259·5
		1563		10·85		
15	26,670		185·2		12·35	200·2
		1612		11·20		
12	21,840		151·7		12·64	163·7
		1645		11·42		
10	18,550		128·8		12·88	138·8
		1688		11·72		
8	15,170		105·4		13·17	113·4
		1723		11·97		
7	13,450		93·43		13·35	100·43
		1740		12·13		
6	11,710		81·30		13·55	87·30
		1780		12·34		
5	9,930		68·96		13·79	73·96
		1822		12·63		
	8,110		56·33		14·08	60·33

EXPLANATION OF THE TABLES OF THE PROPERTIES OF SATURATED STEAM.

In order to apply theoretical principles to questions relating to the steam engine, tables are indispensable, and it will be convenient here to make some observations respecting the tables given in this book, and to give examples of the method of using them.

Tables Ia, Ib.—Tables connecting the pressure and temperature of steam may be arranged either by equal intervals of temperature, or by equal intervals of pressure. The second method is not more practically useful than the first—for, in the results of experiment, pressures of an even number of pounds per square inch do not often occur—and it has certain disadvantages which render the first method preferable. To avoid interpolation as much as possible, and to render the use of logarithms unnecessary when it cannot be avoided, the table extends to every degree. For many purposes a determination to the nearest degree is quite sufficient, but when greater accuracy is desired, the tabulated differences are used, as follows, to obtain more exact results:

Example 1. Find the pressure corresponding to a temperature of 318°·4.

Here
$$\Delta p = 1\cdot 25 \text{ for } 1^\circ$$
$$= \cdot 5 \text{ for } \cdot 4^\circ;$$
$$\therefore p = 87\cdot 45 + \cdot 5 = 87\cdot 95.$$

Example 2. Find the temperature corresponding to a pressure of 124 lbs. per square inch.

Here the temperature lies between 343° and 344°, and at 344° the pressure is 124·89. Difference = ·89. $\Delta p = 1\cdot 64$ for 1°.

$$\therefore \text{Difference of temperature} = \frac{\cdot 89}{1\cdot 64} = \cdot 54,$$
$$\therefore \text{Temperature} = 344^\circ - \cdot 54^\circ = 343^\circ\cdot 46.$$

The accuracy with which the pressure is determined for a given temperature depends mainly on the accuracy with which temperature can be measured. Regnault states that some of his thermometers were capable of measuring $\frac{1}{200}$th of a degree Centigrade; if with Dixon we regard $\frac{1}{50}$th of a degree Fahrenheit as the limit of accuracy, the maximum probable error will be found by dividing the corresponding tabular difference by 50: this amounts to ·07 lb. per square inch at the highest pressure given in the table, but to less than ·01 below 28 lbs.; in no case does the probable error reach $\frac{1}{10}$th per cent.

The values of the differences given are not nearly so accurate, proportionally the last place of decimals not being always reliable: one of the formulæ given in the Appendix is therefore to be used when great accuracy is necessary. For the purpose of interpolation they are always sufficiently accurate.

As stated in the text, the standard atmosphere adopted in Dixon's table, from which the present table is reduced, is 30 inches of mercury at 32° at the sea-level at the Equator. This corresponds to 14·73 lbs. per square inch at the Equator, 14·66 at the Pole, and 14·7 in latitude 44°. It is the last value which has been adopted in the present work, and the table is therefore exact in latitude 44°, and amply sufficiently approximate at all other points on the earth's surface.

Tables IIa, IIb.—To avoid the necessity of interpolation altogether in determining the total and latent heat of evaporation, it would be necessary that the tables should extend to every degree at least. On the other hand, the differences of the quantities in question vary so slowly that simple interpolation is amply sufficient for wide intervals of temperature. The values are therefore given by the table for every 27°, and for intermediate values interpolation is used. A single example will suffice.

Example 3. Find in foot pounds and thermal units the latent heat for the pressure 70 lbs. per square inch, also the heat necessary to raise the feed water from 100°.

By temperature table the temperature to the nearest degree is 303°. (The nearest degree is always sufficient.)

$$\begin{array}{rrl}
293^\circ & h = & 263\cdot 4 \\
\text{Diff. for } 10^\circ & = & 10\cdot 27 \\
\hline
\therefore\ \text{for } 303^\circ & h = & 273\cdot 7 \\
\text{but for } 100^\circ & h = & 68\cdot 1 \\
\hline
\therefore & h_1 - h_2 = & 205\cdot 6
\end{array}
\qquad
\begin{array}{r}
\Delta h = 1\cdot 027 \\
10 \\
\hline
10\cdot 27
\end{array}$$

Again, in foot-pounds—

$$\begin{array}{rr}
293^\circ\ h = & 203{,}300 \\
\text{Diff. for } 10^\circ = & 7{,}930 \\
\hline
 & 211{,}230 \\
 & 52{,}550 \\
\hline
\therefore\ h_1 - h_2 = & 158{,}680
\end{array}
\qquad
\begin{array}{rr}
104^\circ\ h = & 55{,}650 \\
\text{Diff. for } 4^\circ = & 4 \times 775 \\
= & 3{,}100 \\
\hline
 & 52{,}550
\end{array}$$

A similar process obtains the latent heat of evaporation in foot-lbs or thermal units.

Table III.—The volume of steam varies so rapidly with the pressure that it could only be determined directly by a very extensive table: it is therefore preferable to tabulate the weight of a cubic foot of steam, instead of the volume of 1 lb. The differences in this case vary so slowly that wide intervals are sufficient, especially at high pressures. In the table the interval varies so as not to exceed one-fifth of the pressure itself, and the differences given are the mean differences per lb. within the interval.

Example 4. Find the weight (w) of a cubic foot of dry steam at the pressure 33 lbs. per square inch. Here

$$p = 30 \qquad w = \cdot 07413 \qquad \Delta w = \cdot 002297$$
$$\therefore \text{ difference for 3 lbs.} = 3 \times \cdot 002297 = \cdot 006891$$
$$w = \cdot 07413 + \cdot 00689$$
$$= \cdot 08102$$

The reciprocal of w is the volume in cubic feet of 1 lb. of steam, and is readily found by a table of reciprocals or division to be 12·34 in this example.

At pressures less than 5 lbs., or when great accuracy is required, it is better to use Table IV., as explained below.

Tables IVa, IVb, IVc.—In these tables the results are given for every 27°, for the same reasons as in Table II., and the process of interpolation by means of which results are obtained for intermediate temperatures is precisely similar, so that it need not be further illustrated. Table IV*a*, however, may be used for other purposes, as follows:

First. To find the specific volume of steam at low pressures, or when special accuracy is desirable.

Example 5. Find the specific volume of steam at 70 lbs. pressure.

Here the corresponding temperature is 303°, and the value of Pu by the table is

$$\mathrm{P}u = 60{,}670 + 530 = 61{,}200$$
$$\therefore u = \frac{61{,}200}{70 \times 144} = 6 \cdot 071$$
$$v = 6 \cdot 071 + \cdot 016 = 6 \cdot 087$$

Secondly. To calculate the value of $(\mathrm{P}_1 - \mathrm{P}_2)\,\mathrm{V}$[illegible] [illegible]rmal units, where the pressures are given in lbs. per squ[illegible] [illegible]nd V_3 is the volume of dry steam corresponding to some [illegible] which may or may not be the same as either of the [illegible]

Here we might proceed by finding the [illegible] square foot and the volume V_3 separately, [illegible]

and division by 772 the result would be obtained. It is better, however, to proceed thus:

$$(P_1 - P_2)V_3 = \frac{P_1 - P_2}{P_3} \cdot P_3V_3 = \frac{p_1 - p_2}{p_3} \cdot P_3V_3,$$

which can now be calculated by use of Table IV*a* with facility. This method is constantly used in the text.

Table V.—The meaning of the term "internal-work-pressure," and the use of this quantity in the graphical treatment of questions in the theory of the steam engine, have been sufficiently explained in the text. In order that a small table may suffice, the tabulated intervals are unequal, being arranged so as not to exceed one-fifth of the pressure under consideration.

Example 6. Find the internal-work-pressure during the formation of steam at 65 lbs. pressure.

By Table V. we find

$$\begin{array}{lll} p = 60 : \bar{p} = 633 \cdot 4 & : & \Delta p = 9 \cdot 21 \\ \text{Diff. for 5 lbs.} = \underline{46 \cdot 05} & & \underline{\quad 5} \\ \therefore \text{for } p = 65\ \bar{p} = 679 \cdot 45 & & 46 \cdot 05 \end{array}$$

Nearly the same results may be obtained direct from the temperature table, by use of the tabulated values of the differences. For by Art. 47,

$$\bar{p} = T \Delta p - p.$$

Example 7. Find the equivalent pressure at 12 lbs. on the square inch.

By Table $t = 202^\circ \therefore T = 663^\circ$,

$$\Delta p = \frac{\cdot 249 + \cdot 245}{2} = \frac{\cdot 494}{2} = \cdot 247,$$

$$\begin{aligned} \therefore \text{Pressure equivalent} &= 663 \times \cdot 247 - 12 \\ &= 163 \cdot 76 - 12 = 151 \cdot 76. \end{aligned}$$

By table the result is $151 \cdot 7$, showing practical identity.

The ratio which this pressure bears to the external pressure is

$$\frac{\delta\{\log. p\}}{\delta\{\log. T\}} - 1.$$

Example 8. Find the ratio for pressure 5 lbs. on the square inch.

$$\begin{array}{lll} t = 163^\circ & \log. p = \cdot 70586 & \log. T = \cdot 79518 \\ t = 161^\circ & \log. p = \underline{\cdot 68529} & \log. T = \underline{\cdot 79379} \\ & \qquad\ \ \cdot 02057 & \qquad\ \ \cdot 00139 \end{array}$$

$$\therefore \text{Ratio} = \frac{20570}{139} - 1 = 13 \cdot 8.$$

TABLE OF HYPERBOLIC LOGARITHMS.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1·01	·00995	1·36	·3075	1·71	·5365
1·02	·01980	1·37	·3148	1·72	·5423
1·03	·02956	1·38	·3221	1·73	·5481
1·04	·03922	1·39	·3293	1·74	·5539
1·05	·04879	1·40	·3365	1·75	·5596
1·06	·05827	1·41	·3436	1·76	·5653
1·07	·06766	1·42	·3507	1·77	·5710
1·08	·07696	1·43	·3577	1·78	·5766
1·09	·08618	1·44	·3646	1·79	·5822
1·10	·09531	1·45	·3716	1·80	·5878
1·11	·1044	1·46	·3784	1·81	·5933
1·12	·1133	1·47	·3853	1·82	·5988
1·13	·1222	1·48	·3920	1·83	·6043
1·14	·1310	1·49	·3988	1·84	·6098
1·15	·1398	1·50	·4055	1·85	·6152
1·16	·1484	1·51	·4121	1·86	·6206
1·17	·1570	1·52	·4187	1·87	·6259
1·18	·1655	1·53	·4253	1·88	·6313
1·19	·1739	1·54	·4318	1·89	·6366
1·20	·1823	1·55	·4382	1·90	·6418
1·21	·1906	1·56	·4447	1·91	·6471
1·22	·1988	1·57	·4511	1·92	·6523
1·23	·2070	1·58	·4574	1·93	·6575
1·24	·2151	1·59	·4637	1·94	·6627
1·25	·2231	1·60	·4700	1·95	·6678
1·26	·2311	1·61	·4762	1·96	·6729
1·27	·2390	1·62	·4824	1·97	·6780
1·28	·2469	1·63	·4886	1·98	·6831
1·29	·2546	1·64	·4947	1·99	·6881
1·30	·2624	1·65	·5008	2·00	·6931
1·31	·2700	1·66	·5068	2·01	[illegible]
1·32	·2776	1·67	·5128	2·02	[illegible]
1·33	·2852	1·68	·5188	2·03	[illegible]
1·34	·2927	1·69	·5247	[illegible]	[illegible]
1·35	·3001	1·70	·5306	[illegible]	[illegible]

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
2·06	·7227	2·46	·9002	2·86	1·0508
2·07	·7275	2·47	·9042	2·87	1·0543
2·08	·7324	2·48	·9083	2·88	1·0578
2·09	·7372	2·49	·9123	2·89	1·0613
2·10	·7419	2·50	·9163	2·90	1·0647
2·11	·7467	2·51	·9203	2·91	1·0682
2·12	·7514	2·52	·9243	2·92	1·0716
2·13	·7561	2·53	·9282	2·93	1·0750
2·14	·7608	2·54	·9322	2·94	1·0784
2·15	·7655	2·55	·9361	2·95	1·0818
2·16	·7701	2·56	·9400	2·96	1·0852
2·17	·7747	2·57	·9439	2·97	1·0886
2·18	·7793	2·58	·9478	2·98	1·0919
2·19	·7839	2·59	·9517	2·99	1·0953
2·20	·7884	2·60	·9555	3·00	1·0986
2·21	·7930	2·61	·9594	3·01	1·1019
2·22	·7975	2·62	·9632	3·02	1·1053
2·23	·8020	2·63	·9670	3·03	1·1086
2·24	·8065	2·64	·9708	3·04	1·1119
2·25	·8109	2·65	·9746	3·05	1·1151
2·26	·8154	2·66	·9783	3·06	1·1184
2·27	·8198	2·67	·9821	3·07	1·1217
2·28	·8242	2·68	·9858	3·08	1·1249
2·29	·8286	2·69	·9895	3·09	1·1282
2·30	·8329	2·70	·9933	3·10	1·1314
2·31	·8372	2·71	·9969	3·11	1·1346
2·32	·8416	2·72	1·0006	3·12	1·1378
2·33	·8459	2·73	1·0043	3·13	1·1410
2·34	·8502	2·74	1·0080	3·14	1·1442
2·35	·8544	2·75	1·0116	3·15	1·1474
2·36	·8587	2·76	1·0152	3·16	1·1506
2·37	·8629	2·77	1·0188	3·17	1·1537
2·38	·8671	2·78	1·0225	3·18	1·1569
2·39	·8713	2·79	1·0260	3·19	1·1600
2·40	·8755	2·80	1·0296	3·20	1·1632
2·41	·8796	2·81	1·0332	3·21	1·1663
2·42	·8838	2·82	1·0367	3·22	1·1694
2·43	·8879	2·83	1·0403	3·23	1·1725
2·44	·8920	2·84	1·0438	3·24	1·1756
2·45	·8961	2·85	1·0473	3·25	1·1787

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3·26	1·1817	3·66	1·2975	4·06	1·4012
3·27	1·1848	3·67	1·3002	4·07	1·4036
3·28	1·1878	3·68	1·3029	4·08	1·4061
3·29	1·1909	3·69	1·3056	4·09	1·4085
3·30	1·1939	3·70	1·3083	4·10	1·4110
3·31	1·1969	3·71	1·3110	4·11	1·4134
3·32	1·2000	3·72	1·3137	4·12	1·4159
3·33	1·2030	3·73	1·3164	4·13	1·4183
3·34	1·2060	3·74	1·3191	4·14	1·4207
3·35	1·2090	3·75	1·3218	4·15	1·4231
3·36	1·2119	3·76	1·3244	4·16	1·4255
3·37	1·2149	3·77	1·3271	4·17	1·4279
3·38	1·2179	3·78	1·3297	4·18	1·4303
3·39	1·2208	3·79	1·3324	4·19	1·4327
3·40	1·2238	3·80	1·3350	4·20	1·4351
3·41	1·2267	3·81	1·3376	4·21	1·4375
3·42	1·2296	3·82	1·3403	4·22	1·4398
3·43	1·2326	3·83	1·3429	4·23	1·4422
3·44	1·2355	3·84	1·3455	4·24	1·4446
3·45	1·2384	3·85	1·3481	4·25	1·4469
3·46	1·2413	3·86	1·3507	4·26	1·4493
3·47	1·2442	3·87	1·3533	4·27	1·4516
3·48	1·2470	3·88	1·3558	4·28	1·4540
3·49	1·2499	3·89	1·3584	4·29	1·4563
3·50	1·2528	3·90	1·3610	4·30	1·4586
3·51	1·2556	3·91	1·3635	4·31	1·4609
3·52	1·2585	3·92	1·3661	4·32	1·4633
3·53	1·2613	3·93	1·3686	4·33	1·4656
3·54	1·2641	3·94	1·3712	4·34	1·4679
3·55	1·2669	3·95	1·3737	4·35	1·4702
3·56	1·2698	3·96	1·3762	4·36	1·4725
3·57	1·2726	3·97	1·3788	4·37	1·4748
3·58	1·2754	3·98	1·3813	4·38	1·4770
3·59	1·2782	3·99	1·3838	4·39	1·4793
3·60	1·2809	4·00	1·3863	4·40	1·4816
3·61	1·2837	4·01	1·3888	4·41	1·4839
3·62	1·2865	4·02	[illegible]	4·42	1·4861
3·63	1·2892	4·03	[illegible]	[illegible]	1·4884
3·64	1·2920	4·04	[illegible]	[illegible]	·4907
3·65	1·2947	4·05	[illegible]	[illegible]	·4929

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
4·46	1·4951	4·86	1·5810	5·26	1·6601
4·47	1·4974	4·87	1·5831	5·27	1·6620
4·48	1·4996	4·88	1·5851	5·28	1·6639
4·49	1·5019	4·89	1·5872	5·29	1·6658
4·50	1·5041	4·90	1·5892	5·30	1·6677
4·51	1·5063	4·91	1·5913	5·31	1·6696
4·52	1·5085	4·92	1·5933	5·32	1·6715
4·53	1·5107	4·93	1·5953	5·33	1·6734
4·54	1·5129	4·94	1·5974	5·34	1·6752
4·55	1·5151	4·95	1·5994	5·35	1·6771
4·56	1·5173	4·96	1·6014	5·36	1·6790
4·57	1·5195	4·97	1·6034	5·37	1·6808
4·58	1·5217	4·98	1·6054	5·38	1·6827
4·59	1·5239	4·99	1·6074	5·39	1·6845
4·60	1·5261	5·00	1·6094	5·40	1·6864
4·61	1·5282	5·01	1·6114	5·41	1·6882
4·62	1·5304	5·02	1·6134	5·42	1·6901
4·63	1·5326	5·03	1·6154	5·43	1·6919
4·64	1·5347	5·04	1·6174	5·44	1·6938
4·65	1·5369	5·05	1·6194	5·45	1·6956
4·66	1·5390	5·06	1·6214	5·46	1·6974
4·67	1·5412	5·07	1·6233	5·47	1·6993
4·68	1·5433	5·08	1·6253	5·48	1·7011
4·69	1·5454	5·09	1·6273	5·49	1·7029
4·70	1·5476	5·10	1·6292	5·50	1·7047
4·71	1·5497	5·11	1·6312	5·51	1·7066
4·72	1·5518	5·12	1·6332	5·52	1·7084
4·73	1·5539	5·13	1·6351	5·53	1·7102
4·74	1·5560	5·14	1·6371	5·54	1·7120
4·75	1·5581	5·15	1·6390	5·55	1·7138
4·76	1·5602	5·16	1·6409	5·56	1·7156
4·77	1·5623	5·17	1·6429	5·57	1·7174
4·78	1·5644	5·18	1·6448	5·58	1·7192
4·79	1·5665	5·19	1·6467	5·59	1·7210
4·80	1·5686	5·20	1·6487	5·60	1·7228
4·81	1·5707	5·21	1·6506	5·61	1·7246
4·82	1·5728	5·22	1·6525	5·62	1·7263
4·83	1·5748	5·23	1·6544	5·63	1·7281
4·84	1·5769	5·24	1·6563	5·64	1·7299
4·85	1·5790	5·25	1·6582	5·65	1·7317

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
5·66	1·7334	6·06	1·8017	6·46	1·8656
5·67	1·7352	6·07	1·8034	6·47	1·8672
5·68	1·7370	6·08	1·8050	6·48	1·8687
5·69	1·7387	6·09	1·8066	6·49	1·8703
5·70	1·7405	6·10	1·8083	6·50	1·8718
5·71	1·7422	6·11	1·8099	6·51	1·8733
5·72	1·7440	6·12	1·8116	6·52	1·8749
5·73	1·7457	6·13	1·8132	6·53	1·8764
5·74	1·7475	6·14	1·8148	6·54	1·8779
5·75	1·7492	6·15	1·8165	6·55	1·8795
5·76	1·7509	6·16	1·8181	6·56	1·8810
5·77	1·7527	6·17	1·8197	6·57	1·8825
5·78	1·7544	6·18	1·8213	6·58	1·8840
5·79	1·7561	6·19	1·8229	6·59	1·8856
5·80	1·7579	6·20	1·8245	6·60	1·8871
5·81	1·7596	6·21	1·8262	6·61	1·8886
5·82	1·7613	6·22	1·8278	6·62	1·8901
5·83	1·7630	6·23	1·8294	6·63	1·8916
5·84	1·7647	6·24	1·8310	6·64	1·8931
5·85	1·7664	6·25	1·8326	6·65	1·8946
5·86	1·7681	6·26	1·8342	6·66	1·8961
5·87	1·7699	6·27	1·8358	6·67	1·8976
5·88	1·7716	6·28	1·8374	6·68	1·8991
5·89	1·7733	6·29	1·8390	6·69	1·9006
5·90	1·7750	6·30	1·8405	6·70	1·9021
5·91	1·7766	6·31	1·8421	6·71	1·9036
5·92	1·7783	6·32	1·8437	6·72	1·9051
5·93	1·7800	6·33	1·8453	6·73	1·9066
5·94	1·7817	6·34	1·8469	6·74	1·9081
5·95	1·7834	6·35	1·8485	6·75	1·9095
5·96	1·7851	6·36	1·8500	6·76	1·9110
5·97	1·7867	6·37	1·8516	6·77	1·9125
5·98	1·7884	6·38	1·8532	6·78	1·9140
5·99	1·7901	6·39	1·8547	6·79	1·9155
6·00	1·7918	6·40	1·8563	6·80	1·9169
6·01	1·7934	6·41	1·8579	[illegible]	[illegible]
6·02	1·7951	6·42	1·8594	[illegible]	[illegible]
6·03	1·7967	6·43	1·8610	[illegible]	[illegible]
6·04	1·7984	6·44	1·8625	[illegible]	[illegible]
6·05	1·8001	6·45	[illegible]	[illegible]	[illegible]

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
6·86	1·9257	7·26	1·9824	7·66	2·0360
6·87	1·9272	7·27	1·9838	7·67	2·0373
6·88	1·9286	7·28	1·9851	7·68	2·0386
6·89	1·9301	7·29	1·9865	7·69	2·0399
6·90	1·9315	7·30	1·9879	7·70	2·0412
6·91	1·9330	7·31	1·9892	7·71	2·0425
6·92	1·9344	7·32	1·9906	7·72	2·0438
6·93	1·9359	7·33	1·9920	7·73	2·0451
6·94	1·9373	7·34	1·9933	7·74	2·0464
6·95	1·9387	7·35	1·9947	7·75	2·0477
6·96	1·9402	7·36	1·9961	7·76	2·0490
6·97	1·9416	7·37	1·9974	7·77	2·0503
6·98	1·9430	7·38	1·9988	7·78	2·0516
6·99	1·9445	7·39	2·0001	7·79	2·0528
7·00	1·9459	7·40	2·0015	7·80	2·0541
7·01	1·9473	7·41	2·0028	7·81	2·0554
7·02	1·9488	7·42	2·0042	7·82	2·0567
7·03	1·9502	7·43	2·0055	7·83	2·0580
7·04	1·9516	7·44	2·0069	7·84	2·0592
7·05	1·9530	7·45	2·0082	7·85	2·0605
7·06	1·9544	7·46	2·0096	7·86	2·0618
7·07	1·9559	7·47	2·0109	7·87	2·0631
7·08	1·9573	7·48	2·0122	7·88	2·0643
7·09	1·9587	7·49	2·0136	7·89	2·0656
7·10	1·9601	7·50	2·0149	7·90	2·0668
7·11	1·9615	7·51	2·0162	7·91	2·0681
7·12	1·9629	7·52	2·0176	7·92	2·0694
7·13	1·9643	7·53	2·0189	7·93	2·0707
7·14	1·9657	7·54	2·0202	7·94	2·0719
7·15	1·9671	7·55	2·0215	7·95	2·0732
7·16	1·9685	7·56	2·0229	7·96	2·0744
7·17	1·9699	7·57	2·0242	7·97	2·0757
7·18	1·9713	7·58	2·0255	7·98	2·0769
7·19	1·9727	7·59	2·0268	7·99	2·0782
7·20	1·9741	7·60	2·0281	8·00	2·0794
7·21	1·9755	7·61	2·0295	8·01	2·0807
7·22	1·9769	7·62	2·0308	8·02	2·0819
7·23	1·9782	7·63	2·0321	8·03	2·0832
7·24	1·9796	7·64	2·0334	8·04	2·0844
7·25	1·9810	7·65	2·0347	8·05	2·0857

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
8·06	2·0869	8·46	2·1353	8·86	2·1815
8·07	2·0881	8·47	2·1365	8·87	2·1827
8·08	2·0894	8·48	2·1377	8·88	2·1838
8·09	2·0906	8·49	2·1389	8·89	2·1849
8·10	2·0919	8·50	2·1401	8·90	2·1861
8·11	2·0931	8·51	2·1412	8·91	2·1872
8·12	2·0943	8·52	2·1424	8·92	2·1883
8·13	2·0956	8·53	2·1436	8·93	2·1894
8·14	2·0968	8·54	2·1448	8·94	2·1905
8·15	2·0980	8·55	2·1459	8·95	2·1917
8·16	2·0992	8·56	2·1471	8·96	2·1928
8·17	2·1005	8·57	2·1483	8·97	2·1939
8·18	2·1017	8·58	2·1494	8·98	2·1950
8·19	2·1029	8·59	2·1506	8·99	2·1961
8·20	2·1041	8·60	2·1518	9·00	2·1972
8·21	2·1054	8·61	2·1529	9·01	2·1983
8·22	2·1066	8·62	2·1541	9·02	2·1994
8·23	2·1078	8·63	2·1552	9·03	2·2006
8·24	2·1090	8·64	2·1564	9·04	2·2017
8·25	2·1102	8·65	2·1576	9·05	2·2028
8·26	2·1114	8·66	2·1587	9·06	2·2039
8·27	2·1126	8·67	2·1599	9·07	2·2050
8·28	2·1138	8·68	2·1610	9·08	2·2061
8·29	2·1150	8·69	2·1622	9·09	2·2072
8·30	2·1163	8·70	2·1633	9·10	2·2083
8·31	2·1175	8·71	2·1645	9·11	2·2094
8·32	2·1187	8·72	2·1656	9·12	2·2105
8·33	2·1199	8·73	2·1668	9·13	2·2116
8·34	2·1211	8·74	2·1679	9·14	2·2127
8·35	2·1223	8·75	2·1691	9·15	2·2138
8·36	2·1235	8·76	2·1702	9·16	2·2148
8·37	2·1247	8·77	2·1713	9·17	2·2159
8·38	2·1258	8·78	2·1725	9·18	2·2170
8·39	2·1270	8·79	2·1736	9·19	[illegible]
8·40	2·1282	8·80	2·1748	9·20	[illegible]
8·41	2·1294	8·81	2·1759	9·21	[illegible]
8·42	2·1306	8·82	2·1770	[illegible]	[illegible]
8·43	2·1318	8·83	2·1782	[illegible]	[illegible]
8·44	2·1330	8·84	2·1793	[illegible]	[illegible]
8·45	2·1342	8·85	2·1804	[illegible]	[illegible]

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
9·26	2·2257	9·51	2·2523	9·76	2·2783
9·27	2·2268	9·52	2·2534	9·77	2·2793
9·28	2·2279	9·53	2·2544	9·78	2·2803
9·29	2·2289	9·54	2·2555	9·79	2·2814
9·30	2·2300	9·55	2·2565	9·80	2·2824
9·31	2·2311	9·56	2·2576	9·81	2·2834
9·32	2·2322	9·57	2·2586	9·82	2·2844
9·33	2·2332	9·58	2·2597	9·83	2·2854
9·34	2·2343	9·59	2·2607	9·84	2·2865
9·35	2·2354	9·60	2·2618	9·85	2·2875
9·36	2·2364	9·61	2·2628	9·86	2·2885
9·37	2·2375	9·62	2·2638	9·87	2·2895
9·38	2·2386	9·63	2·2649	9·88	2·2905
9·39	2·2396	9·64	2·2659	9·89	2·2915
9·40	2·2407	9·65	2·2670	9·90	2·2925
9·41	2·2418	9·66	2·2680	9·91	2·2935
9·42	2·2428	9·67	2·2690	9·92	2·2946
9·43	2·2439	9·68	2·2701	9·93	2·2956
9·44	2·2450	9·69	2·2711	9·94	2·2966
9·45	2·2460	9·70	2·2721	9·95	2·2976
9·46	2·2471	9·71	2·2732	9·96	2·2986
9·47	2·2481	9·72	2·2742	9·97	2·2996
9·48	2·2492	9·73	2·2752	9·98	2·3006
9·49	2·2502	9·74	2·2762	9·99	2·3016
9·50	2·2513	9·75	2·2773	10·00	2·3026

Explanation of Table.

The table gives the hyperbolic logarithm of all numbers between 1 and 10, increasing by ·01, from which can be derived all multiples by 10 by adding 2·3026 once, and all multiples by 100 by adding the same number twice.

Example 1. Find $\log._\epsilon 53\cdot 1$:

$$\begin{aligned} \log._\epsilon 5\cdot 31 &= 1\cdot 6696 \text{ (by table)} \\ &\ \underline{2\cdot 3026} \\ \log._\epsilon 53\cdot 1 &= 3\cdot 9722 \end{aligned}$$

Example 2. Find $\log._\epsilon 531$:

$$\begin{aligned} \log._\epsilon 5\cdot 31 &= 1\cdot 6696 \text{ (by table)} \\ &\ 2\cdot 3026 \\ &\ \underline{2\cdot 3026} \\ \log._\epsilon 531 &= 6\cdot 2748 \end{aligned}$$

APPENDIX.

Note A.—Formulæ connecting the Pressure and Temperature of Saturated Steam. Temperature of Steam from Brine. (Art. 2.)

A knowledge of some of the numerous formulæ which have been devised for the purpose of representing the elastic force of steam is desirable, not so much for the purpose of obtaining numerical results, which can usually be done more easily and accurately by a table, as for use in various theoretical investigations in which a knowledge of the algebraical relation between pressure and temperature is required. If saturated steam were a perfect gas, it would be possible to obtain such a formula from the known approximate formula for the latent heat of evaporation by means of the equation

$$L = u\,T\,\frac{dP}{dT},$$

proved in the concluding articles of Chapter V. A formula so obtained, however, is not even approximately correct except at low pressures, and it is necessary to resort to empirical formulæ found by trial to correspond with the table to a greater or less degree of approximation. For a full account of all the formulæ proposed up to the date of publication (1849) I must refer to Dixon's 'Treatise on Heat,' already cited in the text. I shall here only mention two of the principal forms, and add one other, introduced at a later period.

Tredgold's Formula.—Among the earliest and simplest forms we find

$$p = \left(\frac{a+t}{[illegible]}\right)^{n} [illegible]$$

where a, b, n are constants deter[illegible] comparison with the table. Thus, for example, T[illegible] one of the first to use this formula, expr[illegible] steam by writing $a = 100$, $b = 177$, [illegible] and inches of mercury.

All formulæ of this class are very defective in form, and consequently apply only to a limited portion of the table for which their constants have been specially determined. Thus, the French commissioners appointed to investigate the elastic force of steam some years previously to Regnault's researches, employed Tredgold's formula up to the pressure of four atmospheres, while for higher pressures they found it advisable to replace the index 6 by 5. Notwithstanding its deficiencies, the simplicity of this formula renders it for some purposes very convenient.

Roche's Formula.—A far more exact representation of the law connecting the elastic force of steam with its temperature is obtained by expressing not p itself, but the logarithm of p in terms of the temperature. The simplest example of this kind is the formula

$$\log. p = a - \frac{b}{t + c},$$

which has been used with various values of the constants by many writers, among the earliest of whom was M. Roche, about 1830. As an example of Roche's formula, we may take that used by Mr. Tate (Fairbairn's 'Millwork,' Part I., p. 201), viz.

$$\log. p = 5 \,.\, \frac{t - 212}{t + 367},$$

where p is given in atmospheres of 14·7 lbs. per square inch. This formula represents the results given by the table with very considerable accuracy, especially at high pressures; and it may be used, without fear of serious error, in case the pressure of steam should be required at a somewhat higher temperature than that of 432°, to which the table extends.

By differentiation and reduction, a formula is obtained for the rise of pressure corresponding to a rise of 1° in temperature, namely,

$$\Delta p = \frac{6666}{(t + 367)^2} \cdot p,$$

which may sometimes be used with advantage when an accurate value of Δp is required at a particular temperature.

Rankine's Formula.—After Dixon's treatise was published, Rankine proposed, partly on theoretical grounds, the formula

$$\log. p = A - \frac{B}{T} - \frac{C}{T^2},$$

where T is the *absolute* temperature, and A, B, C are constants. This formula represents the experimental results very closely, not only for the vapour of water, but also, with suitable modifications in the constants, in the case of other liquids. It is convenient for theoretical investigations, because it is expressed in terms of the absolute temperature, and many of the derived formula are thereby simplified. The values of the constants employed by Rankine were

$$A = 6{\cdot}1007 : \log. B = 3{\cdot}43642 : \log. C = 5{\cdot}59873,$$

in which it is to be noted that the pressure is supposed in pounds per square inch, and that the temperature must be reckoned from a zero 461°·2 below the zero of Fahrenheit's scale.

Hence by differentiation we find

$$k + 1 = \frac{T}{P} \cdot \frac{dP}{dT} = \frac{B'}{T} + \left(\frac{C'}{T}\right)^2,$$

where B′ C′ are other constants easily derived from B and C, their values being

$$\log. B' = 3{\cdot}79864 : \log. C' = 3{\cdot}13099.$$

The meaning of this number k, and its connection with the internal-work-pressure, are explained in Chapter V., page 122. It was by this formula that Table V. was calculated.

Again, it was shown in Chapter V. that

$$L = u T \frac{dP}{dT},$$

where u is as usual written for $v - s$

$$\therefore \ u = \frac{L}{T} \cdot \frac{dT}{dP}.$$

Proceeding as above, a formula is obtained by which u is found in terms of the latent heat of evaporation L. It was in this way that Rankine calculated the densities of steam registered in his tables, from which Table III. in this book has been calculated by interpolation.

Temperature of Steam from Brine.—The experiments on t temperature of the steam from brine are very discrepa case of ebullition from a surface exposed to the atm temperature of the steam often appears to be about 2 the brine has a much higher temperature. The r

The v
density is

at any othe

where m is a
by a table fo
repeat here.
appear to be
results: *

3
2
21

The value of s the
and to ·01857 at
be taken into accou
the error so introd
probable error of the

of water from 32° to $t°$ was determined by Regnault in the following way:—The boiler used for the experiments on the elastic force of steam was filled three-quarters full, and maintained at a steady pressure by use of the air chamber as before (Art. 2, p. 3). A small pipe, curved downwards into the water, and provided with a stop-cock, connected the boiler with a vessel, filled with water and open to the atmosphere, which served as a calorimeter: the pipe projected inwards into this vessel, and was pierced with small holes, the end of the pipe being closed. On opening the stop-cock, the difference of pressure between the boiler and the atmosphere causes hot water to flow into the calorimeter, the rise of temperature of the water in which furnishes a measure of the heat given out by the cooling water. Rankine has pointed out * that the value of h thus found by Regnault is too large because the energy exerted by the difference between the boiler pressure and the atmospheric pressure will be employed in generating kinetic energy and overcoming frictional resistances, and will appear as heat in the calorimeter. The energy in question will—neglecting the dilatation of water—be given by

$$\text{Energy} = \frac{\cdot 016\,(P - P_0)}{772} \text{ thermal units per pound,}$$

where P is the boiler pressure, P_0 the atmospheric pressure, both in pounds per square foot; which becomes, reducing the pressures to pounds per square inch,

$$\text{Energy} = \frac{p - 14\cdot 7}{336} \text{ thermal units per pound.}$$

Rankine accordingly diminishes the values of h given by Regnault by small quantities calculated in this way, and gives a formula representing the corrected results. It is not, however, clear that the whole amount of this correction ought to be subtracted, for the pipe being long and small, the greater part of the energy exerted will be spent in overcoming frictional resistances, and it does not seem evident that all the heat thus generated will reach the calorimeter. Moreover, Rankine's formula at the lower temperatures appears to deviate more from the experiments than is justified by the correction in question.

In any case the correction is small, as the above form
and is not the only correction to which the values

* 'Transactions of the Royal Society of Edinburgh,'

strictly speaking, be subjected, though no doubt it is the most important; I have therefore employed Regnault's values of h, as has been done by other writers on the subject, notwithstanding that they are slightly too large at high pressures.

Note C.—Geometry of the Curves $P V^n$ = Constant. With Applications.

The curves given by the equation $P V^n$ = const. where P is the ordinate, V the abscissa, and n an index, occur so frequently in the theory of the steam engine, that it is convenient to investigate their geometrical properties without reference to the particular application to be made of them.

Area.—Let A B (Fig. 29, p. 340) be any plane curve, L N, K M two ordinates, P_1, P_2, the corresponding abscissæ O N, O M being V_1 V_2, then the area L K M N, comprised between the curve, the ordinates, and the axis, is given by the formula

$$\text{Area} = \int_{V_1}^{V_2} P\,d\,V;$$

but in the present case

$$P\,V^n = P_1\,V_1^n,$$

whence by substitution

$$\text{Area} = P_1\,V_1^n \int_{V_1}^{V_2} \frac{d\,V}{V^n}.$$

Performing the integration, we find

$$\text{Area} = P_1\,V_1^n \,.\, \frac{V_2^{1-n} - V_1^{1-n}}{1-n},$$

which, remembering that

$$P_1\,V_1^n = P_2\,V_2^n,$$

may be written

$$\text{Area} = \frac{P_1 V_1 - P_2 V_2}{n-1},$$

which is the result so frequently used in the text. When $n = 1$ the formula fails: the curve in that case is a common hyperbola, and its area is given by

$$\text{Area} = P_1\,V_1 \,.\, \log._\epsilon \frac{V_2}{V_1}.$$

If to the area thus calculated we add the rectangle D $\tilde{N}$ we obtain

$$\text{Area D L K M O} = P_1 V_1 + \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

$$= \frac{n}{n-1} \cdot P_1 V_1 - \frac{1}{n-1} \cdot P_2 V_2.$$

When the curve is regarded as an expansion curve, the ratio $V_2 : V_1$ is the ratio of expansion, usually denoted by r : P_1 is the initial, and P_2 the terminal pressure, so that we have

$$P_2 = \frac{P_1 V_1^n}{V_2^n} = P_1 \cdot r^{-n},$$

and if P_m be the mean forward pressure,

$$P_m = \frac{\text{Area D L K M O}}{V_2} = \frac{n}{n-1} \cdot \frac{P_1}{r} - \frac{1}{n-1} \cdot P_2.$$

For example, suppose the expansion curve to be the saturation curve, then $n = \frac{17}{16}$,

$$P_m = 17 \frac{P_1}{r} - 16 P_2.$$

When $n = 1$, the rule fails, and we have

$$P_m = P_1 \cdot \frac{1 + \log._e r}{r}.$$

Geometrical Construction.—The curve may be constructed approximately by the following general method (see Fig. 29). Starting from L, draw any horizontal line, Q C, at a small distance below L, then the question is to find S, the point on the curve which lies on Q C. For this purpose, set downwards

$$\text{O T} = \frac{\text{D O}}{n-1} : \text{O R} = \frac{\text{O Q}}{n-1},$$

and complete the rectangle N T as shown in the figure, also draw the horizontal line R F to meet the ordinate L N H in F, as shown in the figure. Then bisect D Q in Z, join Z F, and prolong it to meet the horizontal through T in E: a vertical through E will be the new ordinate very approximately, and by its intersection with Q C will determine S.

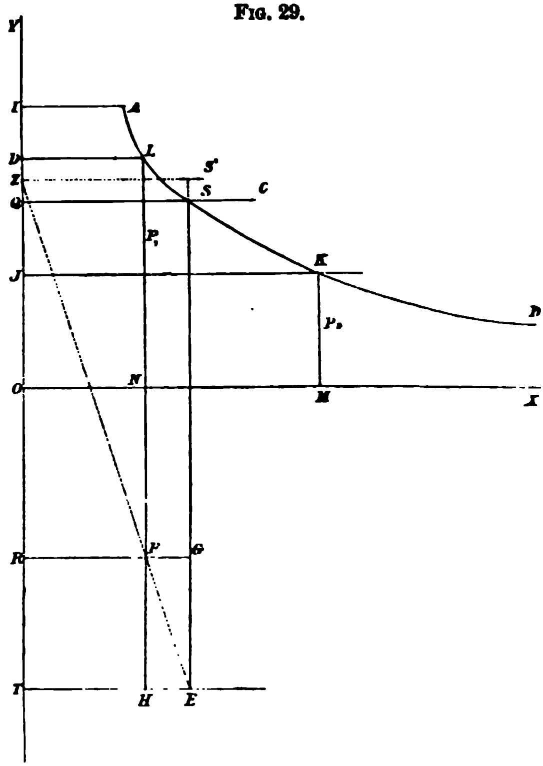
For, completing the rectangle $T S^1$, as shown by the dotted lines in the figure, the rectangles $Z H$, $Z G$ are equal, and $Z N$ is common,

$$\therefore \ O H = O G + N S^1;$$

that is to say,

$$\text{Rectangle } N S^1 = \text{Rectangle } O H - \text{Rectangle } O G.$$

FIG. 29.



Now if the points L S be taken near enough together, the area of the rectangle $N S^1$ may be made to differ as little as we please from the area of the strip of the curve L S N, and the rectangles O H, O G are equal to $P_1 V_1$, $P_2 V_2$, respectively divided by $n - 1$; hence, referring to the formula for the area given above, it is clear that we have determined S, so that it lies on the curve $P V^n =$ constant very approximately. The construction can now be repeated

as many times as we please, so as to obtain more points on the curve. The physical interpretation of this construction, when applied to the adiabatic curve for air, will be understood on referring to Art. 69 in Chapter VII.

The mean pressure is found graphically by inverting the construction. For let us suppose L S to be two given points not necessarily near together, but anywhere on the curve: then inverting the construction the line $Z S^1$ will be determined, the ordinate of which must be the mean pressure during expansion.

Deviation from an Hyperbola and from the Saturation Curve.—All curves of this class resemble an hyperbola the more closely the nearer n is to unity. When n is nearly equal to 1 the deviation is very small, and it is often important to know it, the more so as the construction just given then becomes cumbrous.

Taking the equation

$$P V^n = P_1 V_1^n,$$

to find the vertical distance between two curves, differing slightly in the value of n, we have only to differentiate, assuming V constant and n variable, then

$$\delta P = -P \,.\, \log._\epsilon V \,.\, \delta n + P \,.\, \log._\epsilon V_1 \,.\, \delta n,$$

the curves being supposed to start from the same initial point indicated by the suffix 1. Or if r be the ratio of expansion $V : V_1$,

$$\delta P = -P \,.\, \log._\epsilon r \,.\, \delta n,$$

which becomes, if one of the curves be an hyperbola,

$$\delta P = (1 - n) P \,.\, \log._\epsilon r.$$

For example, the vertical distance of the saturation curve of steam from an hyperbola is approximately

$$-\delta P = \frac{1}{16} \,.\, P \log._\epsilon r,$$

where r is the ratio of expansion, reckoned from an initial point, from whence the two curves start.

The horizontal distance of two curves [illegible] each other may in like manner be found by differentiation, [illegible] and n V variable; whence is obtai[illegible]

$$\delta V [illegible]$$

a formula which may be used to find the quantity of steam condensed in adiabatic expansion. Let us suppose the steam initially dry, and write

$$n = \frac{17}{16} : \delta n = 1\cdot135 - 1\cdot0625$$
$$= \cdot0725$$

then

$$-\frac{\delta V}{V} = \cdot0725 \times \frac{16}{17} . \log_{\cdot\epsilon} r;$$

or if $1 - x$ be the f condensed,

$$1 \quad . \log_{\cdot\epsilon} r;$$

this formula is only proximation, but its simplicity renders it convenient. le, in an adiabatic expansion of 4 : 1 the fraction of ste ed is roughly approximately

$$1 - \quad g_{\cdot\epsilon} 4 = \cdot094.$$

It has been fully explai he text that in an actual steam cylinder the action of the sides such that the central mass of steam expands adiabatically, though the whole mass is far from doing so. The amount of suspended moisture at the end of the stroke may therefore be found by a modification of the foregoing formula, in which the datum will be not the ratio of expansion, which will be unknown, but the ratio of pressures, as shown by the indicator diagram. Let r^1 be the ratio of pressures, then

$$r^1 = r^n : \log_{\cdot\epsilon} r^1 = n . \log_{\cdot\epsilon} r,$$

whence by substitution and reduction

$$1 - x = \cdot0642 . \log_{\cdot\epsilon} r^1.$$

In applying this formula it must be remembered that the fraction $1 - x$ is the fraction of the central mass of steam which is condensed, not the fraction of the whole mass of steam contained in the cylinder.

Note D.—Specific Heat of Steam. (Art. 52.)

The conventional meaning ascribed to this expression is fully explained in the text, and its mean value found approximately. A
la will now be given, by means of which an exact value can
ained at any temperature.

If Q be the heat supplied during expansion,

$$\Delta Q = \Delta I + P \Delta v,$$

where ΔI is the increment of intrinsic energy consequent on expansion through the volume Δv. But we know that for dry saturated steam

$$I = H - P u,$$

$$\therefore \Delta I = \Delta H - P \Delta u - u \Delta P,$$

where as usual u is written for $v - s$, and consequently Δu is the same thing as Δv, hence

$$\Delta Q = \Delta H - u \Delta P;$$

but it was shown in Art. 40 that

$$u \Delta P = \frac{L}{T} \cdot \Delta T;$$

therefore dividing by ΔT, and proceeding to the limit,

$$\frac{dQ}{dT} = \frac{dH}{dT} - \frac{L}{T} = \cdot 305 - \frac{L}{T}.$$

The specific heat is the heat necessary to *raise* the temperature 1°, and is therefore $+\dfrac{dQ}{dT}$

$$\therefore \text{Specific Heat} = \cdot 305 - \frac{L}{T} \text{ thermal units.}$$

For example, at temperature 278°

$$T = 278 + 461 = 739 \;:\; L = 917 \cdot 6,$$

$$\text{Specific Heat} = \cdot 305 - 1 \cdot 242$$
$$= - \cdot 937.$$

This calculation shows that the specific heat of steam is negative, as shown in the text. By multiplication by 772 we obtain t value in foot pounds, that is $-723 \cdot 4$ foot pounds. The differe between this result and -764 given in the table, page 134, a mean value between 266° and 293°, is due to the na culation in the text, which is not suited to secure

comb

For i..

Hence by substitution,

$$\Delta h + \Delta(\rho x) + \mathrm{P} \,.\, \Delta(u x) = 0.$$

Neglecting the variation of the specific heat of water we may replace Δh by $772\,\Delta \mathrm{T}$ where T is the absolute temperature: while for ρ we may write its value $\mathrm{L} - \mathrm{P}\,u$, whence by substitution and reduction

$$772 \,.\, \Delta \mathrm{T} + \Delta(\mathrm{L}x) - u x \,.\, \Delta \mathrm{P} = 0.$$

Now divide by Δ T and proceed to the limit, and we have

$$772 + \frac{d}{d\mathrm{T}}(\mathrm{L}x) - u x \frac{d\mathrm{P}}{d\mathrm{T}} = 0.$$

But the equation given above enables us to substitute for u in terms of L, and we find

$$772 + \frac{d}{d\mathrm{T}}(\mathrm{L}x) - \frac{\mathrm{L}x}{\mathrm{T}} = 0,$$

an equation between $\mathrm{L}x$ and T, which can be integrated if we divide by T, for it becomes

$$\frac{772}{\mathrm{T}} + \frac{1}{\mathrm{T}} \cdot \frac{d}{d\mathrm{T}}(\mathrm{L}x) - \frac{\mathrm{L}x}{\mathrm{T}^2} = 0,$$

which is equivalent to

$$\frac{772}{\mathrm{T}} + \frac{d}{d\mathrm{T}}\left\{\frac{\mathrm{L}x}{\mathrm{T}}\right\} = 0.$$

Now integrate, and we find

$$772 \log. \mathrm{T} + \frac{\mathrm{L}x}{\mathrm{T}} = \text{const.},$$

which is the fundamental equation of Art. 76

Note F.—VALUE OF THE MECHANICAL EQUIVALENT OF HEAT.

Since the text was in type, the Report of the *British Association* for 1876 has been published, containing (p. 275) *the first* report of a committee appointed for the [illegible] *of determining* the mechanical equivalent of heat. From [illegible] report *it appears* that Dr. Joule has [illegible] his experiments on the *friction of*

water, the mean result of sixty of which is 774·1, "subject to a small correction, possibly amounting to $\frac{1}{400}$ on account of the "thermometric scale error."

Note G.—Absolute Scale of Temperature. (Chapter V.)

The conception of an absolute scale of temperature is due to Sir W. Thomson, and the fundamental experiments by which the close coincidence of this scale with that of an air thermometer was experimentally demonstrated, were made by him in conjunction with Dr. Joule, and described in a paper on the "Thermal Effects of Fluids in Motion," Part II., published in the 'Philosophical Transactions' for 1854, p. 353. These are the experiments cited on p. 77 as showing that the absolute zero is 460°·66 below the zero of Fahrenheit's scale.

LONDON: PRINTED BY WILLIAM CLOWES AND SONS, STAMFORD STREET AND CHARING CROSS.